CIRCULARITY IS STILL SCARY

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Abstract

Cook (forthcoming) presents a paradox which he says is not circular. I see no reasons to doubt the non-circularity claim, but I do have some concerns regarding its paradoxicality. My point will be that his proposal succeeds in offering a formalization, but fails in providing a formal paradox, at least of the same type and strength as the Liar.

KEY WORDS**:** Paradox; Circularity; Yablo's formalization.

Resumen

Cook (en prensa) presenta una paradoja que según él no es circular. No veo motivos para cuestionar la pretensión de no circularidad, pero sí me resulta algo problemática la cuestión de su paradojicidad. El punto que intentaré defender será que la propuesta de Cook es exitosa en ofrecer una formalización, pero fracasa en proveer una paradoja formal, al menos del mismo tipo y fuerza que el Mentiroso.

PALABRAS CLAVE**:** Paradoja; Circularidad; Formalización de Yablo.

1. Natural and Formal Paradoxes

As it was said in the introduction to this issue, Cook's proposed paradox is framed in the infinitary language Lp, and it consists of the set $\{S_n\}_{n\in\omega}$ under the denotation function $\delta(S_n)= \wedge\{F(S_m)\text{: } m\in\omega\ m>n\}$. This sentences are indeed contradictory, but there is more than contradiction in a semantic paradox. Some of them emerge in the natural language. As I see it, the characteristics of those cases are as follows:

— The sentences involved seem to be declarative.
— We should be able to evaluate declarative sentences as true or false.
— But, for some reason, we cannot do it.

The development of formalized languages has split the paradox problem in two. On one hand, we are provided with a tool to represent those original paradoxes in the form of arguments. This is useful, first, because it gives insight into the structure of the problem, and second, because it is a nice way to test the system itself. Logic is intended to capture the notion of consequence, so, in order to see if it can deliver, we analyze its verdict regarding the hard cases (or the "unusual" ones). Yet on the other hand, it is not all good news, since now we see that the theories built upon these formal languages have paradoxes of their own. Here, what happens is that:

— We want the language to have certain expressive power.
— The intent to reach that power results in a trivialization of the theory (or some other unwanted result).

Of course, this distinction is vague, and it is not supposed to provide necessary or sufficient conditions, but merely to shed some light on two separated projects: formalizing a natural paradox, and solving a formal one. If we accomplish the first task, what we get is just a formalization. As we can see, the Liar is a queen among paradoxes: it is, at the same time, a natural paradox and a formal one. When we try to incorporate a truth predicate to the language of arithmetic, we can obtain a contradiction, via diagonalization theorem. The same thing happens with Curry, or with the Russell/barber shop combo. On the contrary, it can be argued that Cantor is only a formal paradox, and Sorites is only a natural one.

2. The first project

I think it is plausible to consider Cook's paradox as a formalization of the Yablo paradox. In fact, it is one of the most successful ones we have so far. It is well known that first order theories deem the original argument invalid, since they are compact. We do have ω-inconsistency in first order, but if our present interest is to represent some argument we consider prima facie valid, then if the theory says it is not, we have a problem, no matter what other, bizarre things happen with the semantics. So, concerning the ability to capture logical consequence, the dilemma is: either we choose to say that logic taught us what we believed was wrong, and the argument was not sound after all (a path no one appears to have followed), or we stick with the claim that the argument is valid, and move on to second order logic.

This road —which I will call option 1— consists of adding the numerical instances of $Y(x)\leftrightarrow\forall y>x\ \neg T\ (<Y(dot(y))>)$ to PA2. It has the advantage over Cook's of being closer "grammatically" to the natural presentation, and of generating less infinitary concerns (although it is not without them). As a downside, its non-circularity is, at least, call into question (Picollo, in this issue, claims that it is not). Although no instances of the fixed-point theorem are used in the proof, we could consider the biconditionals as partial definitions of the predicate Y(x). But because all of them include the predicate itself -in a non-trivial way- there is not a circular sentence involved, yet there is still a circular predicate.

Cook's proposal, on the other hand, takes the lead because it is not questionable on its non-circularity, being its primary downside the fact that it is presented in a less accepted framework. Deriving a syntactic contradiction is not, from my point of view, a major improvement. If we follow the characterization we provided earlier, we get that the natural language phenomenon is fundamentally a semantic one, so an insatisfiability result seems to be just fine. Even so, if someone insists on obtaining falsum, it is always possible to do it by means of the ω-rule.

This formalization certainly is kind of deviant regarding the syntax of the original one. However, Russell taught us that grammatical appearances are sometimes misleading in order to capture logical form. Despite being true that Yablo did not propose infinitely long sentences, the main interest of the case was its lack of circularity, achieved via sentences referring to infinite others, and that feature is well preserved.

3. The second project

In order to provide the reasons why I think Cook's proposal does not constitute a formal paradox, it will be useful to compare it again with other options. In the first place, we have the circular version of Yablo, that is, getting a predicate Y(x) in PA via fixed point theorem and adding a uniform principle of truth for it. This alternative meets our criteria: we try to augment the expressive power of arithmetic by adding a partial truth predicate, whose principles we thought harmless, and are rewarded with a contradiction. The problem is, of course, that what we have here seems to be something like a complicated version of the Liar, which we could doubtfully identify with the original intention. In the second place, there is option 1, that we already considered as a formalization. It could be the case that this option constitutes at the same time a formal paradox, like it happens with the Liar. Nonetheless, I think it does not. Recall that, in order to avoid using the fixed point theorem to get the predicate Y(x), it is necessary to add to PA2 the numerical instances of $Y(x) \leftrightarrow \forall y>x\ \neg T\ (<Y(dot(y))>)$, and we don't have an independent motivation to do so. Someone could argue that, like in the natural language case, we must be able to evaluate every arithmetical sentence. But it is important to notice that the Yablo sentences are not formulated in arithmetical language, because we need to add the new, non-recursive, predicate. Of course, the addition was harmless when we were chasing the first goal. Since now we are not, PA2 ∪ {Y} must have some intrinsic value in order to obtain the formal paradox. All this could be seen as a variation on Priest's (1997) objection "how do we know it exists?", in the line of "why do we want it to exist?".

Cook's antinomy suffers, I think, a similar problem, and hence should not be regarded as a formal paradox, or at least not one as strong as the Liar. It is true that it shows that certain patterns of reference lead to contradiction. But if this result is to be considered a formal paradox, we should maintain this desideratum:

(D) An infinitary language like LP should be capable of consistently expressing every denotation function.

But (D) is not something one can demand without providing an argument. Someone could say that we had, prior to Cook's paradox, no reason to banish non self-referential sentences, and hence, no reason to mistrust the kind of denotation function involved. Nonetheless, it is not uncommon in the literature to state the fear of circularity as being truly a fear of ungroundedness. The Yablo paradox never claimed to be a grounded antinomy, but a non-circular one. That was shocking mostly because we thought it was the only mean by which ungroundedness could arise in traditional theories, like PA ∪ {T}, or naïf set theory. But we knew that if we went beyond these theories, the link faded away. The clearest example of this are the border line cases of vague predicates in natural language. There is nothing circular about them, although it seems that, like in the self-referential ones, there is some kind of misadjustment between those sentences and the world. The immediate objection is that the Liar not only has no truth value, but sort of oscillates between them. However, the same thing happens with borderline cases. Our judgment that this particular shade is blue fluctuates as we compare it with adjacent shades in a forced-march Sorites.

So, it is not that surprising to find this function problematic. Of course, it is a very interesting project to learn which of them are safe. But what was so strong about the Liar was that we had very good reasons to have the full T-schema in an arithmetical theory. It looks like it should represent the most basic functioning of the predicate. Do we have those with the δ function? I think we do not.

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