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ARTIGO ORIGINAL

Cognitive skills as predictors of elementary students' understanding of arithmetic concepts

Habilidades cognitivas como preditoras da compreensão conceitual da Aritmética de alunos de anos iniciais

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Abstract: Quantitative reasoning is a fundamental skill for mathematical performance since it requires conceptual comprehension of mathematical operations. Thus, the purpose of this study was to look at the function of cognitive abilities as predecessors of arithmetic conceptual knowledge. This study included 127 third and fourth graders who were tested for two broad domain skills: working memory and phonemic awareness, as well as two narrow domain skills: number transcoding and number estimative. A quantitative reasoning exercise was also used to measure these pupils' conceptual grasp. Multiple regression analysis shows that working memory, number transcoding and number estimation are the cognitive predictors of quantitative reasoning achievement. These findings support the need of developing such abilities in children beginning in kindergarten to prevent problems and aid in the development of conceptual arithmetic learning.

Keywords: Mathematics teaching; Mathematical reasoning; Arithmetic; Concept formation; Elementary school.

Resumo: O raciocínio quantitativo é uma habilidade fundamental para o desempenho matemático, pois envolve a compreensão conceitual das operações matemáticas. Este estudo se propôs a investigar o papel das habilidades cognitivas como precursoras da compreensão conceitual da aritmética. Participaram 127 alunos de 3º e 4º anos do Ensino Fundamental, os quais foram testados em duas habilidades de domínio geral: memória de trabalho e consciência fonêmica, e em duas habilidades de domínio específico: transcodificação numérica e estimativa numérica. Além disso, a compreensão conceitual dos alunos foi avaliada por meio de uma tarefa de raciocínio quantitativo. A partir de uma análise de regressão múltipla, os resultados indicam que memória de trabalho, transcodificação numérica e estimativa numérica influenciam o desempenho no raciocínio quantitativo. Esses resultados corroboram a importância de estimular essas habilidades desde primeiros anos da escola para prevenir dificuldades e auxiliar no desenvolvimento da aprendizagem da aritmética conceitual pelas crianças.

Palavras-chave: Ensino de matemática; Raciocínio matemático; Aritmética; Formação de conceitos; Ensino fundamental.

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Introduction

Mathematics is a vital component of academic learning and as a daily living skill. Both improved academic achievement and successful integration into society depend on the right development of math skills. Some studies seek to identify the cognitive abilities that are crucial for the growth of mathematical comprehension. Additionally, research is currently being done to determine which capabilities best predict mathematical success in order to comprehend the relationships between various skills and avoid learning challenges in this area. These cognitive abilities include those in the general domain, which are connected to a variety of academic competencies (such as reading, writing and mathematics). Examples of these skills are working memory and phonological awareness. Additionally, we may take into account knowledge-related domain-specific abilities, such as number transcoding and number line estimating.

Some studies emphasize the value of working memory for mathematical achievement in relation to general domain skills, indicating that it is a precursor skill to initial mathematical learning (ARAGÓN *et al*., 2019; GEARY, 2011; PASSOLUNGHI; VERCELLONI; SCHADEE, 2007; PASSOLUNGHI; LANFRANCHI, 2012; XENIDOU-DERVOU *et al*., 2018) and demonstrating significant improvements in children's performance since kindergarten and remaining with positive effects throughout elementary school. Working memory is a memory system that allows people to temporarily regulate, store, and process relevant information while performing cognitive tasks (BADDELEY, 2011; RAGHUBAR; BARNES; HECHT, 2010). Evidence indicates that working memory appears to be connected to solving problems in mathematics and arithmetic (GATHERCOLE; ALLOWAY, 2004; PASSOLUNGHI; VERCELLONI; SCHADEE, 2007). Specifically, there is a substantial ability in preschool children's performance while solving numerical problems, quantity comparison, informal computation, and informal mathematical principles (ARAGÓN *et al*., 2019). Working memory, on the other hand, has less of an influence on academic achievement as pupils go through the school grades, becoming more demanding solely on new or more complicated activities (CHU; VANMARLE; GEARY, 2016; GEARY, 2011).

Phonological awareness, or the ability to recognize and manipulate phonemes that compose up words, is linked to fundamental mathematical abilities such as recalling arithmetic facts and solving arithmetic problems (DE SMEDT; BOETS, 2010; HECHT *et al*., 2001; SÁNCHEZ; MATILLA; ORRANTIA, 2017), as well as number transcoding (LOPES-SILVA *et al*., 2014). Evidence also suggests that phonological awareness is a particular predictor of arithmetic computation (HECHT *et al*., 2001; SÁNCHEZ; MATILLA; ORRANTIA, 2017). However, studies differ in the way they consider phonological awareness ability, with some assessing phonological processing, which includes phonological memory, phonological awareness, and processing speed (HECHT *et al*., 2001; SÁNCHEZ; MATILLA; ORRANTIA, 2017), while others considered only phonological awareness, using tasks at different levels to assess it. For example, in the research by Simmons, Singleton and Horne (2008) only the syllabic awareness task was used as a representative of phonological awareness, whereas in Lopes-Silva *et al*. (2014) only the level of phonemic awareness was considered. Likewise, these studies show that phonological processing, phonological awareness, and, more especially, phonemic awareness have a considerable impact on mathematical performance. As a result, because phonemic awareness is the most advanced level of phonological awareness (GILLON, 2017), it is understood that activities at this level serve as indicators of children's phonological awareness. Therefore, in this study, phonemic awareness was used as a proxy for measuring phonological awareness.

Some domain-specific cognitive skills are also recognized as essential for good mathematical achievement, such as number transcoding (HABERMANN *et al*., 2020; MALONE; BURGOYNE; HULME, 2020; MOURA *et al*., 2015) and number line estimation (ARAGÓN *et al*., 2019; GILMORE *et al*., 2018; LINK; NUERK; MOELLER, 2014). There is already data demonstrating a substantial relationship between number transcoding, which is the act of translating verbal and written representations in the Arabic number form (i.e., translating the verbal information 'fifty-seven' into the Arabic notation '57'), and mathematical ability (GEARY; HAMSON; HOARD, 2000). Furthermore, evidence indicates that this skill can be used to predict arithmetic achievement at the start of school (HABERMANN *et al*., 2020; MALONE; BURGOYNE; HULME, 2020; MOELLER *et al*., 2011). It can be deduced from this that understanding of Arabic numerals is vital for the subsequent development of arithmetic abilities, and that acquiring number system symbols and their labels (number names) is a key capacity for the development of numerical knowledge.

Number line estimation, or the capacity to roughly determine the physical location of a number on a number line, is also mentioned in the literature as an important skill for mathematical understanding, notably for comprehending number magnitude and the number system (ASHCRAFT; MOORE, 2012; BOOTH; SIEGLER, 2008; SIEGLER; THOMPSON; OPFER, 2009). This ability is also linked to arithmetic skills, such as computations with the four fundamental math operations (LINK; NUERK; MOELLER, 2014), which are the primary focus of the elementary school curriculum. Evidence also suggests a link between it and working memory and quantitative reasoning (NOGUES; DORNELES, 2020) – the general and specialized domain skills considered in this study. Moreover, while exploring the causative relationship between number estimation and mathematical performance, it was discovered that the ability to precisely find numbers on a number line had explanatory value for subsequent mathematical learning (GEARY, 2011; SASANGUIE; VAN DEN BUSSCHE; REYNVOET, 2012). Aside from that, number estimation was the only significant predictor of conceptual arithmetic performance, which included the understanding of additive composition (GILMORE *et al*., 2018).

The majority of these research concentrated on understanding the abilities that underpin mathematical accomplishment, especially arithmetic achievement, or children's ability to do calculations. However, we know that in order to 'do math', we must have both procedural knowledge (knowing how to manipulate numbers) and conceptual knowledge (understanding the concepts and relations involved). In reality, knowing how to accurately carry out processes and understanding the concepts involved are two separate but complimentary abilities (CANOBI, 2004; GELMAN; GALLISTEL, 1978; GILMORE; GÖBEL; INGLIS, 2018; NUNES *et al*., 2016), both of which are required for mathematical competency. A learner may perform well in an algorithm but not comprehend the principles being employed (CANOBI, 2004; GELMAN; GALLISTEL, 1978). In this sense, learning mathematics becomes more important when linked to understanding the quantities and relations involved (NUNES *et al*., 2016), which may be possible through problem-solving activities that promote reasoning about concepts and implied relations.

The capacity to grasp the relationships between the numbers provided in a mathematical problem – that is, to know how to detect whether the quantities involved in a calculation are rising or decreasing, for example – was examined in this study using the quantitative reasoning method (NUNES *et al*., 2016). It is a particularly significant talent for mathematical accomplishment (NUNES *et al*., 2007), as it requires the mental comprehension of mathematical operations. Therefore, understanding the development of calculus skills in children means understanding the development of reasoning behind arithmetic operations. As previously stated, reasoning about mathematical relations and doing an arithmetic computation are two distinct abilities that may be used to a wide range of mathematical situations. In this sense, recognizing the relationships between quantities is the core portion of most mathematical problems to be solved, with the correct computation to be completed only later (NUNES *et al*., 2012).

Two quantitative relations are sufficient to characterize the distinct sorts of issues to be studied by children in terms of quantitative reasoning: addition and multiplication. Additive reasoning is based on part-whole relationships between quantities and incorporates the addition and subtraction operators. Regarding multiplicative reasoning, is based on one-to-many correspondence or a ratio of quantities (NUNES *et al*., 2016; NUNES; BRYANT, 1997). This classification simplifies the analysis of issue kinds and offers a significant contribution to the organization of the school curriculum (NUNES *et al*., 2016). Quantitative reasoning is characterized as thinking about and deciding on the relationships between quantities, as well as utilizing numbers to represent quantities and the right operation to draw conclusions about quantities (NUNES *et al*., 2016).

There is evidence to suggest that developing quantitative reasoning in early school years adds considerably and predictively to subsequent mathematical performance (CHING; NUNES, 2017; NUNES *et al*., 2007, 2012). Research indicates that assessing children' numeric thinking in the early years of primary school has a significant impact on later mathematics achievement, even when controlling for students' intellectual ability and working memory capacity (NUNES *et al*., 2007, 2012). Furthermore, when individual effects in arithmetic calculations and quantitative reasoning are compared, the latter will create a better return on pupils' mathematical performance than simply knowing arithmetic (NUNES *et al*., 2012).

So far, studies have identified quantitative reasoning as a necessary skill for mathematical achievement (CHING; NUNES, 2017; NUNES *et al*., 2007, 2012), emphasizing the importance of understanding it prior to learning arithmetic procedures (NUNES *et al*., 2016), as it involves concepts and relations that must be acquired in order to develop a good conceptual understanding of arithmetic (GILMORE; GÖBEL; INGLIS, 2018).To be more explicit, in order to do well in math, one must have strong quantitative thinking skills. As a result, understanding the development of this talent is also required in order to devise strategies for promoting its teaching in schools. In this regard, given the relevance of numeric reasoning accomplishment for mathematical competency, one alternative is to explore the cognitive skills linked with it. Knowing some of the abilities that impact its performance might thus be valuable in promoting treatments and activities that attempt to encourage the development of quantitative reasoning.

Therefore, additional research is required to understand the development of students' quantitative reasoning or, more broadly, arithmetic conceptual knowledge. In this regard, given the relevance of quantitative reasoning ability for mathematical competency, one alternative is to explore the cognitive abilities linked with it. For this, we developed this study to evaluate the relationships between cognitive abilities and conceptual mathematics comprehension. Given its strong relationship with mathematical knowledge (CHING; NUNES, 2017; NUNES *et al*., 2007, 2012), we hypothesize that the assessed abilities will be connected with quantitative reasoning accomplishment. Furthermore, such abilities would have a causative relationship in this performance, i.e., they would have a prediction value for the conceptual arithmetic performance.

Method

Participants

This study included 127 children from the third (n=55, 43.3%) and fourth (n=72, 56.7%) grades of primary school, 79 girls (62.2%) and 48 boys (37.8%), with a mean age of 9.3 years (SD = 0.7). All participants are kids from public schools in Porto Alegre (RS), Brazil, and are part of a larger study on the predictors of mathematical achievement in elementary school, which is being directed by one of the writers of this paper. It is worth noting that the participating pupils, their guardians, as well as the instructors and schools that provided time and space for data collecting, signed conditions of agreement and were informed of the full assessment procedure for conducting the research. The sample was drawn from the assessment of non-verbal reasoning – Raven's Colored Progressive Matrices – (ANGELINI *et al*., 1999), with the cutoff point set at the 25th percentile. In addition, the participants were evaluated in two general domain skills: working memory and phonological awareness, as well as two domain-specific: number transcoding and number line estimation.

Instruments

Working memory task. Working memory was assessed using the Working Memory Test Battery for Children (PICKERING; GATHERCOLE, 2001). This material was utilized to test the visuospatial sketchpad using the Block Memory task, and the Digit Memory task in direct and inverse sequence. Tasks are divided into stages, with each sequence steadily increasing the quantity of information to be retained. The number of sequences successfully repeated is used to assess performance on this exercise, which is halted if the child makes three mistakes at the same level.

Phonemic awareness task. A phoneme suppression task was used to measure phonemic awareness, in which the child hears a word and must state what new word is formed when a particular phoneme is deleted, for example: 'casa' [house] without /k/ is 'asa' [wing], which is equivalent to 'cup' without /k/ being 'up' (LOPES-SILVA *et al*., 2014, 2016). The challenge consists of 28 words ranging from 2 to 3 syllables in length, with the phonemes to be suppressed located in various locations within the words. The total number of right answers is used to determine correction, and there are no interruption criteria.

Number Transcoding Task. Participants in this exercise are required to write the numbers given to them in Arabic numerals. Thus, the capacity of number transcoding is assessed by converting an aural (phonological) input to a verbal stimulus (written). The task entails 28 integers with one to four digits each, and the correction is giving points for each accurate response regardless of the interruption condition (MOURA *et al*., 2013).

Number Estimation Task. The number-to-position test (SIEGLER; OPFER, 2003), which involves asking the participant to mark the location of a number on a straight line divided by the numbers 0 (to the left) and 100, was used to assess the ability of number line estimation (to the right). In a notebook with a number line and a number to be guessed on each page, participants were required to estimate 22 numbers that were given in a random sequence. Performance was assessed by computing the percentage of absolute error (PAE) for each kid, which measures the accuracy with which pupils estimate each required number. This calculation was adapted from Siegler and Booth (2004) and is performed by dividing the difference in absolute value between the child's estimate and the number to be estimated using the straight-line scale. In other words, if a child is asked to estimate the number 60 but makes it as 40, the percentage of absolute inaccuracy will be 20%. This value is equivalent to the result of $|40–60|/100$. The number estimation variable was treated as '1 – PAE' for analysis purposes, which means that in the example above, the considered accuracy would be 80% ($1 - 0.20 = 0.80$). As a result, the closer to 1 (100%), the more correct the child's response. This assignment was completed without regard for interruption criteria.

Quantitative reasoning task. The quantitative reasoning task was based on Nunes (2009), which is employed in this study to assess conceptual grasp of arithmetic. The challenge is based on solving simple arithmetic problems with one mathematical operation and integers up to 20. The quantitative relations involved include additive reasoning (which covers scenarios of amount composition, transformation, and comparison) and multiplicative reasoning (which includes instances of direct and inverse relationships between quantities and product of measurements). The task included 18 problems, 9 of which required additive reasoning (3 of quantity composition, 3 of transformation, and 3 of comparison) and 9 of which required multiplicative reasoning (3 of direct relation, 3 of inverse relation and 3 of product of measures). The only written information in the notebooks that each participant received was the illustrations of the problems, with one problem per page. The evaluator gave instructions orally, so understanding the utterance did not rely on the students' reading ability, but rather on their phonological awareness. The task did not present interruption criteria. The proportion of right answers to total questions was taken into account while making the correction.

Procedure

Participants were evaluated in two sessions. The first session assessed domain-specific cognitive abilities (number transcoding and number estimate), followed by an assessment of students' conceptual arithmetic comprehension. This session lasted approximately 50 minutes in the classroom with all students, with the first two activities lasting 20 minutes and the final task lasting 30 minutes. The second session assessed general domain cognitive skills (working memory and phonemic awareness), which were performed independently in a separate room with each student for approximately 20 minutes. All chores were completed during the school shift.

Data Analysis

Descriptive statistical analyses were carried out for data treatment taking into account the students' performance on the assessed tasks. Cronbach's alpha coefficient (α), which takes into account the total number of questions in each task and the students' responses for each item, was also calculated to assess the internal consistency of the tools used. Then, using the stepwise forward model, a multiple linear regression test was conducted to see if there were any possible relationships between the variables and to see if these skills had any effect on conceptual arithmetic performance.

Results

The purpose of this study was to examine the predictive usefulness of domain-general and domain-specific cognitive abilities for comprehending conceptual arithmetic. Initially, the pupils' achievement was compared by school grade, as shown in **table 1**. The independent t-test revealed that fourth graders performed substantially better than third graders on the following tasks: number transcoding (t(94.33)=-4.501; p<0.05), number estimating (t(125)=-2.154; p<0.05) and quantitative reasoning (t(125)=-2.154; p< 0.05). Cronbach's alpha (α), a measure of task reliability, was also examined, and it was found that the higher the value of α, the more in line the tasks are with what they are meant to assess. Thus, the tasks had acceptable values and a very good reliability, with α values ranging from 0.8 to 0.9, allowing for a more consistent interpretation of the results.

Table 1 – Descriptive analysis of assessed skills

		3rd grade (n=55)		**4th grade (n=72)**		**T-test**
	α*	**Mean (SD)**	**Min.-Max.**	**Mean (SD)**	**Min.-Max.**	**p-value**
Working memory	0.91	13.67 (3.19)	13.67-28.67	19.97 (3.05)	12.33-31.33	=0.263
Phonemic awareness	0.89	21.16 (5.96)	5-28	21.35 (5.11)	5-28	=.0849
Number transcoding	0.89	22.56 (4.64)	13-28	25.94 (3.45)	13-28	**<0.05**
Number estimation	0.85	0.90 (0.05)	0.77-0.96	0.91 (0.04)	0.81-0.97	**<0.05**
Quantitative reasoning	0.82	8.87 (4.12)	0-16	11.35 (3.88)	0-18	**<0.05**

*Cronbach's alpha coefficient (α): values between 0.41 and 0.60 mean a moderate strength of agreement; between 0.61 and 0.80 mean substantial; and between 0.81 and 1.0 an almost perfect strength of agreement.

Source: Landis and Koch (1977).

The Pearson correlation test findings revealed that all of the variables are directly and strongly associated; **table 2** has further information. Number transcoding has the highest index of connection with quantitative thinking (r=0.63, p<0.01), whereas phonemic awareness has the lowest (r=0.35, p<0.01).

Table 2 – Correlations between cognitive skills

	WM	**PA**	**NT**	**NE**
Working memory (WM)	.			
Phonemic awareness (PA)	0.46**	.		
Number transcoding (NT)	0.39**	0.41**	.	
Number estimation (NE)	0.22*	0.20*	0.54**	.
Quantitative reasoning	0.43**	0.35**	0.63**	0.48**

*(p<.05); **(p<.01)

Source: Prepared by the authors.

Thus, we may emphasize that all of these skills are vital for conceptual arithmetic performance, since the better the students do in each of these abilities, the better their numeric reasoning achievement. Then, using the stepwise forward approach, a multiple linear regression analysis was performed to evaluate the influence relationship between these abilities and quantitative reasoning. In this procedure, the variables are inserted into the regression model one at a time, in decreasing order of effect on the result variable. The resulting model (**table 3**) confirmed that number transcoding ($\beta=0.43$, $p<0.05$), working memory ($\beta=0.30$, $p<0.05$) and number estimation ($\beta=19.21$, $p<0.05$) explain 46.3% of the variance in quantitative reasoning performance, but phonemic awareness did not show significant influence values, thus it is not included in the final model of the regression analysis.

Table 3 – Multiple linear regression model (stepwise forward)

	Quantitative reasoning achievement				
Predictive variables	**R^2 partial**	**ΔR^2**	**β**	***CI***	**p-value**
Number transcoding	0.399	0.399	0.43	0.27–0.58	**<0.05**
Working memory	0.439	0.040	0.30	0.11–0.49	**<0.05**
Number estimation	0.463	0.024	19.21	4.14–34.27	**<0.05**

R^2 adjusted = 0.463, F(3,120) = 36.35, p<.001

Source: Prepared by the authors.

From these results, we confirmed that only three variables appeared as predictors of conceptual arithmetic ability, with number transcoding having the highest influence value ($\beta=0.43$, $p<0.05$). According to the resulting model, this same variable alone explains 39.9% ($\Delta R^2=0{,}399$) of the variance in quantitative reasoning achievement, while working memory contributes an additional 4% ($\Delta R^2= 0.040$) and number estimation contributes 2.4% ($\Delta R^2=0.024$) in explaining students' conceptual arithmetic performance.

The same multiple linear regression analysis was then performed, again using the stepwise forward model, to confirm any variations in predicting ability between school grades. As can be seen in **table 4**, the number estimation variable was introduced as a significant predictor in the model only for the fourth grade.

Table 4 – Multiple linear regression model (stepwise forward) by school grade

		Quantitative reasoning achievement				
	Predictive variables	**R^2 partial**	**ΔR^2**	**β**	***CI***	**p-value**
3rd grade[1]	Number transcoding	0.435	0.435	0.48	0.28 – 0.69	**<0.05**
	Working memory	0.476	0.041	0.33	0.03 – 0.63	**<0.05**
4th grade[2]	Number transcoding	0.252	0.252	0.38	0.12 – 0.63	**<0.05**
	Working memory	0.293	0.041	0.31	0.57 – 0.93	**<0.05**
	Number estimation	0.331	0.038	24.97	47.76 – 0.79	**<0.05**

[1] 3rd grade: R^2 adjusted = 0.476, F(2,51) = 25.04, p<.001
[2] 4th grade: R^2 adjusted = 0.331, F(3,66) = 12.36, p<.001

Source: Prepared by the authors.

Thereby, it appears that the models converge between the variables with stronger explanatory power of the students' quantitative reasoning skills. In terms of values, the resultant model for the third grade explained 47.6% of the variation in conceptual arithmetic, whereas the model for the fourth grade explained 33.1%. Regarding the predictor variables, number transcoding continued with the largest portion of influence for both the 3rd grade ($\beta=0.48$, $p<0.05$) and the 4th grade ($\beta=0.39$, $p<0.05$), responsible for explaining alone 43.5 % and 25.2% of the variation in quantitative reasoning achievement between third and fourth graders, respectively. Working memory was next tested in both school grades (3rd grade: $\beta=0.33$, $p<0.05$; 4th grade: $\beta=0.31$, $p<0.05$), with a 4.1% explanation value in both models. Just in the fourth grade does number estimation provide a little amount of effect to the model ($\beta=24.97$, $p<0.05$), explaining only an additional 3.8% of the variation in quantitative reasoning achievement.

Discussion

In this study, we looked at the links between cognitive abilities and primary school children' conceptual arithmetic understanding. Working memory and phonemic awareness were evaluated as domain-general skills, whereas number transcoding and number estimating were evaluated as domain-specific skills. All of these abilities showed a strong relationship with quantitative reasoning, which was regarded as a measure of conceptual arithmetic knowledge.

According to the findings, all of the abilities evaluated in this study are directly and strongly connected to quantitative reasoning. Furthermore, the data revealed that certain of the abilities, notably number transcoding, working memory, and number line estimation, in this order of significance to the investigated regression model, were predictors of conceptual arithmetic understanding. Even while phonological awareness had a moderate link with numeric reasoning accomplishment, the task was insufficient to impact its performance. This finding contradicts previous research that has found phonemic awareness, and more generally, phonological processing, to be a key predictor of arithmetic proficiency (HECHT *et al*., 2001). However, this finding supports previous research that found no substantial link between phonological awareness and mathematical performance (PASSOLUNGHI; VERCELLONI; SCHADEE, 2007; PASSOLUNGHI; LANFRANCHI, 2012). Such contradictory result could be attributed to the task used as an outcome, which requires more working memory to create solution strategies than the phoneme processing during task instruction receipt. Given that both working memory and number transcoding have higher demands for phonological processing, which includes phonemic awareness, it is also possible that this skill underpins both of those abilities.

It was feasible to confirm that, when conceptual arithmetic performance was broken down by school grade, fourth graders performed much better than third graders on the tasks of number transcoding, number estimation and quantitative reasoning. Since students in higher school grades often have more understanding of the curriculum, it was always anticipated that they would score better on the activities used to evaluate their performance. When assessing the predictive value of the abilities evaluated, again split by school grade, number transcoding and working memory tasks had a considerable effect on the model for both school grades, although the number line estimating ability had an extra contribution for the 4th grade. This may be explained by the fact that, as previously said, fourth graders have a stronger knowledge of numerical concepts, as well as having demonstrated higher performance in this skill when compared to third graders.

These cognitive abilities and their application to mathematical tasks may be explained in a few different ways. For example, number transcoding refers to the knowledge of number system and place-value (units, tens, hundreds, etc.), which is required for doing arithmetic calculations. This study's findings are consistent with earlier studies indicating the relevance of digit reading and writing expertise for subsequent mathematics learning (HABERMANN *et al*., 2020; MALONE; BURGOYNE; HULME, 2020). Furthermore, understanding the number system and, more specifically, understanding the place-value of numbers is an important predictor of the development of arithmetic skills (MOELLER *et al*., 2011). In fact, one might assume that this numerical competence is the essential foundation for more complicated mathematical understanding. In terms of conceptual arithmetic, it is necessary to translate the 'spoken' format of the number (number name) into its written format in digits in order to then carry out the proper positioning of the numbers according to their values (units, tens) when solving a calculation either mentally or in writing format.

It is plausible to argue that number estimation may be related to magnitude representation and the ability to recognize and employ diverse ways to problem resolution. The study's predictive value backs with previous research that found a link between number estimate competence and mathematics proficiency (ARAGÓN *et al*., 2019; GILMORE *et al*., 2018; SASANGUIE; VAN DEN BUSSCHE; REYNVOET, 2012), specifically when it comes to conceptual arithmetic knowledge (GILMORE *et al*., 2018). Thus, it is clear that the ability to correctly estimate the location of numbers on a straight line necessitates knowledge of the number system and additive composition, which refers to the notion that every natural number is made up of the addition of smaller numbers. The assignment utilized in this study to measure conceptual arithmetic skills included additive and multiplicative relations, as well as students' understanding of additive composition. Because the number system follows an ordinal structure based on the principle of additive composition (GILMORE *et al*., 2018), comprehending the number system is required for a greater accomplishment in number estimation, and hence for better performance in conceptual arithmetic.

On the other hand, working memory plays a role in the capacity to remember and manipulate information in order to adhere to a resolution strategy and complete an arithmetic computation. There is strong evidence that working memory affects math performance (ARAGÓN *et al*., 2019; GEARY, 2011; NUNES *et al*., 2007; PASSOLUNGHI; LANFRANCHI, 2012), which is consistent with the results of this study. The results of this study support the idea that when solving story problems, working memory aids children in remembering the quantitative relation underlying the problem by identifying the operations that must be performed, while long-term memory is accessed to retrieve arithmetic facts or prior knowledge to formulate a solution strategy and, ultimately, solve the problem. Working memory capacity can directly contribute to efficient mathematical success; however, poor capacity of this cognitive function might create difficulty in mathematical activities.

It is then easier to understand how these skills are related when dealing with arithmetic problems because they call for not only the correct application of a technique but also an understanding of the structure of the number system and the concepts necessary for the interpretation and selection of the procedure to be used. A consistent conceptual framework helps students build more effective problem-solving techniques and equips them with the right logic to approach more challenging problems in an understandable way (CHING; NUNES, 2017). This demonstrates the necessity of nurturing these skills in kids starting in elementary school in order to support the development of their conceptual mathematics learning.

As study limitations, we highlight the lack of additional assessment tasks that might aid in a more complete understanding of the relationships between the cognitive skills examined. Furthermore, the absence of procedural performance evaluation makes it hard to compare conceptual and procedural performance, both of which are important for mathematical comprehension. Thus, further research is needed to investigate student performance between these two types of knowledge, as well as their causal relationships with future mathematical ability.

The results discussed help in the teaching and learning of mathematics in primary school because they show how important domain-general and domain-specific skills are for growth in mathematical understanding. This study also helps us better understand how conceptual arithmetic knowledge affects math accomplishment and helps us (re) think how to teach math, which should give kids a consistent foundation for conceptual understanding of numbers and operations.

Finally, the findings suggest that exercises may be used in the classroom to increase children's comprehension of the number system structure, which could also help pupils gain a deeper conceptual understanding of arithmetic. Thus, it is essential to design assignments that encourage the development of these predictive skills beginning in the early years of elementary school in order to promote the prevention of later mathematical difficulties.

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