

# Mathematical model for quay crane assignment problem with tidal constraints

## Modelo matemático para la asignación de grúas pórtico a buques considerando el efecto de las mareas

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### Abstract

**Objetivo**— The operations associated with loading and unloading of container ships demand the use of quay cranes that represent one of the most expensive resources of a maritime terminal. Therefore, the assignment of cranes to ships must be optimized. This article proposes a mathematical model to optimize the decision of assignment of cranes to ships, considering the behavior of tides, which is a not commonly considered factor in the scientific literature for similar problems.

**Metodología**— A mixed integer linear mathematical model was designed and tested for the actual case of a container terminal in Buenaventura-Colombia with satisfactory results.

**Results**— With the available capacity the model allows to mobilize up to 2800 containers per half day, while the number of containers per ship in the real case does not exceed 2000 units.

**Conclusions**— The consideration of tides, combined with downtime penalty cost can allow using smaller number of cranes with savings in energy cost.

**Keywords**— Tides; quay assignment; mixed integer programming; container terminal

### Resumen

**Objetivo**— Las operaciones asociadas con la carga y descarga de buques portacontenedores exigen el uso de grúas de muelle que representan uno de los recursos más caros de una terminal marítima. Por lo tanto, la asignación de grúas a barcos debe optimizarse. Este artículo propone un modelo matemático para optimizar la decisión de asignación de grúas a barcos, considerando el comportamiento de las mareas, que no es un factor comúnmente considerado en la literatura científica para problemas similares.

**Metodología**— Se diseñó y probó un modelo matemático lineal entero mixto para el caso real de una terminal d

e contenedores en Buenaventura-Colombia con resultados satisfactorios.

**Resultados**— con la capacidad disponible, el modelo permite movilizar hasta 2800 contenedores por medio día, mientras que el número de contenedores por barco en el caso real no supera las 2000 unidades.

**Conclusiones**— La consideración de las mareas, combinada con el costo de penalización por tiempo de inactividad puede permitir el uso de un menor número de grúas con ahorros en el costo de energía.

**Palabras clave**— Mareas; asignación de grúas; programación entera mixta; terminal de contenedores



## I. INTRODUCTION

According to some research [1], a container ship invests 60% of its useful life in ports, and therefore there is a need to minimize this amount of time by increasing the performance of berth operations and reducing operational delays. At this point, the use of cranes becomes relevant, since the proper assignment and programming of cranes allows to minimize the time that ships spend on the quay. One of the well known problems around this important resource is the Quay Crane Scheduling Problem (QCSP), which seeks to optimally assign the quay cranes to a ship in order to minimize the total time of loading and unloading containers while minimizing the time invested on the quay.

In the literature there is much research related to optimization of quay crane operations. Some researches [2]-[6], propose models that seek to determine the sequence of tasks with the aim of reducing the number of movements and the time spent on a quay. Other studies presents new strategies for loading and unloading of containers [7]-[9], who consider a new method called double cycling that minimizes the movements without containers (simultaneous loading and unloading). Chinese researchers [10], [11] consider additional elements such as the interference between quay cranes, and others [12], proposes to minimize the time of ships departure by controlling the energy consumption of the assigned cranes.

For a ship to leave the port, it is necessary to use the access channel which connects the port with the open sea. The access channel must comply with the width and depth restrictions required by each vessel (draft), and depending on the geographical location of a port it can be affected by tidal changes. Fig. 1 shows the map of the world with the patterns of tidal changes in the different regions.

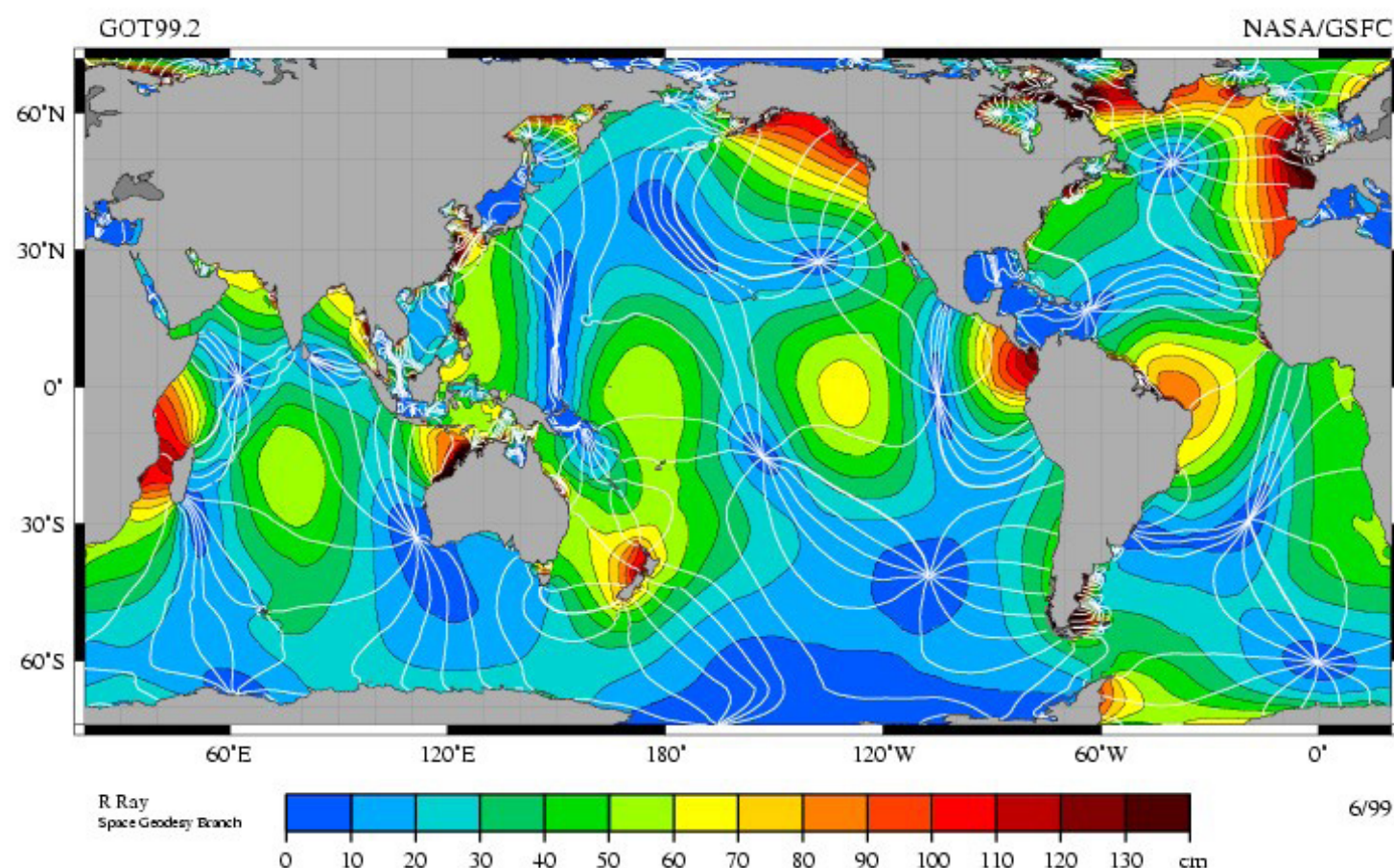


Fig. 1. Worldwide tidal patterns.  
Source: NASA, <https://svs.gsfc.nasa.gov/stories/topex/tides.html>

Changes in tides affect the draft of the access channels, and therefore the exit of the ships once they finish their operations on the quay will be affected too. To date there are few articles addressing the QCSP with tidal considerations. Propose a nonlinear programming model that determines the number of cranes to be assigned to each ship considering tidal variations with a 12-hour cycle and also generating savings in fuel consumption [13]. On the other hand, developed a mathematical model that considers limitations in the access channel and variations in the water depth in berths [14]. Another authors mentioned the importance of tides in the modeling of port operations [15], [16].

Unlike other article [13], this article deals with the problem of assigning cranes to ships by explicitly considering the penalty of downtime due to delays induced when the ship finishes operations at low tide, and at the same time it needs high tide to exit the port. This analysis is critical in certain container terminals in the world according to their location, where the tides and specifically the variation of the access channel depth restricts the entry and exit times of larger vessels, thereby affecting the decision to assign cranes, as is the case of the Colombian Pacific region.

The objective of port operators is to perform the ship operations in the shortest possible time. However, it is possible that even assigning all available cranes to the ship, the operations will end at a time when the tide is not high enough to cross the channel, and therefore the ship must wait. When this situation is foreseen, the port personnel verifies and modifies, if necessary, the number of cranes to be assigned to guarantee the departure of the vessel at the appropriate time and to generate savings in energy consumption.

This article analyzes the operation of a real container terminal in Colombia and evaluates the effect of applying an optimization model to assign quay cranes to ships, considering tidal conditions and minimizing the costs associated with the consumption of energy and penalization costs when waiting because of tides. This document is developed as follows: in Section 2, the nonlinear programming model is described, in Section 3 a real case study is exposed, Section 4 shows the results of the optimization model, and in Section 5 some conclusions are presented.

## II. PROPOSED MODEL

This section describes the gantry crane assignment model for quayside operations. In order to decide the variables and parameters to use in the model [2], [11], [12], [13], [14], were relevant and in addition, interviews were conducted with the Director of Planning and Operations Control, the statistician and personnel who work in the operation of the port under study.

With the abovementioned, it was decided to consider the duration times of the operations, loading and unloading times of each crane, as well as its energy consumption and costs during a time period and the objective is to minimize the cost associated with the use of the cranes, taking into account the behavior of the tides and the penalties associated with waiting times.

In this research, a 24-hour tidal cycle is considered that results in 2 tides per day, one high and one low. The above is an assumption of the model which is explained in the description of it and is complemented with Fig. 2 to give greater clarity. This assumption is a simplified version of reality but It does allow the problem to be solved considering tidal variations and opens possibilities to consider other tidal cycles and frequencies in future research.

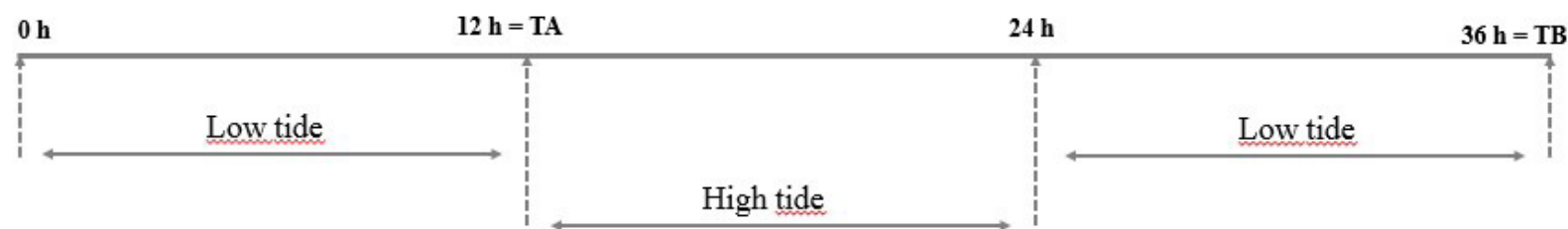


Fig. 2. Period of application of the model.  
 Source: Authors.

In this section, we present a model that considers the following assumptions and characteristics that define its scope:

- A day is divided into two periods of tides, a period with low tide (without loss of generality, it is assumed to start at 00:00 hours) and the other with high tide, as can be observed in Fig. 2.
- The service to ships begins when the ship has docked and the operation is not interrupted until the ship leaves the terminal.
- Only one ship is served at time at the dock.
- The vessel can enter or exit through the access channel only at high tide.

- The ship needs one hour to prepare its exit and 2 hours to traverse the access channel. Therefore, if a vessel is scheduled to depart before 24:00 hours (the moment at which low tide starts, according to Fig. 2), operations must be completed before 21:00 hours.
- The number of quay cranes assigned to a ship do not change during the stay of the ship on quay, and the travel times of cranes along the berth are not considered.
- It is assumed that all cranes have the same capacity and energy consumption.
- It is assumed that the time of loading and unloading a container is equal to an average loading time regardless of the number of movements.

The parameters that we will use in the model are the following:

$CGrua$ : Energy consumption of the crane (Kwh).

$CEn$ : Cost of energy (USD/Kwh).

$CPEN$ : Penalty cost associated with finishing operations in low tide condition (USD).

$TCU$ : Average time of loading or unloading a container (h).

$Carg_j$ : Number of containers to be loaded on the ship  $j$ .

$Desc_j$ : Number of containers to be unloaded on the ship  $j$ .

$TA$ : Moment of beginning of the high tide.

$TB$ : Moment of beginning of the next high tide.

$HLB_j$ : Time of arrival of vessel  $j$  to the anchoring area.

The list of decision variables is the following:

$E_j$ : Downtime of ship  $j$ .

$A_j$ : Docking time of ship  $j$  on berth.

$S_j$ : Time to start operations on ship  $j$ .

$F_j$ : Time to end operations on ship  $j$ .

$NCont_j$ : Number of containers to be handled in ship  $j$ .

$Y_{ij}$ : 1 if the crane  $i$  is assigned to the ship  $j$ , 0 otherwise.

$W_j$ : 1 if the parameter is greater than or equal to  $TA$ , 0 if it is less.

$R_j$ : 0 if the time to complete the operations ( $F_j$ ) is greater than 21, 1 otherwise.

In order to properly define the relationship among the variables, in Fig. 3 we depict the variables and their connection with time specifications when serving a container ship.

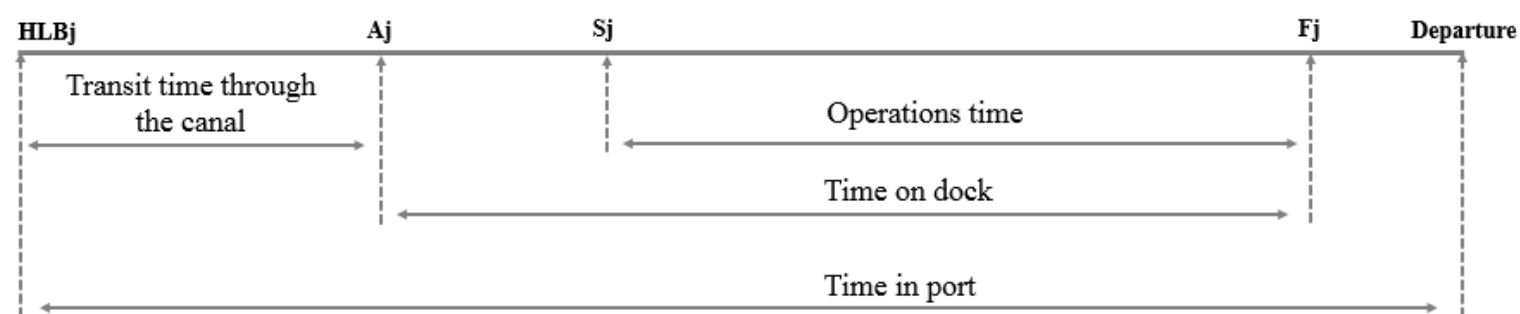


Fig. 3. Outline of the crane assignment model.  
Source: Authors.

The mathematical model is presented below, and after that a brief explanation will be shown. Minimize (1):

$$\sum_j \sum_i Y_{ij} * CEn * CGrua + \sum_j E_j * CPEN + \sum_j [S_j - (A_j + 1)] * CPEN \quad (1)$$

Subject to:

$$F_j = S_j + \frac{TCU * NCont_j}{\sum_i Y_{ij}} \quad \forall j \quad (2)$$

$$E_j = (TB - F_j) * (1 - R_j) \quad \forall j \quad (3)$$

$$NCont_j = Carg_j + Desc_j \quad \forall j \quad (4)$$

$$1 \leq \sum_i Y_{ij} \leq 4 \quad \forall j \quad (5)$$

$$A_j \geq (TA + 2) \quad \forall j \quad (6)$$

$$A_j \leq (TA + 2) + M * W_j \quad \forall j \quad (7)$$

$$A_j \leq (HLB_j + 2) + M * (1 - W_j) \quad \forall j \quad (8)$$

$$A_j \geq (HLB_j + 2) \quad \forall j \quad (9)$$

$$S_j \geq A_j + 1 \quad \forall j \quad (10)$$

$$F_j \leq 21 + M * (1 - R_j) \quad \forall j \quad (11)$$

$$F_j + E_j \leq TB \quad \forall j \quad (12)$$

$$Y_{ij}, R_j, W_j \in \{0,1\} \quad \forall i, j \quad (13)$$

The objective function seeks to minimize the energy cost associated with the use of cranes and also the penalty cost for ending operations at low tide. The motivation of this penalty is that the model allocates a number of cranes considering two possible cases: a) the model can decide to allocate all available resources (cranes) to comply with the tasks in order to end operations at high tide; b) the model can decide to allocate fewer cranes (saving energy cost) because if all cranes were used then the vessel could end its operation in a low tide condition.

The nonlinear constraint (2) defines the variable  $F_j$  as the sum of the time at which the operation begins ( $S_j$ ) and the ship's operating time. It can be observed that the larger the number of cranes assigned, the shorter the operating time (assuming that all cranes have the same consumption and the same performance), and the operations' completion time is less.

Likewise, constraint (3) defines the downtime as the period between the moment of completion of operations ( $F_j$ ) and the start of the next high tide period (TB). This downtime only exists if the operations end between the 21:00 and 36:00 hours, that is, if the ship must wait until the next high tide period to sail as shown in Fig. 4. Note that if the binary variable  $R_j$  takes the value of 0, then  $E_j$  exists, otherwise  $E_j = 0$ .

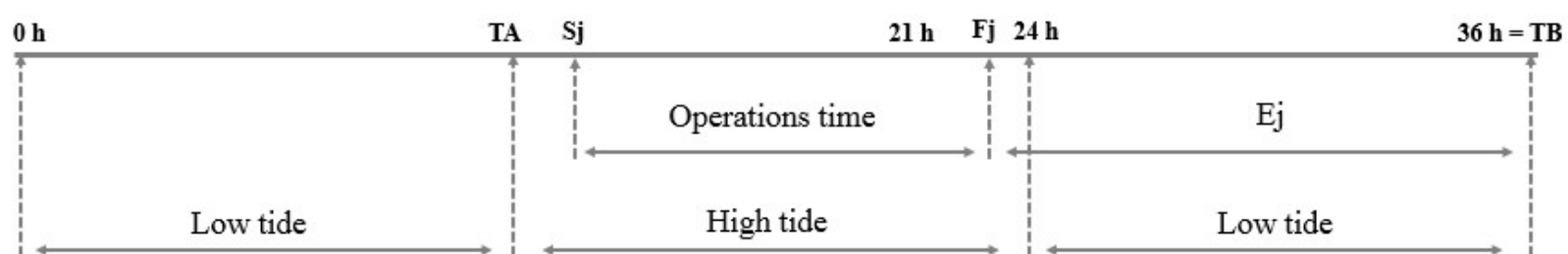


Fig. 4. Definition of  $E_j$ .  
 Source: Authors.

### III. CASE STUDY

The previous model was tested in the case of a container terminal in the city of Buenaventura, Colombia. As shown in Fig. 1, this city is one of the world's regions with major tidal changes. For ships to access this terminal, they must cross an access channel which currently represents a limitation for the competitiveness of the companies installed there. Since the access channel depth at low tide is not

enough to receive all type of ships, then those with a draft greater than 12.5 meters should enter or leave as long as there is enough time at high tide.

The limitation regarding the depth of the access channel implies longer waiting times for vessels to enter or leave the port. This means that in case of delays caused by tides, vessels must navigate faster through the channel, consuming more fuel.

Because of this kind of limitation, that implies cost overruns for all players, the shipping lines have requested to the terminal to pay a delay charge.

In addition to the cost overruns, the decision to assign cranes for the movement of containers is also affected by tides. Assigning too many cranes to a ship can imply finishing operations at low tide condition, when the ship needs to wait until the next high tide period. In this case, maybe it can be more convenient to assign fewer cranes, obtaining savings in fuel consumption.

The information used to run the model was directly obtained from the website of the selected container terminal. The data referring to the arrival schedule of vessels was taken for a period of one month, and during this time a total of 18 vessels were served. In Table 1 a brief sample of the traffic data used for this work is presented. The information corresponds to the month of November 2016.

TABLE 1. CONTAINER TRAFFIC IN NOVEMBER 2016.

ETA		VESSEL		MOVEMENTS			REMARKS	
DATE	TIME		LENGTH	DIS.	LOA.	TTL.	SAIL	HIGH TIDE
07/11/2016	06:00	MAERSK BOGOR	223	750	750	1500	07/11/2016	3.1 m 08:43 H.
Monday							21:50 H	3.2 m 21:22 H.
08/11/2016	07:00	MAERSK UTAH	292	300	300	600	08/11/2016	3.0 m 09:44 H.
Tuesday							21:50 H	3.2 m 22:26 H.
09/11/2016	08:00	SKAGEN MAERSK	346	1000	1000	2000	10/11/2016	3.1 m 10:56 H.
Wednesday							06:19 H	3.3 m 23:35 H.
12/11/2016	17:00	APL TURKEY	293	841	805	1646	13/11/2016	3.7 m 01:39 H.
Saturday							20:40 H	3.7 m 14:11 H.
14/11/2016	06:00	MAERSK BINTAN	222	650	650	1300	14/11/2016	4.3 m 03:23 H.
Monday							20:00 H	4.2 m 15:54 H.
15/11/2016	14:00	MAERSK LAUNCESTON	266	1000	1000	2000	16/11/2016	4.4 m 04:12 H.
Tuesday							02:00 H	4.4 m 16:43 H.

Source: Authors.

According to the information provided by the container terminal, the average energy consumption of the quay cranes subjected to a constant work is 66 kWh, data that is assumed equal for all cranes. Also, the estimated penalty cost per hour due to delays is 1 000 USD. This cost is used for both penalties.

The model was developed and executed in LINGO software. The computational time needed to obtain the optimal solution was 0.73 seconds with 88 iterations.

### IV. RESULTS DISCUSSION

Table 2 shows a summary of the results. Eighteen ships were served, and a total of 25 600 containers were moved. In addition, according to the analysis on the number of containers and the arrival time of each vessel, it was determined that using two or three cranes minimizes the objective function in a proper way. Note that the objective function ranges from 20.24 USD (in this case there is no downtime) to 9020.24 USD (9 hours of downtime  $E_j$ ).

TABLE 2. SUMMARY OF THE OPTIMAL SOLUTION OF MODEL.

Ship	Containers moved	Cranes assigned	Arrival time (hours)	A <sub>j</sub>	S <sub>j</sub>	F <sub>j</sub>	E <sub>j</sub>	R <sub>j</sub>	Result (USD)
1	1500	2	06:00	14:00	15:00	11:30	0.5	0	520.24
2	600	3	07:00	14:00	15:00	21:00	0	1	30.35
3	2000	3	08:00	14:00	15:00	11:00	1	0	1030.35
4	1400	3	18:00	20:00	22:00	12:00	0	0	1030.35
5	1300	2	14:00	16:00	17:00	10:30	1.5	0	1520.24
6	2000	3	14:00	16:00	17:00	10:00	2	0	2030.35
7	2000	3	14:30	16:30	17:30	10:30	1.5	0	1530.35
8	1000	2	18:00	20:00	21:00	12:00	0	0	20.24
9	2000	3	06:30	14:00	15:00	11:00	1	0	1030.35
10	600	3	07:00	14:00	15:00	21:00	0	1	30.35
11	1600	3	08:30	14:00	15:00	07:00	5	0	5030.35
12	1300	3	18:00	20:00	21:00	10:00	2	0	2030.35
13	2000	3	13:00	15:00	16:00	12:00	0	0	30.35
14	700	2	13:30	15:30	16:30	03:00	9	0	9020.24
15	2000	3	14:00	16:00	17:00	10:00	2	0	2030.35
16	1000	2	18:00	20:00	21:00	12:00	0	0	20.24
17	2000	3	06:00	14:00	15:00	11:00	1	0	1030.35
18	600	3	07:00	14:00	15:00	21:00	0	1	30.35

Source: Authors.

In all cases, the starting time of operations ( $S_j$ ) was equal to the moment of berthing ( $A_j$ ) plus the time of inspection spent by port authorities, except for ship number 4. In this case,  $S_j$  was determined 2 hours later beyond the moment of berthing, because the model tried to finish at 36:00 hours (Fig. 4), in order to avoid the penalty for ending at low tide.

According to the above, each ship entering to the dock of the port has a maximum stay time of 36 hours. This time may be that high because the frequency of visits of ships to the port is not daily, and the arrival times include morning and afternoon hours. The latter means that it may be the case that one vessel leaves the port and is navigating across the channel, while another is sailing toward the port under study.

In most cases, the completion time ( $F_j$ ) is the day after the vessel arrival, and this is because they have a large number of containers to move, and due to their arrival time it is not possible release them before 21:00 hours, despite assigning all cranes. In the case of ships 2, 10 and 18, note that  $F_j$  occurs in the same day of the arrival. This is because these are the ships with the least amount of containers to move. Here you can see the coherence between  $F_j$  and  $R_j$ , where the latter is the binary variable that takes value 0 if the ending time of operations ( $F_j$ ) is greater than 21 and 1 otherwise.

For the downtime indicator  $E_j$ , the ship 14 takes the highest value. In this case, it arrives at 13:30 hours with 700 containers to move, berths at 15:30 hours and starts operations at 16:30 hours, according to the that causes the ship to start loading and unloading as soon as possible. In the case of this ship, the model should consider four situations for the minimization of the objective function:

- *Assign all capacity (four cranes).* When the model assigns all the cranes it is trying to finish the operations before 21:00 hours to take advantage of high tide. However, in this case, using all capacity, the operations can be finished at 21:45 hours. Considering that the ship needs 2 hours to traverse the access channel and 1 additional hour for inspection, the ship would not reach out at high tide, and the downtime ( $E_j$ ) is set at 14 hours 15 minutes which is penalized in the objective function. Therefore, in this case the model will try to reduce the penalty in the objective function.
- *Assign three cranes.* If three cranes are assigned, then the operations on the ship would end at 23:30 hours, which decreases the downtime ( $E_j$ ) to 12 hours 30 minutes, and this represents an improvement over the previous situation.

- *Assign two cranes.* In this case, the operations would end at 27:00 hours with a downtime ( $E_j$ ) of 9 hours. This strategy is the one that most reduces the value of the objective function, and it was chosen by the model.
- *Assign one crane.* In this scenario, the operating time is 21:00 hours, which exceeds the next high tide ( $TB$ ) period and therefore the scope of the model (Fig. 4).

For the rest of ships, the decisions taken by the model behave similarly.

According to certain studies [17], the most important indicators to consider are those related to the times of ships at the port. Therefore, the following indicators were selected to analyze the results of the model: operation time of ships ( $F_j - S_j$ ), time spent at the port (time since  $HLB_j$  until the departure time of a ship) and time spent at the berth. All of them will be explained below. The results are shown in Table 3.

TABLE 3. STAY TIME OF VESSEL  $J$ .

Operation time (hours)	Time spent at port (hours)	Time spent at berth (hours)
20.5	30	22
7	14	7
20	28	22
14	18	16
17.5	22	20
17	22	20
17	21.5	19.5
15	18	16
20	29.5	22
6	14	7
16	27.5	22
13	18	16
20	23	21
10.5	22.5	20.5
17	22	20
15	18	16
20	30	22
6	14	7

Source: Authors.

#### A. Indicator of operation time

This indicator is defined as the time of completion of operations ( $F_j$ ) minus its starting time ( $S_j$ ). Table 3 shows the number of hours spent in operations in each ship. This indicator is very important in maritime operations, because it allows us to know the performance of the port and to make strategic decisions regarding capacity or planning.

For ships 1, 3, 9, 13 and 17 the amount of containers to be moved is the highest, and it ranges from 1 500 to 2 000 containers, representing the highest operating times (20 hours or more). The model verifies if the assignment of all the available capacity causes the ship to leave before 21:00 hours. However, since it is not possible to achieve this goal, the amount of resources to be assigned is modified until the minimum in the function objective is obtained. In this case, two cranes were assigned for the first vessel and three for the rest of them.

#### B. Indicator of time spent of ships at the port

It represents the time elapsed between the arrival hour of the ship ( $HLB_j$ ) until its departure time. This is equal to the completion time if it ends at high tide or at the time when the next high tide ( $TB$ ) begins.

Since the earliest time a ship can arrive at the port is at 6:00 am, the maximum time a ship stays is 30 hours, and this would be the time elapsed from 6:00 to 36:00 hours ( $TB$ ).



As shown in Table 3, vessels 1 and 17 have the longest time at the port. This is because they arrive at low tide, they must wait until 12:00 hours to cross the access channel, and they must move at least 1000 containers. The ships 1, 2, 3, 9, 10, 11, 17, and 18 have waiting times because they arrive at low tide.

### C. Indicator of time spent of ship at the berth

The time spent at the berth is the period between the moment of berthing and the moment of the ship departure. Vessels 2, 10 and 18 are the only ones that leave before 21:00 hours, showing the shortest time at the berth. Each one of them arrives with 600 containers to move. The other ships have longer times of staying at berth because the number of containers is higher, and it is not possible to complete the loading and unloading operations before the time limit.

In order to know the amount of containers that could be moved depending on the arrival time of the ship and the number of cranes that are assigned, a final analysis was carried out to force the model to finish operations at 21:00 hours, as a time limit. It was identified that  $S_j$  can take values between 15:00 and 20:00 hours, assuming that a ship never reaches the port with less than 100 containers to be moved.

Fig. 5 shows the maximum number of containers with which a ship can reach the port, in order to obtain a final operational time that determines an exit time less than 21:00 hours.

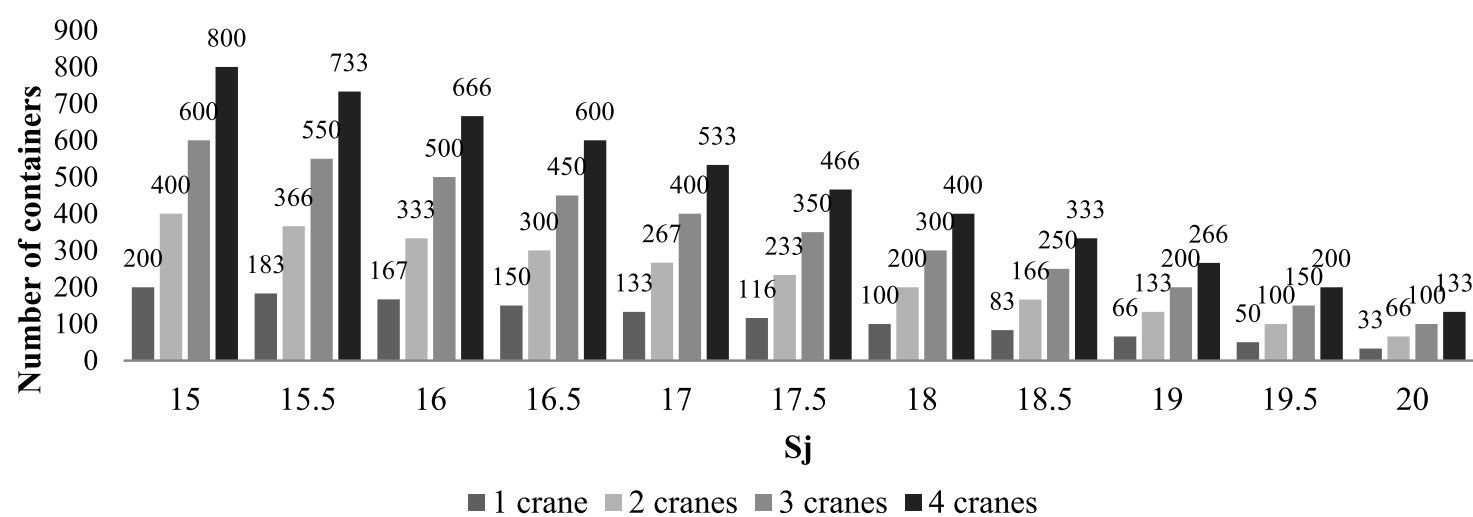


Fig. 5. Containers to move - subject to  $S_j$  and  $F_j = 21:00$  hours.  
 Source: Authors.

If the operations on a ship start at 18:00 hours, the ship cannot bring more than 400 containers to move if it wishes to finish operations at 21:00 hours as a maximum time limit. In this case, the four cranes available for operation would be assigned.

On the other hand, if only two cranes were available and it is desired to finish operations at most at 21:00 hours, the ship which arrives at 18:00 hours. cannot bring more than 200 containers to move.

Extending this analysis, if the ship 14 had arrived at most at 12:45 hours, assigning the four available cranes would be able to complete operations at 21:00 hours, and this would avoid falling into downtime and decrease the objective function.

## IV. CONCLUSIONS

This work proposes a model to solve the problem of assigning cranes considering restrictions in tide depth, and this work was motivated by changes in tidal conditions occurring in certain ports in the world and in the increase in ship dimensions. The model proposes a new approach where the problem considers an adequate assignment of cranes that minimizes the time of the ships' stay in the port, but also synchronizes the completion of operations with the tides that allow the departure of the ship according to its draft. With the available capacity the model allows to mobilize up to 2 800 containers, as long as the ship arrives at the port between 0 and 12:00 hours. When comparing this capacity with the case study, the number of containers per ship in the real case does not exceed 2000 units.

The model requires loading and unloading of the containers as close as possible to the moment of berthing of the ship, which considerably reduces the idle time at the start of operations. Results show that most of the operations started once the ship docked and inspections were completed by the port authorities, and this condition resembles what actually happens in the port, where it is necessary to start operations as soon as possible so as not to incur extra expenses for waiting. With the penalty imposed on the objective function for each hour incurred in downtime ( $E_j$ ), the model tries to assign all the capacity to finish operations before 21:00 hours. When this is not possible, it tries to reduce the cost of the penalty by extending the number of hours of operation for the loading and unloading tasks to conclude at a time close to 36:00 hours ( $TB$ ). To get this, the model assigns a smaller number of cranes in addition to minimize the cost of the penalty and the associated energy cost. The Crane Assignment Model is focused on reducing the main cost related to cranes, energy consumption, considering the penalties for delays. In addition, it is characterized by dividing the day into two periods, one at low tide and the other at high tide. However, other tidal variations can be explored. For the development of the crane assignment model, it was considered that the energy consumption of cranes was the same for all of them and that their capacity was the same. However, in real life, this situation may not arise, since the constant investment in technology and equipments has made it increasingly seek to reduce the energy consumption of cranes. For this reason, this problem is suggested as a suitable extension. Finally, we can say that models with proper consideration of tides, combined with downtime penalty cost can allow using smaller number of cranes with savings in energy cost.

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