LEARNING REPORTED BY THREE TEACHERS AND THE TEACHING OF ALGEBRA IN THE FIRST GRADES

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In this paper, our aim is to understand how teachers perceive their professional learning in an in-service teacher education process, as well as the way they related this learning to their teaching practices regarding algebraic thinking in the first grades of elementary school. It is a qualitative-interpretative study, whose data were analyzed considering the Grounded Theory. The results indicate signs of professional learning identified in teaching practice were a glimpse of possibilities to approach algebraic thinking in first grades; changing the planning for students to select and develop mathematical tasks; adopting new methodological strategies with elements of the exploratory teaching approach.

Keywords: Early Algebra; First grades of Elementary School; Grounded Theory; Teacher Education; Teacher's professional learning

Aprendizaje reportado por tres docentes y enseñanza de álgebra en los primeros grados

En este artículo, nuestro objetivo es comprender cómo los docentes perciben su aprendizaje profesional en un proceso de formación docente en servicio, así como también cómo relacionan este aprendizaje con sus prácticas docentes sobre pensamiento algebraico en los primeros años de la enseñanza básica. Se trata de un estudio cualitativo-interpretativo, cuyos datos fueron analizados a la luz de la Grounded Theory. Los resultados indican que los indicios de aprendizaje profesional identificados en la práctica docente fueron vislumbrar posibilidades de abordaje del pensamiento algebraico en los primeros grados; cambiar el horario para que los estudiantes seleccionen y desarrollen tareas matemáticas; adopción de nuevas estrategias metodológicas con elementos de la enseñanza exploratoria.

Términos clave: Álgebra inicial; Aprendizaje profesional docente; Formación de profesores; Primeros grados de la Enseñanza Primaria; Teoría fundamentada

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Aprendizagem relatada por três professoras e o ensino de álgebra nos anos iniciais

Neste artigo, nosso objetivo é compreender como os professores percebem sua aprendizagem profissional em um processo de formação docente em serviço, bem como a forma como relacionam essa aprendizagem com suas práticas de ensino sobre o pensamento algébrico nos anos iniciais do ensino fundamental. Trata-se de um estudo qualitativo-interpretativo, cujos dados foram analisados à luz da Grounded Theory. Os resultados indicam que os indícios de aprendizagem profissional identificados na prática docente foram vislumbrar possibilidades de abordagem do pensamento algébrico nos anos iniciais; mudar o planejamento para os alunos selecionarem e desenvolverem tarefas matemáticas; adoção de novas estratégias metodológicas com elementos da abordagem de ensino exploratório.

Palavras-chave: Álgebra inicial; Aprendizagem profissional do professor; Formação de Professores; Primeiras séries do Ensino Fundamental; Teoria fundamentada

Different studies on Mathematical Education analyze formative processes involving teachers. In a worldwide literature review with papers published from 2000 to 2010, about teachers' professional development, Avalos (2011) identified only 10 papers in a universe of 111 that explicitly addressed the effectiveness of in-service teacher education process, focusing on changes in knowledge, teachers' beliefs, and practice. In Brazil, Fiorentini et al. (2016), by analyzing 246 dissertations and theses referring to in-service teacher education, have not identified any related to the signs of professional development processes in teaching practices after a period without following the teacher. In another study, when reviewing 35 studies on teacher professional development carried out in the last three decades in the USA, Darling-Hammond et al. (2017) show us that well-designed in-service teacher education processes can, when effectively implemented, lead to desirable changes in practice teacher and student outcomes.

In contrast, the study by Liu and Phelps (2020), when discussing the effectiveness of professional development programs, highlighted that investigating the conditions that support the stability of acquired knowledge is as important as studying the levels of acquired knowledge. In a survey conducted in Brazilian and Latin American publications, regarding the efficiency of in-service teacher education initiatives from 2007 to 2017, Moriconi et al. (2017) have found no studies approaching this theme.

In the field of Early Algebra, the mathematical theme of this study, although this topic has been addressed and discussed in several countries since the 1990s (Ferreira et al., 2020; Pincheira & Alsina, 2021), there are few Brazilian PhD

theses referring to the teachers' professional development of the first grades of Elementary School that deal with algebraic thinking. In a recent study, referring to the Brazilian theses defended from 2007 to 2018, Silva and Bianchini (2020) have identified that only one of them approached the theme of algebraic thinking in the first grades (Gladcheff, 2015).

In this context, our study[1] proposed to contribute to overcome this gap, seeking to understand how teachers perceive their professional learnings in an in-service teacher education process, as well as how they related these learnings with their teaching practices regarding algebraic thinking in the first grades of elementary School. For that end, we aim to answer the following research questions: Which professional knowledge regarding the algebraic thinking have the three teachers reconstructed after the in-service teacher education process? and how has the professional knowledge regarding the development of algebraic thinking in the first grades been expressed in the teachers' practices after the in-service teacher education process?

This article begins with discussions on the teacher's professional learning, the different views of the teaching practice, and the development of algebraic thinking in the first grades of elementary school. After that, we will describe the context of the study and the methodological and data collection procedures, discuss the data analyses; and discuss the results in light of the literature. Lastly, we present the conclusions and final remarks.

Theoretical Framework

Teacher's Learning and Professional Knowledge

Teaching is a complex, demanding and multifaced profession (White et al., 2013) and it demands theoretical and practical knowledge of many domains. Besides knowing your area of expertise, teachers must have pedagogical knowledge; be able to design and analyze project, classes, and teaching materials; identify and diagnose the learning problems of their students, of the school they work in and of the community they belong to; and develop the ability of analyzing and making practical decisions (Ponte, 1999).

Besides, professional challenges appear as changes happen in different scopes. Ponte (1999) shows us some of them, such as: curricular innovations motivated by the advances in academic productions; different instructional processes, related to the planning, development, and evaluation of the class, among others.

The studies by Shulman (1987) present a base of knowledge needed to be a teacher, distributed into the following domains: (a) content knowledge; (b) general pedagogical knowledge, including generic classroom management principles; (c)

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curriculum knowledge, including materials and programs; (d) pedagogical knowledge about the subject; (e) knowledge on the students; (f) knowledge of educational contexts, communities, and cultures; and (g) knowledge of educational purposes.

Besides the knowledge explained above, Ponte (2012) has identified that teacher knowledge is essentially guided towards action, and their education must, consequently, have as premise promoting reflection through situations that enable examining, questioning, and evaluating their own practices or the one of other teachers, so to contribute for teachers to become able to analyze and face daily classroom situations.

In their turn, Opfer and Pedder (2011) have constructed and widened the concept of teacher's professional learning, combining multiple and fragmented literature aspects about professional development, teaching and personal learning, the organizational learning, and changes in teachers. Based on the theory of complexity, these authors have constructed a professional learning concept that is composed by three overlapping and recursive concepts, which intersect and interact among themselves, namely:

- "the individual teacher", which includes their former experiences, orientation, and beliefs regarding learning and their previous knowledge (p. 384).
- "the school", which approaches collective orientations and beliefs regarding learning, practices or norms of practices that exist at school and the collective capacity of achieving shared learning goals (p. 384).
- "the educative activity", which involves learning activities, tasks, and practices in which teachers take part.

A significant contribution regarding the effectiveness of professional development processes was presented by Liu and Phelps (2020), when listing factors that may support teachers' learning after an in-service teacher education process. They are: the formation's design, the school environment where the teacher works, the partnership with other teachers, the support of the management team, personal characteristics of the teacher, formations that promote the understanding of student thinking.

Based on these studies, we defend that an in-service teacher education process[2] must reverberate *on* practice and be constructed *based on* it, as indicated by Ball and Cohen (1999). It, the practice, enables teaching and learning at the same time. Practice will always be "bigger" and more complex than learning opportunities, since it happens in the interaction between a student and the teacher, with particular

[2] In this paper, we use the term "in-service teacher education process" as a sequence of meetings between teacher educators and participant teachers that focus on approaching robust content, through active and collaborative learning strategies, aligned with the current curriculum and educational policy, which takes place for a period that is sufficient to provide opportunities for participants to learn (Desimone, 2009).

ideas and circumstances (Ball & Cohen, 1999). Still regarding the characteristics of in-service teacher education processes, we defend that, just as in the findings by Jesus et al. (2020) and Ferreira et al. (2022), they promote more learning opportunities when teachers are challenged to rethink and question some actions linked to their teaching practices.

Despite the limitations that learning *in* and *from* practices may have, Ball and Cohen (1999) highlight that, in these conditions, teachers may learn: (a) how to measure situations as they happen, discovering what students know and how the teaching was understood as classes are developed; (b) to use the constructed knowledge to examine their own work, learning to challenge themselves, drawing reasonable conclusions, including generalizations destined for future situation; and (c) frame, guide, and review tasks, reformulating their questions and, with that, learning more about the students' ideas and understanding.

Practice and its different understandings

In this section we present and discuss the different understandings of the concept of *teaching practice*, to situate which aspects of teaching practice were prioritized in the data analysis. We justify that our methodological approach, based on the study by Desimone (2009), began by investigating the characteristics of one in-service teacher education process, where we obtained the data that allowed us to establish relationships between the professional learning reported by the participating teachers and the documents related to teaching practices, both individual and collective.

Ponte and Serrazina (2004) indicate that the teachers' professional practices arise from their functions and are classified into three domains: (a) teaching practices, composed by tasks proposed to students, materials used, communication in the classroom, curricular management, and evaluation; (b) professional practices in the institution, with emphasis on the collaboration among teachers; (c) educational practices, aiming at a permanent professional development. This study is focuses on the teaching practice.

The study of teaching practices is a possibility since theory and practice go hand in hand. The elements of professional practice are influenced by theory and dictate theory. According to Ribeiro (2011), "a better understanding of the teaching process necessarily implies a better understanding of teachers' practices" (p. 72).

In the study regarding teaching practices, Ponte et al. (2012) have focused on the teachers' activity observed in the classroom and strictly linked to their plans of action and decisions. The interpretation of teaching practices, according to these authors, considers two fundamental aspects:

- Context: (a) general social, with highlight to curricular guidelines –both official ones and those taken on by the school where the teacher works– and to the way teachers interpret and relate to them; (b) of the class group, in which one must consider the students' interest on Mathematics, the way

they engage with the work, the relation with teachers and the development of the classroom culture.

- Teacher, considering three different dimensions: (a) the professional knowledge, which involves the domains of Mathematics, curriculum, learning processes, Mathematics Didactics, and of the ability to locate and construct resources for the learning-teaching process; (b) know-how, which is translated as the ability to perform the operations needed to materialize practice effectively; and (iii) the teachers' reflexive ability, which is the basis for their learning and for the improvement of the professional development (Ponte et al., 2012).

Within this perspective, we understand planning as a highlighted moment in the teaching practice. Planning is a concrete process, which involves the adoption of certain routines and ways of doing, in a constant interaction between teachers and students (Serrazina, 2017). During it, teachers put in action their notions about what means to teach and learn Mathematics, about mathematical contents, the learning trajectories, and the models present in school manuals; as well as the knowledge on what the students know and how the learn.

Planning is no easy task, since, when doing so based on an exploratory teaching approach, for example, it is necessary to consider the following guidance (Serrazina, 2017): (a) to establish clear and specific goals; (b) to link the class content to the prescribed curriculum; (c) to provide students with significant tasks, in mathematical and non-mathematical contexts; (d) to anticipate and plan considering the students' difficulties; (e) to anticipate possible questions of the teacher and the students' possible answers; (f) to anticipate how students will solve the tasks; (g) to use the necessary resources, so to guarantee that all students understand what is intended with the tasks; (h) to use questioning as a support for student learning; (i) to consider that students will work in groups or individually, according to the established goals; (j) to identify the additional tasks to go deeper and systematize what was learned, or to embrace students that solve tasks quickly; (k) to evaluate the students' learning during the class.

Reflections about the work with algebraic thinking in the first grades

The introduction of Algebra in the first grades, a movement internationally known as Early Algebra (Ainley, 2001; Chimoni et al., 2021; Lins & Kaput, 2004; Rittle-Johnson et al., 2018; Squalli & Bronner, 2017), is mainly focused on student learning and thinking through the exploration of problem situations, the recognition of the different Algebraic functions and an approach that provides a network of meanings (National Council of Teacher of Mathematics [NCTM], 2007).

Blanton and Kaput (2005), in a survey conducted with students since Early Childhood Education until the end of the two first grades, have proven their ability to think algebraically, including in a covariational manner. Besides, the authors

have described that the development of algebraic thinking must be seen as a whole, since its first characteristics until the use of symbolic language to establish generalizations. The most studied forms of algebraic thinking in the first grades are: Generalized arithmetic, which refers to the thinking about operations and the properties related to numbers; and the functional thinking, which involves the exploration and expression of numerical regularities (Chimoni et al., 2021; Pincheira & Alsina, 2021).

Different authors understand algebraic thinking in different and complementary ways. For example, Blanton and Kaput (2005) define algebraic thinking as "a process in which students generalize mathematical ideas from a particular set of examples, establish generalizations through argumentation discourse, and express them, increasingly, in formal, age-appropriate ways (p. 413). On the other hand, Ponte (2006) extends the ideas of Blanton and Kaput (2005), presenting the following definition for algebraic thinking:

> *"...algebraic thinking includes the ability to deal with algebraic calculation and functions. However, it also includes the ability to deal with many other mathematical structures and use them in interpreting and solving problems in mathematics or other domains. The ability to manipulate symbols is one of the elements of algebraic thinking, but so is "symbol sense" (...), that is, the ability to interpret and creatively use mathematical symbols to describe situations and problem solving. That is, in algebraic thinking, attention is paid not only to objects, but also to the relationships between them, representing and reasoning about these relationships as much as possible in a general and abstract way. Therefore, one of the privileged ways to promote this reasoning is the study of patterns and regularities".* (Ponte, 2006, pp. 7-8, our translation)

In her turn, Kieran (2011) defines algebraic thinking as a "way of thinking" and not as a set of techniques that is built from experiences that students have. More recently, Kieran (2022) bring us a contemporary state-of-art of Early Algebra in a paper that she defends early algebraic thinking as a kind of reasoning engaged in by 5- to 12-year-olds from which children can build meaning for the objects and ways of thinking in order to prepare them for the later study of secondary school algebra.

For us, still based on Chimoni et al. (2021), algebraic thinking is an amalgam of the definitions presented, because to develop it, it is not necessary to include new content or advance content that would generally be developed in the final series of the Elementary School, it is about privileging pedagogical proposals that allow students to perceive, generalize, conjecture and justify the mathematical relationships identified, making use of a variety of representations and languages, in short, promoting changes in the way of thinking.

A specific topic that can be approached during the development of algebraic thinking at this teaching level is the different meanings of the equal sign: (a)

operational, meaning a result; (b) equivalence, when it expresses a relation of balance between the terms "before" and "after" the sign; and (c) relational, in which relations are established between expressions, and which implies the understanding and use of the properties of addition and multiplication operations (Barboza et al., 2020; Kieran, 1981).

In more recent proposals, Blanton and Kaput (2011), Ponte and Branco (2013), Torres et al. (2022) defend that the construction of students' algebraic thinking in the first grades should be developed through permanent tasks that create opportunities for (a) the investigation of patterns, (b) the establishment of conjectures, generalizations, justifications for the establishment of mathematical relations, (c) the use of symbols, (d) the promotion of mathematical discussions, based on what the students say, do, and write, among others. Besides the types of tasks, Ponte (2014) warns us of the relevance of the kind of teaching provided to students, presenting as an alternative the "exploratory teaching approach", which is opposed to the teaching based on presenting the concept and some examples and exercised for students to solve, with the goal of settling concepts.

Corroborating the reflections about the type of teaching, Kieran (2011) highlights the role of teachers' proposals and interventions, as well as the selection of mathematical tasks. In its turn, Cyrino and Oliveira (2011) highlight that it is necessary for teachers to be prepared to identify and understand the different kinds of algebraic thinking expressed by students when carrying out mathematical tasks, be it through actions programed by the teacher or spontaneously, as well as they should know how to explore it.

In sum, we highlight in our theoretical framework: (a) teachers learn more efficiently when the activities require their involvement in the teaching practice (Ball & Cohen, 1999); (b) the domains of the teachers' professional practice (Ponte & Serrazina, 2004), which we deem closely connected to the three systems that involve teacher learning (Opfer & Pedder, 2011); (c) the relevance of planning as a possibility for investigating signs of professional learning by the teacher (White et al., 2013); (d) the relevance of the development of algebraic thinking in the first grades (Chimoni et al., 2021; Pincheira & Alsina, 2021).

Methodology

Our research falls within a qualitative approach, in a constructionist epistemology and in the theoretical-interpretative perspective (Estebán, 2010). Data interpretation was based on the Grounded Theory (GT) in its constructivist approach (Charmaz, 2009), whose main characteristics are: not having a research question completely defined; collecting data through semi-structured interviews and textual analyses; analyzing data inductively, based on a field work that is rooted, base, and embedded in data (Tarozzi, 2011).

Context of the study: from the in-service teacher education process[3] to the participants' practice

The context that has enabled the data collection in our study is composed by two articulated phases: (a) the analysis of an in-service teacher education (INTE) process named "Mathematics in the First Grades and the development of Algebraic Thinking", henceforth referred to as "MFGAT-INTE", with the aim of identifying the professional learnings constructed according to the view of the participants, (b) the analysis of documents referring to the teaching practices of the three participants, in the three years after participating in the MFGAT-INTE, with the goal of locating and identifying signs of professional learning constructed during the in-service teacher education process. To complement the information from these phases, the participants took part in stimulated memory interviews, which will be discussed in detail in the following sections.

The MFGAT-INTE, was developed in the facilities of a public university in the State of São Paulo, Brazil, from May to July 2016, aimed to develop mathematical and didactical knowledge of first grades teachers related to the algebraic thinking, and it covered elements that characterize the Generalizing Arithmetic (Blanton & Kaput, 2005), especially number and operation properties, the equal sign as equivalence, sequencies, and patterns. This was the context of a master's research developed by one of the participants of the research group ForMatE[4] (Ferreira, 2017).

The teacher educator planned and developed the in-service teacher education process according to the principles of the Exploratory Teaching Approach (Ponte, 2014), respecting its different phases. During the phase of *task launching*, she would clarify possible doubts and encourage the participants to get involved in the proposal. Besides, she would organize the resource, the time for execution, and define if the task would be performed individually or in small groups. In the *exploration phase*, she would go around the class, encouraging participants and observing how the task was being developed. Lastly, in the *discussion and synthetizing phase*, she would involve the whole group in a collective discussion based on the resolutions presented for the task, and also *guide* the discussion, guaranteeing its mathematical and didactical quality[5].

Regarding the phase referring to the teaching practices of the three teachers, the investigation occurred through the teachers' professional documents, both individual —plannings, reflexive registry, and mathematical tasks proposed to students— and collective ones (Political-Pedagogical Projects, Year/cycle council

[3] Based on Desimone (2009), we chose to start investigating the knowledge built after the in-service teacher education process, through the investigation of the characteristics of the process, as they allow us to identify whether it was effective or not, in providing opportunities for professional learning to teachers and promoting changes in their teaching practices.

[4] http://dgp.cnpq.br/dgp/espelhogrupo/774793

[5] The description and detailed analysis of the MFGAT-INTE can be found in Baldan da Silva, Ribeiro and Aguiar (2022).

minutes[6] and Performance Files[7]). These documents refer to the period from 2017 to 2019, and this will be mentioned again in more detail in the discussion of the study methodology.

Selection of the participating teachers

The selection of participants happened deliberately and based on previously established criteria. The first one was to identify, among the teachers that had concluded the in-service teacher education process, those who worked during the education process —and who kept working until 2019— in the first grades of Elementary School in a certain municipal teaching network in the metropolitan region of São Paulo, Brazil. With that, we had selected five teachers. A second criterion was to identify teachers who had no absences in the in-service teacher education process, having thus experiences all the learning opportunities provided then. Thus, we reduced the number to four teachers eligible to participate in the study. Lastly, we made a survey to locate the school where these teachers were working in 2019 and were able to contact and invite them to participate in this survey. However, due to the organization of the work in one of the schools, one of the teachers was excluded since he was not working in the mathematics areas in 2019, our last criterion. The participating teachers were:

- Helena[8]: Pedagogue with 15 years of experience as a teacher, who has worked in Early Childhood (Nursery and Early Childhood VI) and 3 years as Japanese teacher (Nursery up to the 6-year-old group) in a private school. She has been working for 10 years as teacher in the initial cycle (1st, 2nd, and 3rd grades) in the public network. She used to not have a good relationship with Mathematics, but when she began studying it and delving in it to be able to teach, her relationship improved. She has worked at the same school from 2016 to 2019.
- Kátia: Pedagogue, with experience of 5 years teaching at a public school. She has worked 22 years in a private school, where she performed many functions, among them: assistant secretary, secretary, assistant coordinator, and principal. She has always had a good relationship with Mathematics, a fact she credits to the good learning opportunities when she was a student, as well as to her effort to know this knowledge area increasingly better. She has worked from 2017 to 2019 at the same school, however, it is a different school from the one where she worked in 2016, when she participated in the formative process.

[6] The minutes of the Year/cycle councils serve to register the analyses, reflections, and referrals related to the trimester's teaching-learning process, in the municipal service where the participant teachers work.

[7] Performance Files are documents in which the goals to be developed and evaluated during the teaching trimester are registered, in the municipal service where the participant teachers work.

[8] To preserve the anonymity of the participating teachers, we used fictional names.

- Márcia: Pedagogue with a 14-year teaching experience, most of in Early Childhood in a private school. She has been working in a public school, in Elementary, in the last 5 years. Her relationship with Mathematics had never been very good, especially when she was a student, but, as a teacher, she understands the importance of improving her mathematical knowledge in order to teach better. She has worked in the same school from 2016 to 2019.

In sum, it is worth highlighting that: (a) all participants of our research work in the same teaching municipal network; (b) they have over five years of teaching experience; (c) they have had experiences both in public and private schools, even though sometimes not teaching, as in the case of the teacher Kátia.

Data collection procedures

As aforementioned, data collected for this study were obtained through interviews and documents. In a first phase, we designed a stimulated memory interview script (Falcão & Gilbert, 2005), making use of: (a) protocols of the MFGAT-INTE, such as the development of Professional Learning Tasks[9] (PLT) with the participants; (b) transcriptions of the audios referring to the discussions promoted in the subgroups; and (c) the participants' academic and professional information, as well as their relationship with Mathematics. As an example, we present below an excerpt from the interview script that was used in the first interview with all participants:

> *Researcher*: Now that you have the answers related to the task as a whole, I would like you to answer the following questions: (a) Do you remember how the trainer organized the discussions in the subgroup? Were there moments of socialization? If so, how were they organized? (b) Was there any speech from the trainer after the presentation of the groups? If so, what was it related to? and (c) What do you believe you learned from this assignment?

Considering the principles of Grounded Theory, after conducting the first interview and beginning the data analysis, the researchers noticed the need for complementing the obtained information and decided to conduct a second stimulated memory interview with the three participants. The script for the second interview considered the complement of the information obtained in the first one, as well as the analysis of the professional documents of the participant teachers, both individuals and collective ones, related to the period from 2017 to 2019. To illustrate, we present below an excerpt from the script of the 2nd stimulated recall interview.

[9] Tasks oriented toward teacher professional learning, used in formative processes (Ball & Cohen, 1999).

3 - According to its 2019 planning, on 08/01/2019, it was planned to develop the mathematical task on page 146, of the Buriti More Mathematics textbook, (2nd year), which I reproduce in the figure below. Could you tell us what were the reasons that led you to choose this mathematical task. What were your expectations of student learning using this mathematical task? Could you tell us a little about how the task was developed, with students in your classroom? Do you establish any relationship between your choices – both in terms of the mathematical task and in terms of how it was developed – with the in-service teacher education process of 2016? You can comment?

Figure 1. Excerpt from the interview script

Thee stimulated interview scripts were personalized, consisting of general and specific questions for each participant, as it was constructed from the analysis of the documents of each one of them. In the first interview, we rescued the documents related to MFGAT-INTE and in the second, we selected excerpts from professional documents, which portrayed the practice of each one. Through this script, it was possible to provide opportunities for the participants to talk about their own practice based on the analysis of the documents.

FINDINGS

Aiming to contribute to the reader's understanding of our findings, we present them in two phases: (a) the process of constructing codifications: initial, focalized, and axial; and (b) Core Category[10], identified from the previously constructed codifications.

The first action related to the data analysis was the transcription of the six interviews —both the ones related to the MFGAT-INTE and those related to the participants' teaching practices— two of them with each participant. After that, with the aid of the software *ATLAS.ti*[11], the initial codification of the data was carried out. In this phase, after a thorough reading, 280 citations were selected, and 91 codes were identified.

On a second moment, during the focalized codification, we grouped the 91 codes and 280 citations into themes, which enabled the construction of the 6 categories: Learnings of the MFGAT-INTE, Personal aspects, Participants' development regarding Mathematics and Algebra, Reliving the MFGAT-INTE, Teaching practice after the MFGAT-INTE and School environment. It is worth highlighting that sometimes the same code or citation fitted in more than one category.

[10] It is a concept that expresses an intense analytical potency, it is saturated, dense, but porous, able to integrate the categories because it is extremely ramified into them, it is complete and has a great explanatory power (Tarozzi, 2011, p.140).

[11] Developed by the company Scientific Software Development. The student software license we used was acquired and downloaded directly from the website of the developer company.

After that, the axial codification was done, connecting the categories and its subcategories that were created in the initial codification, a moment when it was possible to specify the properties and dimensions of each one of them. Throughout the last codification phase, the theoretical one, a central category organized a set of other categories, the Core Category, which, in this study, was called "Signs of learning in teaching practice after the MFGAT-INTE." The Core Category was identified from the amalgam of the categories: "MFGAT-INTE Learnings", "Relieving the MFGAT-INTE", and "Teaching practice after the MFGAT-INTE", since it was identified that they showed evidence that allow us answering the guiding questions of this study (Tarozzi, 2011).

In the Core Category, the three participants showed signs that the MFGAT-INTE changed their *perspective* in relation to Mathematics and to the development of algebraic thinking. Evidence of these results can be observed in the dialogues below, extracted from the stimulated memory interviews, since each of the participants had the opportunity to review an excerpt from the evaluation carried out as soon as the course was concluded. Based on the researcher's questions and stimuli, the teachers were reliving their experiences three years after completing their in-service teacher education.

Researcher: After reading your answer at the end of the course, if you did this evaluation today, after three years, what would your answer be? The same? Would you make any adjustments? Which?

Helena: After the course, I began seeing Mathematics better and to captivate more people around me. We [referring to teachers] are the one that put light in the eyes of these children, we have to throw glitter and then children begin liking Mathematics and Algebra. (First stimulated interview)

Márcia: Sometimes we have already worked on many things, but I did not have this perspective, the course brought me a wider view on what to do to teach Mathematics. Teachers can make the difference, offering activities that will enrich the students' knowledge, but many times we stick to the usual activities, like set up and make the count. (First stimulated interview)

Kátia: After the course, I started seeing Mathematics differently, and it makes no more sense to me demanding just the operations technique, I wanted them to understand it. In the same way that I discovered that equality is equivalence (I will use this example because it was the most significant to me). I wanted them [the students] to understand this and other concepts because they would use it later on. (Second stimulated interview)

When referring to the perspective, it seems that the teachers are changing the way of seeing the mathematical teaching in the First Grades. Helena for example, commented that she started to "captivate more people around", presenting the

discipline in a more pleasant way, "throwing glitter" with the aim of having students like Mathematics/Algebra. In return, Márcia reported a change in the selection of the proposed mathematical tasks, being attentive to the need of "offering activities that will enrich the students' knowledge", and not only those who ask them to "set up and make the count". While Kátia presents a similar change to Márcia by reporting that "it makes no sense demanding just the operations technique" but adding that it is necessary that students "understand" Mathematics.

During the interview, Helena pointed out that the MFGAT-INTE was *decent*. Seeking to understand the term used, the researcher questioned which characteristics of teacher education that led her to affirm that:

Researcher: Would you like to make any further comments about the course as a whole?

Helena: It was interesting, there needs to be more... we don't have a decent background in Mathematics. I think we need something done right.

Researcher: Are you saying that this was a "decent formation"? What characteristic does a "decent education" have? Would you like to make any further comments about the course as a whole?

Helena: There were mathematical concepts… The mathematical language is important, we say: plus, but it is addition […]. This kind of difference, if you do not understand it, how do you teach it? (First stimulated interview)

Helena highlights as a condition for the perspective shift the fact that she participated in a formative process that "had mathematical concepts", which reminds of the valorization of knowing the content referring to the algebraic thinking and its development. She makes clear the relation between the use of mathematical terms and concepts and the teaching that is offered, by questioning "if you do not understand it, how do you teach it?"

Kátia also refers to content knowledge, highlighting an activity experienced during the MFGAT-INTE. During the second interview, we presented an excerpt from the record of practice, where the participant described her work during the school quarter and presented the term *mathematical challenges* when asked why she had not used a problem situation (which is more common), she differentiated the two concepts referring to one of the mathematical tasks developed in the MFGAT-INTE.

Kátia: In fact, I used the term mathematical challenges because we don't just work on the problem situation, we work on other things as well. During the course [MFGAT-INTE] an interesting activity was related to the equal sign, which, for me, was just equal and that was it. The trainer taught/explained the meaning of equivalence… After the course, I felt the need to introduce this concept [equivalence] for the students, that

> both sides did not need to be exactly equal, they needed to be equivalent. (Second stimulated interview)

Kátia expresses that it was during the MFGAT-INTE that she learned the equivalence meaning of the equal sign: "for me, was just equal and that was it". She adds that, after learning a new mathematical concept, she felt "I felt the need to introduce this concept [equivalence] for the students", which makes us reflect about a possible transposition she makes of her learning to the pedagogical knowledge of the content.

Márcia shows other indications of learning, by referring to the planning of mathematical proposals, when talking about his ability to plan proposals to develop algebraic thinking, due to the MFGAT-INTE. This statement originated from the questions raised by the researcher regarding the insertion of "Algebra" in the teaching plan prepared by the school that teaches in 2018.

> *Márcia:* Until then, I was only planning plain activities, sometimes I would accept tasks planned by other teachers, and asked for an equal copy. Today is different, I may even take a copy to work on the homework, but I choose very thoroughly, and this happened after the course [MFGAT-INTE]. (Second stimulated interview)

When referring to her planning after the MFGAT-INTE, she explains her care with the selection of mathematical tasks: "Today is different, I may even take a copy to work on the homework, but I choose very thoroughly". This makes us conclude that she chooses those tasks that are related to the established goals, the ones that promote student reflection. Another sign of learning that is highlighted in her speech is "Until then, I was only planning plain activities, sometimes I would accept tasks planned by other teachers", which during the interview, enables us to understand that, before the formative process, she did not have as her planning focus proposals that enabled students to communicate different ways of thinking.

Delving into planning-related issues, based on the identification of proposals for the development of mathematical tasks in groups, the researcher asked Kátia to describe what were her expectations with this choice.

> *Márcia:* I still do this. When the students have the opportunity to talk about the problem situations among them, they feel more comfortable to make mistakes, when they are talking among their peers, they are not afraid to say what they truly had in mind, and I would go around the classroom listening to the discussions and "catching" [original emphasis] speeches here and there, and using it in favor of teaching. I give them the problem situation and determine a time for them to solve it, after that, they socialize the ways they have solved it, they come to the board and explain for the class how they have answered it. The exchange helps learning, because the different ways of thinking complement each other, and in mathematics I have learned that [this strategy] is the "ace in the hole" [original emphasis]. (Second stimulated interview)

In her speech, Kátia explains the importance of providing the students with collective moments in the resolution of mathematical tasks: "When the students have the opportunity to talk about the problem situations among them, … they are not afraid to say what they truly had in mind". It is possible to see that she recognizes that these moments foster mathematical discussion, facilitate the communication of different ways of thinking, promote the exercise of argumentation, as well as the collective construction of knowledge.

Kátia also highlights a very similar procedure to the methodological choices adopted by the trainer during the MFGAT-INTE: she says, "and I would go around the classroom listening to the discussions and … using it in favor of teaching." Beyond the discussions themselves, she emphasizes the moments of presentation of the different ways of thinking that could have appeared: "they come to the board and explain for the class how they have answered it."

By revisiting the evaluation, she had done at the end of the MFGAT-INTE, Kátia reaffirms the following signs of learning:

> *Kátia:* I have learned to plan activities that consider the students' thinking and allow me to make interventions so that they can reflect on what they have done, and this makes them use mathematical properties and their generalizations. (First stimulated interview)

Kátia shows another aspect that seems relevant for planning when she affirms considering the students' production to identify their knowledge and mistakes, and based on that, "to make interventions so that they can reflect on what they have done". This seems to occur with the aim of contributing to the construction of student learning and shows us that Kátia seems to have understood that the planning must be closely connected to the teacher's actions, students' needs, and to the curriculum that must be developed.

Discussion of Results

We have chosen to present the discussion of results in two blocks: in the first one, we bring forth the signs related to the MFGAT-INTE learnings identified in the categories "Reliving the MFGAT-INTE" and "Learning of the MFGAT-INTE"; while the second block highlights signs that refer to the participants' teaching practices, identified in the category "Teaching Practice after the MFGAT-INTE."

First, among the signs of learning referring to content knowledge (Shulman, 1987) throughout their "Reliving MFGAT-INTE" and "Learning of the MFGAT-INTE", even though the participants mention learnings related to Generalized Arithmetic (Blanton & Kaput, 2005; Chimoni et al., 2021; Pincheira & Alsina, 2021) —such as the work based on the concept of numbers and the properties of operations, it seems that the most relevant learning was related to the equivalence meaning of the equal sign (Kieran, 1981; Barboza et al., 2020), as mentioned by Kátia, for instance. Besides constructing the concept, especially in Kátia's case,

the participant teachers started to insert it in their teaching practices (Ponte, 1999), making a transposition from learning about mathematical knowledge to the pedagogical content knowledge (Shulman, 1987). This reminds of a widening in the teacher's "know-how" (Ponte et al., 2012).

Also related to professional knowledge (Shulman, 1987), still in relation to the specific content knowledge, but now, when regarding how teachers become to understand algebraic thinking, we noticed teachers starting to defend the approach of algebraic thinking already in the early years of elementary school (Blanton & Kaput, 2005), what seems to us to be an amalgam between specific knowledge and pedagogical content knowledge. These learnings also have impacts on the curricular knowledge (Shulman, 1987), since teachers recognize that it is possible to include and discuss algebraic thinking in the first grades (Squalli & Bronner, 2017; Rittle-Johnson et al., 2018; Chimoni et al., 2021), especially through the identification of mathematical tasks suitable for this end (Kieran, 2011).

In addition, considering that these knowledge was mobilized based, among other things, on the analyses of the students' responses (Ball & Cohen, 1999), we observe that it is a knowledge guided towards action (Ponte, 2012) or, in other words, built on and based on practice (Ball & Cohen, 1999), which can reverberate on the teachers' practices. Their concern with presenting the mathematical concepts related to the development of the algebraic thinking using suitable terms (Ponte et al., 2012), similar to what was experienced in the MFGAT-INTE (Liu & Phelps, 2020), is a sign of the widening of content knowledge and, at the same time, of the pedagogical content knowledge (Shulman, 1987).

Our results have also shown that the participants had changes their general pedagogical knowledge (Shulman, 1987), for instance, when they express having started to be concerned with the methodological choices, similar to the exploratory teaching approach (Ponte, 2014), specially related to the moments of collective discussion among students (Blanton & Kaput, 2011; Ponte & Banco, 2013; Torres et al., 2022).

Second, in the signs of learnings related to the "Teaching practices after the MFGAT-INTE", we have observed changes related to planning (Serrazina, 2017), such as prioritizing proposals that enriching the students' knowledge, that promoting reflection, and that contribute to understanding Mathematics. We notice also the concern with selecting challenging mathematical tasks (Cyrino & Oliveira, 2011) that enable the expression of different ways of algebraic thinking (Jesus et al., 2020), which always appears connected to the objectives established by the curriculum (Serrazina, 2017). We still observed the promotion of teaching practices that engage students (Ponte et al., 2012) and awaken positive feelings in relation to the development of the algebraic thinking.

In the teachers' reports, it is possible to find elements that agree to the exploratory teaching approach (Ponte, 2014), namely: (a) promotion of discussions in small groups, aiming to encourage students to argue and justify; (b) teachers going around the classroom to observe the work developed in the small groups;

and (c) concern to ensure of collective spaces to socialize the discussion done in the groups, as well as the concern with systematizing the socialized knowledge.

Finally, when basing lesson plans on the students' knowledge and mistakes (Serrazina, 2017) shows that these are the starting and finishing point of the teaching-learning process, since it highlights the importance of content knowledge and its relation to the knowledge about students (Shulman, 1987). Besides that, we observed that teachers were concerned about guiding and reviewing mathematical tasks and reformulating questions (Ball & Cohen, 1999), as a strategy to learn more about the students' ideas and understanding (Cyrino & Oliveira, 2011).

Final Remarks

Aiming to answer the guiding questions of this paper Which professional knowledge regarding the algebraic thinking have the three teachers reconstructed after the in-service teacher education process? and How has the professional knowledge regarding the development of Algebraic thinking in the first grades been expressed in the teachers' practices after the in-service teacher education process?, we have used the Grounded Theory (Charmaz, 2009) in our data analyses and, after many types of codifications, constructed the Core Category, which enable us the understanding of how the teachers perceived they professional learning during the MFGAT-INTE and introduced these learnings in their teaching practices.

Regarding professional knowledge concerning the algebraic thinking that the participants have reconstructed, the results indicate that (a) they have started to understand that Algebraic thinking can be developed in the first grades of elementary school (Chimoni et al., 2021); (b) glimpsed possibilities for that development, based on the exploration of the properties of numbers and operations (Chimoni et al., 2021; Pincheira & Alsina, 2021) and of the equivalence meaning of the equal signs (Barboza et al., 2020; Kieran, 1981); (c) started to also recognize the importance of using mathematical concepts and terms pertinent to the algebraic thinking during the development of their classes (Shulman, 1987).

We also highlight that the teachers perceived the potential of reflecting over students' responses as a strategy to understand their mistakes and knowledge, which can help them propose more adjusted interventions in the classroom (Ball & Cohen, 1999). Based on the experience of the in-service teacher education process, they have perceived not only the effectiveness of participating in mathematical and didactical discussions with peer for the construction of knowledge, but also of the validity of socializing and systematizing the discussions they had among students in the school context (Ponte & Branco, 2013).

Regarding the way the knowledge has been expressed in their teaching practices after the in-service teacher education process, the main sings identified are connected to actions used to plan classes, an important aspect of teaching

practices (Ponte & Serrazina, 2004). Among the actions, teachers have shown concern with selecting and developing the mathematical tasks proposed to students (Torres et al., 2022), the adoption of methodological strategies that contain elements of the exploratory teaching approach (Ponte, 2014), mainly with focus on moments of autonomous work among students and on the collective discussions, as important aspects to socialize and systematize knowledge (Ponte & Branco, 2013).

In sum, we recognize that the design of the MFGAT-INTE seems to have indeed influenced the teaching practices of the participants with focus on mathematical and didactical knowledge. Despite having conducted this research in retrospective, once it sought to recover information of an in-service teacher education process that happened three years before our investigation, we understand that the methodology used to collect (Tarozzi, 2011) and analyze (Charmaz, 2009) data has enabled to investigate both the design and structure of the MFGAT-INTE and the teachers' practices in depth. With that, unlike the results presented by Silva et al. (2022) who have discussed the perceptions of the teacher educator, in this paper it was possible to bring forth the "voice of the participants" in relation to their own perceptions regarding their learning.

Overall, we have observed through this study the relevance of investigating "whether" and "how" the learnings enabled by an in-service teacher education process are incorporated into the teaching practices of the participants, a gap that was initially identified in our literature review (Avalos, 2011; Darling-Hammond et al., 2017; Fiorentini et al., 2016; Moriconi et al., 2017).

Our work is focused on the development of algebraic thinking in the first grades of elementary school, so we would like to encourage other researchers, interested in other mathematical areas and in other teaching levels, to use the methodological design of this study. We believe that this design can emerge as a fruitful tool for other investigations on signs of teacher learning, after participating in in-service teacher education processes, in their teaching practices.

We recognize that, like any other research methods, the interview has advantages and limitations. Even though it has made it possible to study the phenomenon in depth (Gil, 2008; Marconi & Lakatos, 2003), the data collected may be inaccurate to some extent. On the other hand, in line with Desimone (2009), when questions focus on a specific teachers' practices, cover a clearly delineated and understood period of time, as well as has a well-constructed and managed script, interview is one of the methods that can provide explanations, examples, and hypotheses, and can promote critical reflection and deepen the focus.

Finally, despite recognizing the limitations of this study by investigating teaching practices through documents and interviews, we encourage other researchers to develop studies may complement these results by analyzing teachers' practices through observations and/or interventions conducted in the classroom.

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