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LOGIC'S RULE (STAYING IN THE ZONE)

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Abstract. The paper explores a Frege-inspired conception of what concerns the nature of logical laws. A basic idea is that logic must 'take care of itself', i.e., nothing topic-specific could play the role of ground for a logical law. In this Fregean mood, we'll see as isolating logical laws requires to separate what truth is from taken to be true. Such a path will lead us through a discussion on the role of representation in its relation to the true and the false. Logic's proper justification (proof) concerns returning to more basic truths, but justifying such truths is something beyond its scope. According to Frege, logical laws are grounded solely on being true, which brings us back to the question about these laws' relation to topic-specific matters, as well as their revision.

Keywords: logical laws • proof • truth-ground • revision of logic

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When, in 1893, Frege sat down to write the preface to *Grundgesetze* (1), he read as though at the end of patience, unbridling his prose accordingly:

> [T]he laws of being true are but boundary stones set in an eternal ground, which our thinking may, to be sure, overflow, but which are immoveable. And just because they are thus, they are the standard for our thinking if it is to reach the goal truth. They do not relate to our thinking as grammatical laws relate to language, so as to express the essence of our human thinking, and changed with it. (1893: xvi)
>
> One who acknowledges a law of logic has thereby…acknowledged a law which prescribes how judging is to be done, wherever, whenever and by anyone who may engage in it. (1893: xvii)

I distinguish two ideas here. The first one is that "logic takes care of itself":

> Logic can only answer the question why and with what right we acknowledge a logical law as true by leading it back to other logical laws. Where this is not possible, it must remain without an answer. Stepping outside logic one can say [such and such]. Such is not to cite a ground of *truth*, but rather of our *holding* [the law] true. (Ibid)

The laws of logic govern *all* truth-or-falsehood, all countable instances thereof. Therefore, they concern nothing but what belongs to all such countables merely by virtue of their being in the business of being true; merely by what belongs to the phenomenon of being true (or false) as such (as I will put it, only what is 'topic-neutral'). Hence, the idea is, nothing outside of logic itself, and nothing topic-specific could possibly ground any such law (or the truth that it holds). No law of truth *could* turn (or have turned) in any way on how things are otherwise. That form of generalized inference may be suspect. However, as I will argue here, the point holds for logic.

The second idea, as per the second citation, is that we should not look for the grounds of a law of logic in anything peculiar to thinkers of a certain provenance, thus in nothing specifically human, cultural, or psychological. What a law of being true is, the idea is, is no business of any such study. At most, we can uncover the etiology of our recognition of it. Psychology, ethology, anthropology are all useless here precisely because logic is intrinsically thinker-neutral, something in common to all rational beings.

Above, Frege makes his point in part through a comparison between logic and grammar. Grammar, he correctly observes, is, in principle, changeable: what was ungrammatical yesterday may be grammatical today, and vice-versa. A law of logic, by contrast, is what holds timelessly, if it holds at all. Still, even though language and logic are very different phenomena, and do not share laws, words (in a mouth, at least) are means of thought-expression. And their syntax may provide clues to a particular decomposition of a thought in which it is thus being presented. Different enough, syntax might make for clues to materially different things. Or so one might think. On the other hand, one might also think, reasonably, that since logic is thinker-neutral, this observation cannot come to very much.

There is an issue worthy of discussion here, but providing the right setting for this would be laborious beyond the ambitions of this essay, which will thus treat only the first of our two initial ideas.

1. Deflation

Frege's unbridled prose (the privilege of a preface) may make logic seem despotic, ruling all thought with an iron fist. In the *Investigations*, Wittgenstein deflates a similar impression of despotism. On §437, he writes:

> A wish seems to know already what fulfills it, or would; the 'proposition', the *Gedanke*, what makes it true, even when not present at all. Whence this determination of what is not yet there? This despotic demand? "The hardness of the logical must." (Wittgenstein 2009, p. 136)

A short while later (§458) he concludes,

> If an order orders 'Do such-and-such', then following the order is called 'doing such-and-such'. (Wittgenstein 2009, p. 140)

Pia wishes she had a persimmon. Now a certain historical episode occurs. Sid acts. It *may* be that his so acting is, *inter alia*, fulfilling Pia's wish. But how can Pia's wish 'Would that I had a persimmon' dictate that just *that* bit of history is what fulfills it? 'There's despotism for you.' Nevertheless, Wittgenstein points out, it is not like that. What is at stake here is just a rule of grammar: what Sid needs to do to fulfill Pia's wish is just something, whatever it may be, which counts as giving her a persimmon (on relevant understanding of this—a ripe persimmon is not, e.g., something one wants to hold in their hand). The rule is just: it counts as fulfilling a wish for a persimmon just in case it counts as giving her a persimmon (and so she wished). The question what *would* count as such has not yet been addressed. Calling such despotism can only be *zum Spass*.

A grammatical rule, such as the above-mentioned, says too little to exercise any very impressive despotism. Perhaps, despite the impression Frege's prose might give, laws of logic also say too little to be despotic in any impressive sense. Perhaps what they say is unimpressive enough even for them to be granted immunity. Such alternative unbridling points to the idea to be explored here.

2. Frege's Road

To isolate the law-governed in any phenomenon, one must prescind from all the variables over which the laws sought do not generalize, as for Newton one prescinds from color in searching for a law connecting acceleration imparted to force applied. On the road to laws of being true, Frege passes through a series of such steps of extraction, starting from the notion of being true itself.

2.1. Being True

For Frege, the first step on the road to logic's laws is to extract (or detach) the phenomenon of *being* true from that of, as he puts it, *holding* true, that is, standing in some particular way towards some given (suitable) countable engager in the business thus extracted (being true). I rate Frege's separation of these two businesses as the most important contribution he offers to philosophy (although it is seldom taken up). To hold true is to commit. There are two main such ways: saying-so and thinking-so. The strategy (the contribution) is firstly to investigate what there *is* to commit to, before asking what it would be for a being to be so committed (this prior to the further question how such might be enabled). Towards what does one stand

in so committing? For now, I will just say: to a suitable protagonist in the business of being true, namely, to a countable truth-or-falsehood. The next sub-section will expand this.

Being true (there being such a thing as that) comes as a package with a particular notion *to represent*, namely, to represent (something) *as* being such-and-such. It comes thus packaged precisely because such representing is non-factive. Let us contrast a case of factive representing. Sid's lack of ambition may represent a childhood lack of irony in the parent-child communication mix. His burgeoning waistline may represent the depth of his devotion to malt and hops. But only if there was such paucity of irony, or such depth of devotion.

By contrast, Pia is free to speculate whether Sid's childhood was irony-poor or not. Representing as thus makes room for falsehood (for representing things as they are not) and hence for truth (for representing things as they are). Therefore, the package here gives truth one task to perform. Suppose you are informed that Sid's unambition represents childhood irony-deficiency. One thing you end up knowing is that Sid's childhood was irony-deficient. If you are merely told that Pia represented Sid's childhood *as* irony-deficient, so far, maybe it was, maybe not. But if you are informed that Pia represented *truly*, it is then restored. You have been informed that, *inter alia*, his youth was irony-deficient.

Representing-as is the joint work of two parties. There is a representer, and there is that which is thus represented *as* being some given way. (Call these *party of the first* and *party of the second*, respectively.) The representer's contribution to the enterprise is the way the represented is represented as being. The represented's contribution is simply, in being as it is, so to be or not. If to be true is to represent (-as) truly, truth is thus two-valued. That two-party enterprise, that is, where it succeeds in representing-as, has two outcomes: things being, or not, as represented. What is not to exclude the enterprise, on occasion, failing to deliver a result, or the result it delivers being very hard to see. With this much we get Aristotle's platitudes (to be true is to represent things as they are, to be false is to represent them as they are not). Put another way, *true* is an identity under predication—*over truth-or-falsehoods*, that is.

One further note: there is just *one* represented (party of the second) on which *any* two truth-or-falsehoods make their truth turn. It is just this which makes being true *punkt* contrast with being true *of* such-and-such. It may be true of Sid that he is indolent while such is not true of Pia. But if it is true that Sid is indolent, such does not leave any open question, 'True of what?', or at least none whose answer distinguishes this truth from any other. Sid being indolent (if he is) is just the way things are (what can be said of any truth). If it is true *of* the way things are, such can be said equally well of any truth. What is either true or false is, at least in first instance, what represents *things* as they are or not, equally the way things are as being thus and so.

2.2. Der Gedanke

As primary protagonist in the business of *being* true, Frege takes what he calls a *Gedanke*. It is what I call *a* thought here. It enjoys ontological priority: all other protagonists (and there will be more) are to be defined in terms of their relation with it. Its business is to be completely true or false outright. It is important to emphasize: if it is not the only truth-*bearer* (*some* content-*bearers*, e.g., utterances may also be called true), it is distinguished by this: if it is true, it is thus the truth there is. *Mutatis mutandis* for falsehood. Therefore, it is a truth-or-falsehood, and a *thinkable*, something to be thought in thinking true (or so). Hence, it is something over which for *true* to be an identity under predication, as a content-bearer is not.

Frege describes a Gedanke as 'that by which truth can come into question at all'. That is, it provides something determinate of which to pose coherently the yes-no question 'True?'. It is to be *just* what does *precisely* that. It is *one* definite way to make truth turn on how things are. It has no features other than what contributes to that. So, it is not visible or otherwise perceivable. It has no spatio-temporal career. Hence nor can it contain indexicals or demonstratives. There is nothing it looks like—as opposed, e.g., to a sentence, for which, in its role as content-*bearer*, it is crucial that it have some sort of perceivable form (by which, of course, identity under predication is lost).

So, as Frege puts it, *that* the sun is rising, as opposed to its rising, is not something that can be before the eyes, *inter alia*, because it has no spatio-temporal location. There is another way of putting this point. A Gedanke has generality (or is one). Such lies in its business of representing-as. Representing-as is what still could be done even if things were not as represented. Such is its distinctive feature. Where there is successful representing-as, the represented, here the way things are, decides whether there is truth or falsehood, simply in being as it is. But there must be something to decide when things *would* be as represented. What is distinctive about *it* is its capacity to answer questions when things *would* be as represented (and/or when not)—if the thought is that Pia swims, how might things be other than they are while still such that she does, *casu quo* does not.

A Gedanke thus has, or is, a generality: something which determines when things *would*, when not, be as represented. A generality is something to be instanced or not by the particular cases over which it generalizes—in the case of a Gedanke by the way things are, no matter how this might happen to be. Such generalizing is part of the bargain with just that by which there is room for *truth* to come into question at all (the non-factivity of representing-as). And, of course, neither generalizing nor instancing are visible occurrences.

A Gedanke is in the business of representing-as. Its role is to engage in representing. But it does not engage in this in the same aspect of the verb *represent* as does a

thinker (or taker-so). Pia may represent Sid as having been at the lock-in last night. Her doing so belongs to history, it is an episode. She may thus *commit* to his so being, pretend to authority as to that. Pia is the sort of being to assume commitment. A Gedanke is not. It is rather something in particular *for* a being such as Pia to commit to. One could say: it is *the* representing of things as being thus and so, thus something there is to do. Consequently, a Gedanke cannot have an etiology. Pia said so, perhaps, because Zoë asked. What Gedanken do is not brought about by anything.

If there is representing-as, there is truth and falsehood. Accordingly, there are truths and falsehoods. So, there are Gedanken.

2.3. Decomposition

A Gedanke (thought) provides something for laws of being true to govern. But a law needs variables (parameters) over which to generalize. There must be recurrent features in what it governs, to which it may be sensitive. A thought must be relevantly the same as ranges of others in identifiable respects. For this, Frege tells us, a thought must be decomposable into proper parts, where a proper part is a partial doing of what the whole thought does; a recurrent thought-element in that it may be a partial doing of what many thoughts do.

Consider the thought that Pia swims. It makes truth turn on whether Pia swims. It would be a partial doing of this to make truth turn on *who* swims—something done in representing *something* as a swimmer, thus something done in many thoughts, in some perhaps truly, while, for all that, in others not. Therefore, we have a recurrent thought-element. It is a *proper* element in its business the truth *of* and not that of truth outright. With which we are on our way towards a decomposition of the thought that Pia swims.

To complete this partial doing to the relevant whole, we need another partial doing which makes some given item, in this case Pia, the one of whom (in this thought) that other partial doing predicates. With this, truth-of becomes truth outright, so we have a Gedanke. Here, of course, these two partial doings must relate in a given way, so the whole decomposition must be structured accordingly. Therefore, we may say: A decomposition of a thought is a *structured* collection of partial doings, whose joint doings, each so relating to the others, is precisely the doing of that which the whole thought does. The whole thought makes truth turn in a certain way on how things are; the (structured) joint doing of those partial doings which make up the decomposition is doing *just* that.

According to the above, and as Frege insists, neither a designatum (object) nor a predicable (a given way for something to be) is itself a proper thought-part (an element of a decomposition). Each requires something in the thought to *make* it relevant to the thought as some proper part does. It is not on for Pia to be a thought-

element. Her having a spatio-temporal career, for one thing, disqualifies her. But a thought can be *about* her if, but only if, something which *can* be in it, some proper part (some subthought) makes it so.

Frege insists that a Gedanke is, in general, decomposable in many, sometimes mutually exclusive, ways. There are many forms of constructing a way of making truth turn on how things are in *just* that way by which a thought is identified as the one it is. The abovementioned requirement of being a decomposition of a given thought may be satisfied in many, sometimes mutually incompatible, ways. To put it in another way, a thought is not a construction of given 'building blocks', or, again, thoughts do not have a syntax, that is, a domain of thoughts (thinkables) is not generated, as the sentences of a language may be, by given rules from a given vocabulary.

A given decomposition presents a thought as predicating given things of given things (in a certain structured way). A given expression of a thought may also so present it. Such an expression of a thought is called an *Aussage* (at least by Frege). Therefore, we can say: the relation *Gedanke-Aussage* is one-many. At least it makes room for the idea that there may be many different ways of counting thoughts (when one occurs twice, when two occur once), according to the relation between *Gedanken* and *Aussagen* most apt for an occasion.

One thing Frege insists on (*vide* 1919) is that any decomposition of a thought must contain one proper element which is in the same business as the whole thought, namely, the business of representing-as. A representer which is a *proper* thought-element is, by definition, in the business of truth-of and not truth outright. It certainly comes as natural to us to suppose this. But of all his demands, this seems the least apodictic.

2.4. Redaction

The laws of logic—as Frege conceives them, and as we know them today—govern the business of *being* true but do so only insofar as they hold purely by virtue of what being true is as such. "The laws of being true unfold the notion *true*." (1918: 59) Hence their claim to topic-neutrality. Their parameters—the variables over whose values the generalized—are to be found within the bounds of the recurrences (samenesses) we have found so far. But further filtering, so to speak, is called for.

The content of a whole thought (the way it represents things (the way things are) as being) is proprietary to it, *precisely* that which distinguishes it from any other thought. Such was the motivation for decomposition. What a decomposition is meant to deliver, by virtue of the proper thought parts in which it consists, is *recurrent* features of representings-as. For example, the thought that Pia swims is decomposable into a designative subthought and a proper predicative subthought in a familiar way. The proper predicative subthought, like the whole thought, is distinguished from any

other of its kind precisely by the way it represents *an* object as being. Whereas, for all of this (and by this criterion of identity), it recurs in an indefinitely extendible range of thoughts. Thence its presence on a decomposition is a recurrent feature within the domain of truth-or-falsehoods.

Nevertheless, for all of this to last, the just-mentioned recurrence is *not* one which a law of logic, i.e., a law of being true, might be sensitive to, for a law of logic was meant to be a law which held simply by virtue of what being true is as such. (In Frege's terms, it 'unfolds' the content of *to be true*.) But the recurrence just mentioned depends not just on features of being true itself, but also on features of what is predicated—here a topic-specific matter, hence not logic's business. To rectify this situation we need to do something which I will call *redacting*. Starting with a decomposition in which this predicative subthought occurs, we must, as it were, black out, or otherwise prescind from, all that distinguishes this subthought from any other of its ilk (in this case any other one-place proper predicative subthought). And we must perform the same operation on all other proper parts of the decomposition in question.

When we are finished with so redacting, what we are left with is a *sort* of decomposition of a thought found, not just in the thought we started from, but in an indefinitely extensive range of others. Exploiting the decomposition this way gives us a form of prescinding from precisely that in the thought we so decomposed which is proprietary to it; just that by which it is distinguished from any other thought. Now, there is something logic might speak to: a form there is for *a* thought to take—what might reasonably be called a *logical* form.

2.5. Truth-yielding

Frege's most important contribution to philosophy, in my view, is the separation between *being* true and *holding* true, a move which might generalize into a motto: *Grounds* (of the truth of a thinkable) before *etiology* (which enables us (or one) to think it). But a second contribution is a close runner-up. It is his separating out into two different sorts of grounds of proof. In 17 *Kernsätze zur Logik* he puts the matter thus:

> One justifies a judgement either by tracing it back to already recognised truths, or without appeal to other judgements. Only the first case, consequence, is the object of logic. (*Kernsatz* 13 (p. 190))

The kind of justification which he reckons as logic's business is what could be rendered in terms of a sequence of sets of thoughts (Gedanken) with the property that for any adjacent pair in the sequence, for all thoughts in the first member were true, so, also for the next (its successor). If an ordered pair of (countable) sets of

thoughts has this property, let us say that the first relates to the second in a truth-*transmitting* relation (or simply truth-*transmittingly*).

If our interest is *proof*, something which exhibits, or identifies, the grounds on which a truth rests, armed only with the idea of truth-transmission, we may have cast our net too wide. For example, any set of truths of arithmetic transmits truth to any other set. But not any such transition *proves* that to which truth is thus transmitted (on any interesting notion of *proof*). Here logic comes into the breech. Suppose that for each pair in a sequence of truth-transmissions there is a 'basic' law of logic by which the sequence is underwritten as truth-transmitting. (The notion 'basic' can remain open here other than to say that basic laws are few.)

Perhaps our net is still cast too wide. Frege thought so in any event. True, he agrees, there is something we have already characterized. It is what he calls deduction. However, he insists, for a deduction to *prove* what it concludes with, it must start from 'already established truths'. One might carp at such insistence, pointing out that, in the right sort of logic, a *logical* truth can be derived from (or as resting on) nothing. But if we bracket this carp (or logical truth if you like) with Frege's insistence, we arrive at just that phenomenon Frege excuses logic from treating that second case of justification (identifying grounds). To adumbrate, I will call it *truth-yielding*.

Let us rehearse the point we have arrived at. As things now stand, what *laws* of being true (aka logic) govern are not thoughts (Gedanken), but logical forms. Logical forms are neither true nor false (though perhaps any thought of some given form is true, *casu quo* false). To arrive at truth-or-falsehoods, a logical form must be 'unredacted', each proper part invested with proprietary content of a sort whose redaction would leave such a proper part behind. For any logical form (a form for a thought to take), there had better be indefinitely many distinct unredactions thereof if there is to be a (non-degenerate) business of being true for laws of logic to govern.

(Note: Some might think that there can be logical forms which ask for no unredaction, and that in particular logical truths are precisely such forms—*forms* for a thought to take which, as such, are already expressions of a thought. Quantification is the relevant trick here. But if it is meant to obviate the need for unredaction, such can only be at the price of leaving the business of instantiation as an exercise for the reader. (The same engagement with the business of truth-yielding remains.)

With unredaction we enter on business to which precisely that sort of justification from which logic excuses itself is critical. For the sake of example, suppose we unredact that form redacted from the thought that Pia swims (is swimming) into the thought that Sid is hungover. We might hope, for example, that, with suitable finagling with logical form we might arrive at a companion thought, that one who is hungover is in need of a Jeeves special. If, throughout, we have thus unredacted to genuine truth-or-falsehoods, and moreover throughout to *truths*, and if, further, our first unredaction also occurs as first term in the unredaction of the second thought,

logic tells us, it follows that Sid needs a Jeeves special. But as to whether those conditions are satisfied—whether, e.g., it is sufficiently determinate which cases would be ones of someone being hungover, such is a question asked of the phenomenon of truth-yielding, just that which Frege excuses logic from treating.

But suppose we now ask whether this *is* true. To be hungover is (allegedly) a way for an item to be. It is, therefore, generalized (in a presumably determinate way) over particular cases of someone being as he is. There are indefinitely ways an item may be while, for all that, still hungover (thus yielding something to which Frege's basic law 1 applies), *mutatis mutandis* for not being hungover. For what is predicated to be so of Sid, his being as he is must be one such case of the just first-mentioned: for him to be as he is must be (one thing which would count as) an item's being hungover. If it is, then a transition from Sid's being as he is to the thought that he is hungover would end in truth, though beginning with something neither true nor false (call it, if you like 'Sid's present condition'). Such a transition is what I am here calling *truth-yielding*, the 'yielding' meant to distinguish it from truth-transmitting, though both end in truth.

If there are to be thoughts, either true or false, there must be such a thing as truth-yielding, for there *is* such a thing if there is such a thing as what it would be for a given predicative subthought (e.g, what predicates being hungover) to be true, hence if there are to be such things as predicative subthoughts at all. But what are the laws which govern truth-yielding—*laws*, such as those of truth-transmission, and not mere maxims or hints? The answer to this, I suggest, is: 'Few and banal if any at all'. And, I suggest, it is precisely for this reason that Frege excuses logic from concern with this phenomenon.

I will not mount a full defense of this view here. But why should one expect lawlessness to be the rule within the realm of truth-yielding? I have some remarks. Within a decomposition, the distinction between the two terms of the relation of instancing, or being a case of, shows up in the distinction between objects and concepts, or the role in the business of being true assigned to each. (A concept, or better still, what it is of, is here a way for an object of truth-of (as opposed to truth outright) to be). In 1882, Frege assigned this distinction the highest importance for logic. In logical form, it shows up in the distinction between designation (or *designatum*) and n-place predication (i.e., $n \geq 1$). What Frege stresses, at least in 1882, is that relations between an object and a concept are utterly different in kind from those between concepts, the one irreducible to the other.

Suppose we now ask what a *law* of truth-yielding might look like. Just as a law of truth-transmission identifies particular cases of such (the terms of such cases marked by logical forms), so a law of truth-yielding (were there such) would identify cases of such—ordered pairs of an object (or n-tuple) and the content of a (proper) predicate (a way for an object/n-tuple to be) in which first item instanced second. It would do

this given that the terms of the prospective case were each identified by, *inter alia*, marks of the sort which the law was sensitive to (something corresponding to logical form in the case of truth-transmission).

But a *law* is a generalization. It tells us that whenever a certain condition is met, there is a certain consequence. Therefore, a law of truth-yielding might, for example, state a condition under which an object being as it was would confer truth on a predication of it that it was hungover (or was a daybed, or an IPA). An object being as it was *would* do this, according to the law, when it satisfied a certain condition C. However, a *law* is a generality. So is condition C. Then what the law would tell us is that, e.g., Sid's being as he is would count as someone being hungover just in case it counted as satisfying condition C. If condition C is trivial (it is just that Sid's being as he is counts as a case of someone being hungover), we thus never traverse the distance between a generality and its instances. If C is non-trivial, then, bracketing its being unlikely that there are any such non-trivialities, the question is only displaced: a question about one prospective case of truth-yielding is exchanged for another. In Frege's felicitous expression of much the same point,

> One moves around on the same level, but one never steps forth from the one level to the next. (1892: 35)

Elsewhere (e.g., the letter to Linke of 1919) Frege speaks of a 'concept' as having 'Merkmale' by which it is distinguished from any other. The relation between a concept and a Merkmal is a relation between one generality and another. To take Frege's 1919 example, the concept *blue silk ribbon* has the Merkmäle *blue*, another *silk* and *ribbon*. This idea *can* give the false impression that 'the conceptual can take care of itself' (so to speak). That is, for a concept to be what it is (a certain way for something to be), it just *is* for it to occupy a certain place in a structure of concepts, it being, perhaps, one among many.

That this is false shows up, not in some sophisticated theorem of model theory, but in the very idea of such a theory at all. In a standard model theory (regardless of order), a model structure for a given set of formulae of some calculus is (for simple standard first order) an assignment to each 'predicate symbol' occurring in those formulae of some subset of a given set (the domain of the theory) as its extension, together with some rules for calculating in terms of the logical constants in the formulae, the 'truth-value' of each.

The crucial point here is that it matters not at all to any such assignment of truth values *what* objects occur in the domain of the theory (just so long as there are enough of them). Sometimes a model-theoretician may *say* what these items are to be (e.g., letters in an infinite alphabet), and such can have heuristic value. But strictly speaking, it does not matter at all what he says. The truth values to be assigned in a given model structure remain the same for all that. A countably infinite set of letters

and a countably infinite set of cats (if we can construct such a thing) would yield the same theorems. There is much more to say as to what may matter to a concept's identity other than merely its relation to given other concepts (fixed in the same way). Such is a story for elsewhere. But we can by now see Frege's idea about the fundamental difference between ways for objects to relate to concepts and ways for concepts to relate to each other so as to unfold. There is a sort of business, namely, truth-yielding, which is none of logic's concern, though without which there would be nothing for laws of being true to govern.

In sum, logic's laws govern logical forms. The grounds of their holding are to be found in the nature of the constants in terms of which these forms are defined. Their application to a given topic-specific domain requires suitable unredactions of given such forms; unredactions, that is, into what are in fact truth-or-falsehoods. The business of unredaction is left as an 'exercise for the reader'. It is work in the fields of truth-yielding, a 'lawless' phenomenon as per above. Where an application leads to wrong results, the unredacting remains the presumptive culprit pending a coherent challenge to the grounds on which the truth of the laws depends.

Caveat: Frege offers us the conception of logic just presented, explicitly at each point. But the 'logic' he constructs for *Grundgesetze* is far from fitting it, even in proper parts. So, crucial exegetical issues are left undiscussed here. I say only: The conception he puts at stake here is what the present discussion requires.

3. Grounding and Exercises for The Reader

Frege's unbridled prose—the privilege of a preface—may create the impression that logic rules with an iron fist, despotically. If so, then with what right? Justification seems called for. But if nothing topic-specific, so nothing outside logic itself, could bear at all on whether its laws held, then, it might seem, there could be no such grounding. Following Frege's path, what laws of being true turn out to be are laws of truth-transmission whose parameters—what they deal in—are logical forms: a transition from thoughts of given forms to thoughts of given forms is truth-transmitting, such a law tells us, if those first forms relate to those second in a certain way (specified by the law). And what is truth-transmitting by such a law is so, the idea is, purely by what being true (equally, representing-as) is as such. Therefore, we would expect the grounds of such a law to lie somewhere in what being true is as such. And so they do, as well will see now. On which grounds a law of logic (of the sort Frege helped make familiar) is, on the one hand, unassuming, but, on the other hand, conclusive.

Frege, were he heeded, would have performed for us that feat put by Wittgenstein as 'rotating our investigations around the axis of our real needs'. His message, in slogan form, 'Grounds before etiology'. Before a philosopher posits enablers, he

should have a very firm grip on what it is that is to be enabled. We can, though, distinguish two notions of *ground*, or, perhaps, two different things which might be grounded. *Knowledge* that such-and-such may rest on such-and-such grounds. Here the role of a ground is to confer such status on a knower (or to be available therefor). Then, there are grounds of the *truth* of a (true) thought, or of such-and-such being so. In the case of truth-yielding, a case to be made that, e.g., one's being as Sid is *ought* to be counted as one's being hungover.

Grounds of knowledge need not be grounds of truth (or being so). The grounds on which I *know* that Sid is hungover may be that I have it on good authority that he is. (Pia told me.) At least such is so if there is such a thing as knowledge by testimony. But that Pia says so is not a ground on which the *truth* that Sid is hungover rests. I do not mean to suggest, however, that this distinction makes for a neat dichotomy when it comes to sorting particular cases. The second sort is what is at issue here.

3.1. Grounding

A law of logic, Frege tells us, unfolds the notion *true*. It holds, in first instance, logical forms, and then, thereby, any unredactions of those forms into *Gedanken* (truth-or-falsehoods) decomposable into *those* forms (among others). So, the law's parameters (equivalent to force and mass for mechanics) must be drawn from a fixed vocabulary. This vocabulary will contain, firstly, the forms of proper thought-parts into which a *Gedanke* might decompose—e.g., the form of a one-place predicative subthought, or the form of a designative subthought; secondly, one or more 'logical constants'. Such constants may be viewed as operators, operating on one or more forms for a thought to take, to yield a new such form.

The forms on which a constant operates are, in first instance, fully redacted, since it is to such *forms* as such that logic applies in first instance. However, by virtue of such a base case, they apply (at the slightest remove) to any such form given that parts of it are unredacted into proper parts for a decomposition to contain. Unless all unredactable parts are fully unredacted, the result is still a form for *a* thought to take.

The role of a given constant in inference might be thought of as fixed by a rule to introduce it , and a rule to eliminate it, terms in the sequence of truth-transmitting transition for any given deduction. Here, though, we will view the behavior of the constants we will consider as fixed by the relation between the 'truth condition' of the yield of any operation of it to that (or those) of what was thus operated on. (Scare quotes because what constants of present sort form, in first instance, are logical forms (forms *for* a thought to take). These are not in the business of being either true or false. So, there is no condition on *their* truth. But there may be a sort of truth condition on any (full) unredaction of something of that form.

The first sample of a logical constant I will consider here is what I will dub 'Horseshoe' and write '__⊃__'. (One might also write it '⊃(__, __).) Let [Δ] and [Γ] each be a form for a thought to take, and let A fully unredact Δ and idem B, Γ. Then [Δ]⊃[Γ] is a form taken by a thought just in case it represents things as not being such that they are that way Δ represents them, while not that way which B does. So, by definition, the thought yielded by the operation is true unless A is true and B is false.

Here is a law governing Horseshoe. (It might be seen as capturing, in law form, Horseshoe-introduction.) And it might thus be written:

A ⊃ (B ⊃ A)

Since, we have decided, laws govern logical forms; what has been written is the logical form of a thought to take. Strictly speaking, this is neither true nor false, hence not yet the *statement* of a law. But I am supposing it to occur here with a certain intention attached to it (the intention of the author of *a* logic (a theory) in which such writing occurs. The intention is that the occurrence of this string of symbols here is to be understood as committing to the claim that any thought of this form is true.

So understood, *is* the law true? It takes only a very simple calculation—some *very* elementary mathematics—to establish that it is. Therefore, if horseshoe is well-defined, and at least given that very elementary mathematics, the truth of the law is thereby grounded. And on those grounds, it is hardly open to doubt. Basic law 1 is as well-grounded a law of being true as anything could be, thus well-grounded *überhaupt*. The sort of doubt one could have about that elementary mathematics is just the sort that goes after (I am, of course, a fallible human being, but waiving that point,...).

But *is* it well-defined? One might see our definition here as presupposing what is not always so. To consider the following three (as I shall suppose) truth-or-falsehoods:

1. Sid has a place at *Stammtisch* at *der alte Schnorrer*.
2. *Der alte Schnorrer* has no *Stammtisch*.
3. There is no such *Kneipe* as *'der alte Schnorrer'*.

What would horseshoe form from the (candidate) pairs, ⟨1,2⟩, ⟨1,3⟩, ⟨2,3⟩? If either 2 or 3 is true, 1 does not express a thought (there is no such thought as that Sid has a place ...). Therefore, there are no such pairs as ⟨1,2⟩ and ⟨1,3⟩ for horseshoe to operate on. But if both are false, there are such pairs. Similarly, if 3 is true, 2 does not express a thought. Again, there is nothing for horseshoe to operate on. But suppose 2 and 3 are both false. Then there are those pairs ⟨1,2⟩, ⟨1,3⟩. But what does horseshoe form from, e.g., ⟨1,2⟩? However, one reads its issue, what horseshoe forms is *defined* by a truth-table. But in those lines in the table where [2] takes on

the value *true*, there is nothing to occupy the other place. We cannot say: 'If [1] is true and [2] is true, then the thought formed is true'. If [2] is true, then horseshoe forms *no* thought.

Not that [2] is necessarily false, but necessarily, if there is such a thing as [1], then [2] is false. And our definition of horseshoe makes no provision for such a case. What to do? One *might* amend horseshoe's definition. But there is another path. Avoiding such situations could be left as an 'exercise for the reader' (to borrow a term from mathematics). If you are to apply horseshoe to two logical forms of a thought, then, when unredacting, make sure that any pair you unredact to is such that each member would be truth-valued no matter which truth value the other takes on. So, viewing things, if things are as in the case just described, the reader simply has not done their job.

Exercises left for the reader may thus work to insulate a law of logic from being borne on from outside logic itself. They may do so in many ways. Otherwise put, a theory of logical forms must hold for any way of unredacting the forms it deals in into full-fledged Gedanken, or what are so treatable for purposes of a given application of the theory. But conversely, unredacting the forms it provides into Gedanken (Frege's primary protagonists in the business of being true) is the responsibility of the one who aims to apply those laws to specific cases of that business.

Moreover, how logical forms *may* be unredacted—what thoughts there *are* into which to unredact—is a world-involving matter. In order for such a thought as the one that Sid has a place at *Stammtisch* to exist, there must be a relevant *Kneipe* and it must have a *Stammtisch*. If not, there is no such thought. Logic, of course, does not aim to be world-involving. Its aim is to arm a thinker with logical forms by which they can deal with whatever constraints the world imposes. Granted, to apply logic to a given topic, the thinker must choose, from the stock of forms logic speaks to, ones which unredact as needed, given the opportunities for unredaction the world provides.

Now, what features of being true (equivalently of representing-as) are drawn on by horseshoe, and correspondingly ground BL1? Since horseshoe is defined in terms of a truth-table, for it to be well-defined we need that truth is two-valued. We thus draw on this feature of whatever representing-as aims at truth: *success* at this enterprise is, at least, representing things as they are *or not*. A failure at this on the part of a *thinker* (back again in the business of *Fürwahrhalten*) may merit a 'nice try'. But it *is* failure to get into the business of *being* true *tout court*; one to which thought-*expression* is in fact liable. The rest is a bit of elementary mathematics, calculating the value of BL1 for each ordered pair of truth values—mathematics elementary enough hardly to be in need of justification. Courtesy may also demand a note to the reader not to unredact placeholders 'A' and 'B' into pairs in which the existence of one member depends on the truth value of the other.

So, modulo that trivial mathematics, the grounds of the truth of BL1 are found in the notion of *true* itself—in some sense the *one* non-specific topic. And they are about the most solid grounds there could be for anything (or at least so count in any circumstances we can now imagine).

But why think there is such a thing as representing-as? This question, I think, can only draw a blank stare, or, to paraphrase Herman Brusselmans, the response 'How could there not be?' How would a rational being such as we be cracked up to get along without it? How can we have access to that sort of detached self-criticism without thought of how things *might* be (as with D.V., we can, of course, add the meaningless tag to all our assertions, 'but I am in principle fallible')?

Note 1: To repeat, laws of being true govern at the level of logical form. So, for example, what occurs as BL1 above is a logical form, formed from two occurrences of horseshoe, and three occurrences of the most general form for a whole thought to take, two marked as to be filled by a recurring thought. But in the statement of the theory in which this form occurs so labelled, its so occurring is to be understood as an expression of a thought, the thought (by the theory's author) that any Gedanke of that form is *eo ipso* true. Its presence in the theory is to be taken as a commitment to this. It is to be understood here that for each placeholder, 'A' and 'B' occurring here, one is free to substitute any fully redacted thought-decomposition, on the condition that one unredacts every occurrence of the placeholder one unredacts. Or one may unredact (still respecting reoccurrence) any given proper part of such a redacted decomposition, leaving as much else as one likes unredacted. Any such move produces an instance of a form of the truth by virtue of the law in question.

Frege tells us (1919: 274) that any decomposition of a thought must contain at least one proper subthought which is in the same business as a whole thought, to wit, the business of representing-as. Where a whole thought is in the business of truth (or falsehood) *punkt*, such a *proper* subthought could only be in the business of truth-*of*, predicating what is liable to be true of one thing, while not true of another. To complete the decomposition in which this occurs, then, one must somehow restore the mere business of truth-of to that of truth-outright. The most straightforward way of doing so would be to provide an item *of* which the predicate is either true or false (and making the whole thought about *it*.) Here it is important that, by the above, we can simply omit doing this, and nevertheless retain an instance of a term of such laws as BL1, which we are ready to consider a second kind of constant.

Note 2: Horseshoe is, of course, only one of a family of logical constants definable in terms of the truth-values of what they operate on, and which operate on logical forms of one or more thoughts to form a further thought (and thus often inter-definable). For each of these constants we can find laws governing them, and specifically their introduction and elimination. Thus far, horseshoe is meant here only as an illustration.

3.2. Stilling calls for completion

The *point* of Horseshoe, one might hold, is to form compound Gedanken from pairs of other thoughts. But following Frege's path to the law-governed, we arrive at logical forms. It is *these* which laws of logic govern, at least directly. To get to Gedanken we must unredact. It remains open to unredact only some of the elements in a given form, leaving others as they are. So doing is, among other things, a way to arrive at the forms of *compound* proper predicative subthoughts from simpler predications plus logical constants such as Horseshoe or Tilde. For example, where an operation of Horseshoe yields what unredacts to Not (Sid drinks but Pia does not eat), it also unredacts to Not ((undetermined) drinks but (undetermined) does not eat). *Mutatis mutandis* for Tilde. The business of truth-of can only be conducted *in* conducting the business of truth outright. Above we begin to see how these two relate.

I turn now to a logical constant which relates to them. I call it 'Ya' (the sound of 'All' written backwards), and write it, '∀_'. Ya is a way of arriving at a form for a whole Gedanke to take by operating on a proper predicative sub-Gedanke. Its point may be, as Horseshoe's, to operate on proper predications. But, again as with Horseshoe, it operates in first instance on redacted logical forms—here of proper predications. One might write these 'Fa', or (perhaps better) as 'F (_)'. But *if* we do, looks may be deceiving. 'F', for example, may create the impression that what Ya operates on are atomic, simple predications. But as we have seen above, this is not so. It operates on *any* form for proper predication to take.

Similarly, that blank space, '(_)'. Here, too, looks may be deceiving. It may look as though that blank space is to be filled in by a designator for an object. But the distinctive thing about a proper predication is that while it engages in representing-as, like a whole thought, it deals only in the business of truth-of, unlike a whole thought, and not in that of truth outright. It raises questions as to of *what* it is true, answers to which may depend on just what it predicates. Whereas a whole thought leaves no room for such questions. So, as to how that blank space may be filled, the answer is: by any means of turning truth-of into truth (or falsehood) outright.

In Frege's metaphor, a proper predication makes a call for completion. But there are two ways of effecting such completion. One is to answer the call. One does this by specifying of *what* the predication is to be true or false. The result: one form for a whole thought to take. But another way is to cancel the call. One way to do this is essentially to 'say' something to following effect: 'It does not matter. Take your pick. Whatever you like.' An operation of Ya cancels some such call (or calls). So, the blank is one to be also understood as fillable, or erasable (whichever image you prefer) in this way. It is thus fillable in either of two quite different ways.

Following current practice, we might write the result of an operation of Ya on a relevant form, something like this: $\forall\chi \mathrm{F}(\chi)$. But once again we must be careful

in reading that 'F'. Here it stands not for an atomic predication, but for *any* form a predication might take. (See discussion above.) In the general case, a proper predicative subthought may make n distinct calls for completion. For example, that only partially unredacted compound yielded by Horseshoe, __ *drinks* ⊃ __ *eats*, makes two such calls. Where Ya operates on a form of predication which contains n such calls, it may, in one operation, cancel m of them, for any $m \leq n$. If $m < n$, then what Ya yields is another proper predication and not a whole thought. Such may also be operated on by a fresh application of Ya (same rule in force). But then what indexed the first application (say, the letter 'x' must differ from what indexes the second (say, 'y'). (Thus, 'Everyone who drinks eats' differs from 'If everyone drinks, then everyone eats')

Ya, as presented here, (*pace* Frege) generalizes over a determinate range of objects—all those which it attaches to *might* predicate (subject to the restriction that no object is such that its existence excludes that of any other in the range). With it, a concern arises. Is there not always a risk that it will prove to generalize too widely? Perhaps not widely enough to run the risk of falsehood. But what assurance can we have that it does not generalize so widely as for something to fall within the scope of generalization which, as it proves, is neither true nor false.

Taking '∨' (vel) for non-exclusive *or* (a variant on horseshoe), and '∼' (tilde) for negation, $P \vee \sim P$ is an easily proven law of logic. Frege calls the law *tertium non datur,* and remarks that what it did not hold of could be no more than a 'quasi-conceptual construction', something which he tells us cannot be law-governed, either by logic, or by any reasonable substitute. So, does the requirement which Ya imposes on truth not open up an avenue by which, in an uncooperative enough world, the laws of logic might prove, or might have proven, other than we take them to be?

But if Ya's well-definition really does run any such risk, we do not need a *theorem* by which to discover it. A *Gedanke* is, by definition, a truth-or-falsehood. What is *not* a *Gedanke*, or certainly not to logic's eye, which just points, once again, to an exercise left for the reader. When unredacting logic's laws so as to apply logic to projects of deriving given thoughts from given others, the reader must be careful to ensure that there is no risk, in the course of the application, of running into any object of thinking which is neither true nor false—on pain of.

When applying logic, we engage in such things as unredacting some 'F' in a given form into *won at Alma*, we are engaging in what is *not* logic's business (*inter alia*, in the business of truth-yielding), and thus in exercises left for the reader. In these exercises, a demand on *our* exercise which might be expressed in citing *tertium non datur* must bear the rider, 'so far as it matters to the application we are now involved with'.

Perhaps Frege is right that *logic* needs no such notion as *domain of discourse*. But the reader to whom exercises are left surely does. What would it be, one might ask,

for there *not* to be different domains of discourse on different applications of logic—for 'everything' always to be entitled to mean *everything*? To suppose this we would need to suppose, firstly, that there is a sort of thing, an object, whose status as such is its capacity to answer a call for completion; secondly, correspondingly to another sort of item, a way for *a* thing to be, a content by which something issues such a call; and, thirdly, in each of these cases, there is a unique domain, the totality of all such things. (Then, we would also have to suppose the way things are to articulate them in a unique way into a unique stock of ways things are.)

But we can see, on reflection, that these suppositions cannot be correct. I will particularly focus on the notion of *object*, on the idea that this notion is categorical, and on the further idea that objects form a unique domain in which all objects, and only these, are to be found, *fertig*. Firstly, a counterfactual tale (mildly so given Ataturk and Mao). Napoleon carried on in it as he in fact did it up to his last breath. Post-breath, he was, or his remains were dressed in full dress uniform (hat and all), and displayed seated at a camp desk, pen in hand, in a museum, just next to the standard meter.

To logic's eye there are now two ways one might identify an object. In one, that object once wrote to Josephine from his/its tent on the field at Austerlitz. Now, having lost that renowned tactical cunning (and so much more), he (or that object) does little but sits behind the desk, pen in hand, though now entirely unproductive. In the other, he (*casu quo* it) ceased to exist post-last breath. What we find behind that desk are (at best) just some embalmed remains, decked out as above.

Each way here of carving out an object from the way things are is equally determinate *in re* questions of recurrence (when *that* object again). Either of them equally satisfies any demand on meeting a call for completion. But one cannot carve out objects in both ways at once. If the first exists, the second cannot, and vice-versa (the despotic hand of Leibniz' law). Hence one had better not encounter both within any given application of logic—any given way of unredacting to genuine questions of truth-or-or falsehood. So, where logic may have no need for a notion *domain*, no role for it in the laws of being true as such, its applications certainly do need this. Where, in an application we prefix an unredaction with Ya, its scope, and hence the applications, had better extend only so far. Correspondingly, within the application, predicates had better be well enough behaved to be treatable as one would be if *tertium non datur* applied.

Can we ask more than this? Do we need to? When we speak, or think, about Napoleon, or even Ataturk, such questions as above seldom occur to us. And if they do, and we pursue them, we would seldom, if ever, find a determinate compelling answer. But nor when, e.g., admiring Napoleon's cleverness in outfoxing Kutusov, is the truth of our thoughts likely to turn on what such answers might be.

What that famous law of *logic* tells us is that in applying logic, the exercise of

unredacting, what is to be unredacted into what predicates has not been successfully unredacted, unless, for purposes of the application to be made, the outcomes of such unredaction can be treated as though *tertium non datur* applied. Such is part of what it is, *per se*, to unredact forms into thoughts (Gedanken). Such structure is not *per se* violated in unredacting to thought about victory if, as it happens, had Napoleon been subjected to a successful brain transplant we would not know what to say, or about lock-ins if were Sid locked in a closet in the corner of the pub, we would not know whether to say he was *at* the lock-in.

Logic tells us that much. Were it to tell us more, it would, I suggest, say too much to be immune to charges of despotism, or to susceptibility to proving wrong.

3.3. Residual Constants

One might recognize constants of other sorts. Firstly, some predications may be seen as so much a part of being true as such that they should correspond to logical constants. Such a constant would, of course, operate on n-tuples of designative subthoughts (more properly forms of such), for given n, and yield a whole thought of those objects. The most obvious—and perhaps only—case is identity, for which $n = 2$ whether or not it is one object that is designated twice. (The generalizing here is over a certain sort of subthought).

What part does identity play in the notion of *true*? At least if there are to be such things as *laws* of being true, there must be parameters whose *recurrences* those laws concern. Among these are whatever it is that may answer a call a proper predicative subthought makes for completion. Plainly, philosophers have differed in how compelling they find this case.

Higher order quantifiers form another sort of constant not discussed here. These cannot be introduced via the same rationale as first-order ones. They raise, I think, interesting philosophical issues which I bracket here (but on at least some of these, see my 2021). Anyway, the aim of such a quantifier (e.g., at second order) would be to quantify over either proper predicative subthoughts or their contents (concepts, or ways for an n-tuple to be ($n \geq 1$). Precisely what they instantiate to is one of the issues calling for discussion. But the *philosophical* issues they raise are for another place. Mathematically, they are unproblematic. I merely note them without further discussion here.

4. Staying in the Zone

Frege contrasts laws of logic with laws of 'morals' (*Sittlichkeit*). In one way, at least, the comparison is apt. Laws of *Sittlichkeit* are traditionally prohibitive (e.g., something about neighbors' wives). Logic's laws, or at least the above, are *permissive*.

Logic's task, Frege tells us (*vide* 1897: 139) is to answer the question 'How must I think to reach the goal *truth*?' (though only at highest generality). Logic as just sketched answers that question by providing, for each of a set of constants, ways in which such *may* be done, ways to transmit truth purely by virtue of what being true is as such. When it has completed that task, for a suitably extensive range of logical constants, its work is done.

Logic *may* have given permission enough when it has dealt with introduction and elimination for horseshoe, tilde and Ya. But if more is wanted, there are further constants to be spoken to, notably those here-bracketed ones. The relevant laws are grounded in the definitions of the concepts plus some simple calculation. Definitions, to be sure, do not bring anything into existence. Nevertheless, if we can identify what it is in the notion *true* which these definitions appeal to, this plus simple calculation provides as firm grounds for such permissive laws to rest on as one might ever have for anything. By reflection we can see how there is no way for things to be otherwise in the relevant respects. (Of course, as always, we are fallible human beings.)

One might want to view logic as more than merely permissive, though. Roughly, the idea would be that some given set of permissions (say, a familiar first-order logic) is, in some interesting sense, *exhaustive*: wherever an ordered pair of (finite/countable) sets of Gedanken, ⟨A, B⟩, relate truth-transmittingly (first to last) merely by virtue of what being true is as such, the permissions logic issues provide a route from A to B.

I just stated the rough idea in terms of relations between thoughts. However, this, needs to be squared with the present conception of logic on which what laws govern in first instance is forms for a thought to take, and not thoughts themselves. (such is a consequence of the generality Frege demands of it.) At the same time, following Frege, a given thought is decomposable in many diverse ways, not generated merely by any given one. For example, he tells us, what is a singular thought on one way of decomposing it may be a universal generalization on another.

What this means for questions of what *thoughts* stand truth-transmittingly to what others merely by given laws of logic depends on how given thoughts count as decomposable (or, the other way, on which decompositions count as decompositions of the same thought. And, recalling, answering such questions is not logic's business, but an exercise left for the reader.

To illustrate, suppose there were some logical constant, Ex., a binary operator on pairs of thoughts (more properly logical forms), *not* definable in terms of horseshoe, tilde and Ya. Suppose there is a sequence of truth-transmitting moves from one set of thoughts to another, starting from a thought, A, ending with a thought, B, in which, at some steps, certification of truth-transmission depends ineliminably on the introduction and/or elimination rules for Ex. Since, by hypothesis, Ex. is not definable in terms of horseshoe, tilde and Ya, this sequence is not certifiable as truth-transmitting merely by the laws governing these constants. But is there *some* truth-transmitting

sequence, beginning with A and ending with B, each step of which *is* certified as truth-transmitting by one or another of these laws? The question is moot unless and until we say how *else* the *thoughts* A and B are decomposable. But we cannot look to *logic* alone to settle this.

Such can be turned into an entrance to a discussion of *thinker* specificity. In sum, there might be other (non-human) intelligences disposed to decompose thoughts in ways we literally cannot imagine, hence to recognize logical constants we cannot—thus logics, not in conflict with ours, but simply incomparable? This, however, is just the issue I marked as too big for present boundaries. I thus end by returning to issues of *topic*-neutrality.

Our central question so far has been: If a law of logic cannot be grounded in anything topic-specific, how then? Following Frege, I have defended the answer: it can be grounded in being true. But what of the antecedent of the conditional? If a law of logic cannot be *grounded* in anything outside the neutral zone, does it follow that it is *absolutely* immune from being borne on by how things are topic-specific? If it did, such would be an exception to a rule. For, in general, it is not that a thought must foresee all that it is so as to what would bear on its truth—it must explicitly represent things as being this way and, therefore, whether they are could be something on which its truth essentially turned.

Leibniz is the one responsible for the main point here, Putnam for the best account of how it might be applied to questions of logical truth. I will briefly sketch this last, then point to a crucial disanalogy between life within and life without 'the zone'. Firstly, though, the modest moral with which Putnam himself concludes:

> In my view, if we cannot describe circumstances under which we would be prepared to say that B had been confirmed, then we are not presently able to attach a clear *sense* to 'B can be revised'. In such a case we cannot, I grant, say that B is 'unrevisable'. But neither can we intelligibly say, 'B can be revised'. (Putnam 1994, p. 253-254)
>
> [W]hat I am inclined to keep from this story is the idea that logical truths do not have negations we (presently) understand. It is not. . . that we can say that the theorems of classical logic are 'unrevisable'; it is that the question 'Are they revisable?; is one which we have not yet succeeded in giving a sense. (Putnam 1994, p. 256)
>
> In the same way, saying that logic or arithmetic may be 'revised' does not have a sense, and will never have a sense, unless some concrete piece of theory building and applying *gives* it a sense. (ibid)

To 'revise' a (putative) law of logic would be to take, or recognize it *not* to hold, and, perhaps to substitute another for it (recognizing another as the one which *really* holds. So, the idea would be: We cannot say (apparently *pace* Frege) that, with respect

to some familiar law of logic, there is simply no such thing as things being otherwise. But we also cannot say that things *could* have been otherwise, since we can attach no adequate sense to the 'could' in this.

Putnam indicates it, such is where we stand *in re* familiar laws, such as BL1. Not that logic itself might show BL1 not to hold. BL1 is precisely a partial unfolding of logic as we know it. And if some apparent law of logic were mistaken— as, say, Frege was on set existence, or Aristotle on quantifiers, such would be boring for present purpose. Logic turns out to be other than we thought, but merely because we had not yet thought hard enough. What we are looking for here is something for is something for logic to turn on other than itself; something by which it might have been otherwise. The interesting case would be where something without the neutral zone, something topic-specific turned out to matter to what logic is. We cannot now conceive of such a circumstance, have no idea what it might be like. So, following Putnam, we cannot intelligibly say that things *might* be, or *have been*, this way. No clear sense can attach to such modals. But neither can we say, categorically, that they could not have been.

Leibniz (1707/1765) had a view on which what matters to the truth of a given thought—what that thought makes truth turn on—and not just *whether* it is true is susceptible to depending on how things are. If that view extended to logical truth, perhaps there is in it a route to what we seemed to want above. And this idea of Leibniz' plays a central role in Putnam's thought. One of Leibniz' favorite ways of illustrating the view concerns gold. 'We' (humanity) had a certain acquaintance with, dealt with, that substance long before the 17th century. Suppose we wanted to define gold—to say what gold is in at least an extensionally correct way. Then, Leibniz suggests, in his time, at least, we would confront a 'pleonasm'. That is, there are a number of different ways in which we might proceed—different sets of 'marks' jointly peculiar to gold. We might, e.g., say that gold is yellow, malleable, and the heaviest metal (as it then seemed to be). But he tells us, we would, or should, take any such definition to be 'provisional'. Which is to say: there is something to which any definition we now give is answerable. (A definition of *gold* cannot be purely stipulative.) This is because we also suppose the object of our acquaintance—that substance—to have what he calls an 'internal constitution'. And what this is may later prove to be, as of now, yet to be discovered. To speak a bit more generally, our 'provisional' definitions are meant to be understood as identifying certain phenomenon (here a substance) which we enjoy given acquaintance. And this leaves room for the idea that what one would suppose to hold of gold—e.g., (17th century) that it is yellow, proves, on further examination of that object of acquaintance, not to be so.

As he was keenly aware on the eve of battle in 1805, Napoleon did not need to have won at Austerlitz. Had he not, we would still have had Napoleon to kick around. But that he did win gives a thought means to make itself. It may make it a condition

for being the one on whom its truth turns that this may be that very one, the one who won at Austerlitz. Had Napoleon not won, there would have been no such thought as that. So necessarily, if there is such a thought, then there is such a man as Napoleon. But it is not necessary that he won for there to be such a man.

If gold were the heaviest metal, if it were yellow (in pure state), it *might* be so definable. It is neither. So, it is not so definable. Perhaps no substance is. There would then be no such substance as 'the heaviest golden metal'. It would not follow that there would then be no such substance as gold. It would still be *gold* that is not so definable. Leibniz' point in short.

The suggestion now to be considered is *mutatis mutandis* for a law of logic. A thought that BL1 holds may commit to nothing as to how things stand outside the neutral zone. For all that, whether the law *does* hold, may still turn out to depend on something without the zone.

But Leibniz works his idea only outside the neutral zone. The key idea of acquaintance with a phenomenon has its home there. Transferring that idea to a step across the boundary, to what lies outside the zone coming to bear on what lies within, is making a very large leap. And it merely helps itself to denial of that on which Frege insists, to wit, that the boundary around the zone cannot be crossed. It can certainly require, if not proof, at least argument.

At which point a story is told aimed at least to stimulate intuition. It involves mathematics (geometry, conceived as the mathematics of spatial notions such as length (or distance), curvature and direction. It involves developments in mathematics since, let us say, 1803 (the death of the sage of Königsberg) until 1860. In the sage's lifetime, the story goes, it would have been literally impossible to conceive of space as having a geometry other than Euclidean. Therefore, if some philosopher were then to claim that it is, in one sense or another, *a priori* that space is shaped so, that there is no such thing as it, or anyway our experience of it, being otherwise, their claiming would be *eminently* exculpable. By 1860, though, thanks to Gauss and Riemann, we had gained the conceptual tools needed to make sense of space being other than Euclidean. Euclidean geometry would remain as irreproachable mathematics. But a philosopher who claimed that space could not be otherwise shaped might be accused of not having done their homework.

The change in history can be described as follows: Before 1803, it was literally impossible for anyone to conceive of space as non-Euclidean. The conceptual tools required for this had simply not been discovered. But post-1860, those tools were in place. With them, it is not just conceivable that space should not be Euclidean; we may well have discovered that it *is* not. There may be very good reasons to suppose it not to be. Those reasons have their first home in mechanics, a topic-specific subject if there ever was. So, for them to be decisive, it is precisely for something else once held impossible to have taken place, namely, for the boundary between the neutral

zone and outside to have been penetrated. Were there really nothing but Euclidean geometry, no other shape in conceptual space for physical space to take, then that much physics may have belonged within the zone. But the antecedent here is false, and (so far) physics remains outside the zone.

So the story goes. But there are crucial differences between logic and geometry which may scupper analogy here. For a start, Euclidean geometry remains as good mathematics as it ever was. It is just that now it may function in a theory of space (or space-time). And it is in this role that the apodictic becomes negotiable. What sort of transformation has occurred here? Note that in mechanics from Aristotle on basic geometrical quantities had always figured among the parameters to which laws of motion are sensitive. Such parameters as velocity, for example, were supposed definable in terms of distance and time. Direction is a basic notion both of geometry and of mechanics. Though Newton's conception of motion was very different from Aristotle's (force now a *sine qua non* for change of motion), the basic quantities to which mechanics referred (explicitly or implicitly) remain the same.

What changed from Riemann on was basically this: while in the pre-Riemann era, it was left to physics to determine the relations between non-geometric mechanical quantities—e.g., to propose, or confirm, a law such as $F = ma$, it was beyond physics' remit to say how purely geometric quantities related. In the post-Riemann era, *all* parameters which laws of mechanics are sensitive to become fair game for it. Physics is thus given a voice in questions of the geometry of space.

All of this radically contrasts with the case of logic. The primary parameters in logic's laws are the truth-values, *true* and *false*. Might these also be parameters in any natural law (a law of our environment)? I take an example from Carlo Collodi. In his tale, Pinocchio's rhinal dimensions turned on what he said. He said things that were either true or false. But such are not the variables on which the dimensions turned. What they did turn on was whether what he said was a porkie. A porkie has a *mens rea*. Perhaps it is *just* possible that rhinal dimensions can depend on the presence or absence of that *mens rea*.

However, we clearly are not home free yet. Suppose we dispense with *mens rea* and suppose that Pinocchio's rhinal dimensions where really a function of whether he spoke truth or falsehood regardless of what he said, and regardless of his state of mind. Then we would have a genuine natural phenomenon whose parameters (that over which its law generalized) were, *inter alia*, the truth values. I will call such a *Pinocchio phenomenon*. There is an obvious reason (perhaps many) why there should be no Pinocchio phenomena in nature. Abstractly put, where a quantity figures as a parameter in natural laws, for it to be *that* quantity is, at least *inter alia,* for it so to figure. If curvature plays a role in mechanics, then whether we are talking about *curvature* depends on whether we are talking about something which in fact plays that role; a condition on how curvature is definable.

Post-Riemann, geometrical notions, e.g., notions of direction and curvature, are on a par with purely mechanical ones. What it *is, e.g.,* for a path to be *straight* depends, *inter alia*, on what those physical laws which deal in, among others, distance and curvature (e.g., in defining their other parameters) require this to be. For there to be a Pinocchio phenomenon in nature is for truth values to be parameters in natural laws. If so, then for it to be truth and falsehood that was in question would be (*inter alia* perhaps) for it to be those items which so figured in those laws. It would be for these to interact with the other parameters in such laws as the relevant lawlikeness required. So, for example, if there were a phenomenon of rhinal dimensions which was a Pinocchio phenomenon, then if revision were needed in a theory of its laws, such might call for a new notion of length, or it might call for a new notion of truth, depending on what exactly the trouble was (even if logic awaits its Riemann for showing how).

What a Pinocchio phenomenon would thus threaten are such fundamental properties of *true* as that it is an identity under predication (over truth-or-falsehoods), and, correlatively, the 'Aristotelian platitudes'. The notion of *true* would thus acquire a content with which, Frege suggested, we would enter a regress when we tried to say what it was for any given thought to be true, but, put another way, we lose just that other item with which truth comes as a package, namely, the phenomenon of representing-as. With which the possibility of determinate questions of truth, thus determinate truth-or-falsehoods, vanishes.

Such are the reasons for disallowing Pinocchio phenomena, and insisting that what truth is, and thus how it unfolds into the laws of truth, are not liable to be hostage to anything external to thought itself. Within the realm of the thinkable, thus the business of being true or false, the notion *true* stands as a fixed point in terms of which all (potential) revision elsewhere is to be measured. It is the one thing that always stands fast, where to be which is to be *per se* insusceptible to acquiring content from elsewhere. One *could* put the point by saying: Logic takes care of itself. So, it is answerable only to itself. Its laws exclude things being otherwise. And since they are true, it is excluded. To say the same of any other topic could only be to joke. Logic is an exception to this, too.

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