OPTIMIZATION MODEL THAT MINIMIZES THE PENALTY CAUSED BY DELAYED DELIVERY OF CONSTRUCTION PROJECTS

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ARTICLE INFO	ABSTRACT
	Purpose: This work aims to minimize the penalty generated by delays in delivering construction projects operated by a single machine. **Theoretical framework:** The fundamental issues to develop this work are optimization and its application to delays in construction projects operated in their initial stage by a single machine. **Design/Methodology/Approach:** We presented a construction case study, developed an optimization model, implemented a computational optimization tool and obtained the optimal sequence to perform the tasks. **Findings:** The numerical results demonstrated the model's usefulness in minimizing the penalty generated by delays in the delivery of projects. **Research, practical & social implications:** The benefit of this study is to help managers or decision-makers schedule their construction projects with limited resources and deadlines per activity to minimize penalty costs of delay. **Originality/Value:** Considering that the construction sector generates the largest labor force in Peru, the study has an important social value by providing a tool to improve operations and incentivize construction companies to continue operating. In addition, it provides a substantial basis for future work by applying optimization in a specific area of civil engineering. To our knowledge, no researcher or company in Peru has yet addressed this study.

MODELO DE OTIMIZAÇÃO QUE MINIMIZA A PENALIDADE CAUSADA PELO ATRASO NA ENTREGA DE PROJETOS DE CONSTRUÇÃO

RESUMO
Objetivo: Este trabalho visa minimizar a penalidade gerada por atrasos na entrega de projetos de construção operados por uma única máquina.

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Estrutura teórica: As questões fundamentais para desenvolver este trabalho são a otimização e sua aplicação aos atrasos em projetos de construção operados em sua fase inicial por uma única máquina.

Projeto/Metodologia/Abordagem: Apresentamos um estudo de caso de construção, desenvolvemos um modelo de otimização, implementamos uma ferramenta de otimização computacional e obtivemos a sequência ideal para executar as tarefas.

Constatações: Os resultados numéricos demonstraram a utilidade do modelo na minimização da penalidade gerada por atrasos na entrega dos projetos.

Investigação, implicações práticas e sociais: O benefício deste estudo é ajudar os gestores ou decisores a programar os seus projetos de construção com recursos limitados e prazos por atividade para minimizar os custos de penalidade de atraso.

Originalidade/Valor: Considerando que o setor da construção gera a maior força de trabalho no Peru, o estudo tem um importante valor social, fornecendo uma ferramenta para melhorar as operações e incentivar as empresas de construção a continuar em operação. Além disso, proporciona uma base substancial para o trabalho futuro, aplicando a otimização numa área específica da engenharia civil. Pelo que sabemos, nenhum pesquisador ou empresa do Peru ainda abordou esse estudo.

Palavras-chave: Otimização, Penalidade, Projeto de Construção, Modelo Matemático, Projetos com uma Única Máquina.

MODELO DE OPTIMIZACIÓN QUE MINIMIZA LA PENALIDAD CAUSADA POR EL RETRASO EN LA EJECUCIÓN DE PROYECTOS DE CONSTRUCCIÓN

RESUMEN

Objetivo: Este trabajo tiene por objeto reducir al mínimo la penalización generada por los retrasos en la ejecución de los proyectos de construcción gestionados por una sola máquina.

Estructura teórica: Las cuestiones clave para el desarrollo de este trabajo son la optimización y su aplicación a los retrasos en los proyectos de construcción que fueron realizados en su fase inicial por una sola máquina.

Diseño/Metodología/Enfoque: Presentamos un estudio de caso de construcción, desarrollamos un modelo de optimización, implementamos una herramienta de optimización informática y obtuvimos la secuencia óptima para realizar las tareas.

Conclusiones: Los resultados numéricos demostraron la utilidad del modelo para reducir al mínimo la penalización generada por los retrasos en la ejecución de los proyectos.

Investigación, implicaciones prácticas y sociales: El beneficio de este estudio es ayudar a los gerentes o a los responsables de la toma de decisiones a planificar sus proyectos de construcción con recursos limitados y plazos por actividad para minimizar los costos de las sanciones por demora.

Originalidad/Valor: considerando que el sector de la construcción genera la mayor fuerza laboral del Perú, el estudio tiene un importante valor social, pues proporciona una herramienta para mejorar las operaciones y anima a las empresas constructoras a seguir operando. Además, proporciona una base sustancial para el trabajo futuro mediante la aplicación de la optimización en un ámbito específico de la ingeniería civil. Hasta donde sabemos, ningún investigador o empresa peruano ha abordado este estudio.

Palabras clave: Optimización, Sanción, Proyecto de Construcción, Modelo Matemático, Proyectos de Máquina Única.

INTRODUCTION

The construction industry is one of the largest and fastest growing economic sectors in the world; according to a recent market study, it is expected to reach an estimated $10.5 billion by 2023, and an annual growth rate of 4.2% between 2018 and 2023 (Al-Janabi et al., 2020). Construction companies have a significant impact on a country's economy. For this reason, researchers around the world have been developing various models that increase the ability of companies to complete projects and thus improve their financial situation before it worsens

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(Daier et al., 2022). The construction industry is often subject to greater risks and unforeseen events than any other industry. Construction projects require extensive planning and are characterized by developing in unpredictable circumstances that make proper planning difficult (Acharya et al., 2006). Managing construction projects properly is essential to solving society's problems. Optimal planning allows managers to reasonably estimate the project's resources, cost, and time, thus avoiding extra expenses during execution.

Construction projects are increasing exponentially every day, but they require serious management in their execution, even more so when working on large projects where the dependence of one job on another is more complex. Therefore, good management is essential to achieve the desired results (Nasution et al., 2022). The efficient completion of a construction project requires the synergistic functioning of its team, who assume different but complementary tasks such as planning, design, procurement, financing, and construction (Kebede & Tiewei, 2021). Delays occur for various reasons, generating postponement completion, increased costs, and even contract dissolution (Arditi & Pattanakitchamroon, 2006). According to Manzano (2020), delays in the delivery of construction projects are a global problem and that studies carried out in both developed and developing countries show this phenomenon; therefore, it is a complex and increasingly frequent issue for organizations.

There is a lot of uncertainty before the execution of a project; they usually have a very variable duration, which generates a problem when estimating times, costs, and quality. A project is successful when completed in the proposed time, with the budget agreed upon in the first instance, and provides the required quality. However, this success is less common than expected. According to Rudeli et al. (2018), construction delays can lead to losses, disputes between parties, abandonment of the project, and even legal litigation that directly affects organizations' financial statements. Santos (2019), highlights that in the Peruvian Public Procurement Law No. 30225, contractors who do not comply with the delivery term established in the contract will assume penalties that generate cost overruns.

In Peru, small and medium-sized construction companies need more machinery to execute two or more construction projects simultaneously. However, using a single machine to perform tasks corresponding to several projects completed simultaneously is a common practice.

In this work, we applied an optimization model to minimize the penalty generated by delays in completing construction projects operated by a single machine. The results are shown using the model in a case study considering a backhoe machine.

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LITERATURE REVIEW

In this section, we review the fundamental topics required for this study's development: optimization, delays in construction projects, and the optimization model for scheduling projects with a single machine.

Optimization

According to Eren et al. (2017), in mathematics the term optimization refers to the study of problems in which one tries to find the maximum or minimum value that a particular function can reach within a set of constraints. To solve an optimization problem one must have some experience in mathematical modeling, know all the problem's characteristics and apply the following steps: data collection, problem definition and formulation, model development, performance, application validation, and interpretation.

In Business and industry, optimization is used to find the best solution to a specific problem. The results are used by managers in decision-making, minimizing investment risk, production time, penalties and maximizing profits.

In the construction field, according to Essam et al. (2023), scheduling construction projects is a complex process due to the project's various task activities. Academia and science are researching how to organize activities as efficiently as possible within the delivery date and considering available resources.

In this study, we present and address a problem of programming tasks executed by a single machine to achieve the optimal delivery delay penalty cost.

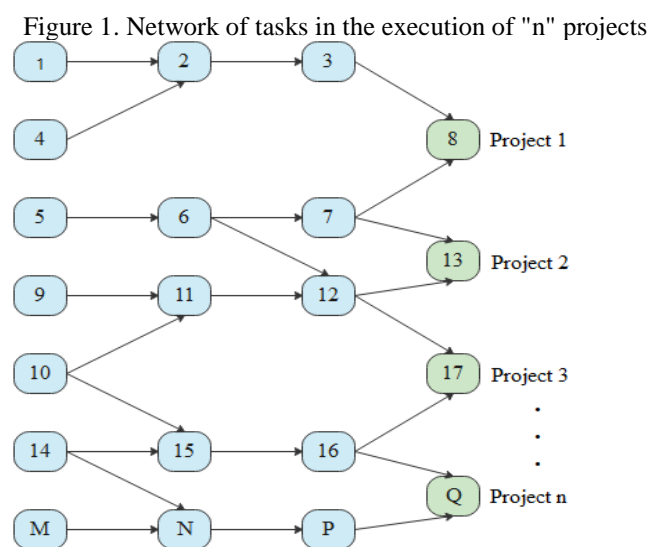### Construction Project Delays

The construction sector is considered one of the most vital and profitable sectors in the world, and its demand continues to grow daily due to the increase in the world's population. Therefore, the topic of improving the performance of construction projects and studying the various factors that generate delays and cost overruns has received increased attention from researchers in the last two decades (Maya et al., 2015). Different risk factors influence schedule performance, cost, and time from the start to the end of a project. Typically, integrated risks are allocated through the evaluation of the project management contract (Mukilan et al., 2021). Three main criteria explain a project's success: completion on time, staying within budget, and following specifications. Currently, construction projects face several problems, and delay is considered among the main ones (Elhusseiny et al., 2021).

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Saving money is critical for the owner, contractor, and subcontractor in a project. Delays and failure to complete work within the given budget are the main reasons for conflicts in construction projects (Vivek & Hanumantha Rao, 2022). Delay is quite common; an excess of time in any activity or operation affects the completion of the work, leading to disagreements and lawsuits (Aravindhan et al., 2021). Procuring items, equipment, machinery, and services constitute an essential part of projects in the construction industry. These supplies are associated with different challenges that need to be identified and controlled throughout the stages of the project lifecycle (Yu & Shen, 2013).

The construction industry faces technological changes evolving from Industry 4.0 that will change how projects are done, driven by technologies such as prefabrication, BIM, automated and robotic equipment, wireless sensors, and 3D printing (Buehler et al., 2018). Incorporating sustainability principles into project planning, design, construction, and management is an indispensable social responsibility of construction organizations (Ershadi & Goodarzi, 2021). According to Neyra (2008), said that a construction project is temporary; that is, it has a beginning and an end. In addition, the life cycle of a project is divided into different stages or phases.

Optimization Model for Single-machine Project Scheduling

The mathematical model that minimizes the penalty generated by delays in the delivery of construction projects is described. Figure 1 shows the task network for a machine in the execution of "n" projects, where "Q" tasks must be performed.

Figure 1. Network of tasks in the execution of "n" projects

Source: Prepared by the authors (2023)

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In Taha (2012), studies mathematical models that minimize penalties by considering non-interference, precedence, and delivery time restrictions.

Non-interference restrictions: Figure 2 ensures that the machine does not execute two tasks at the same time, given:
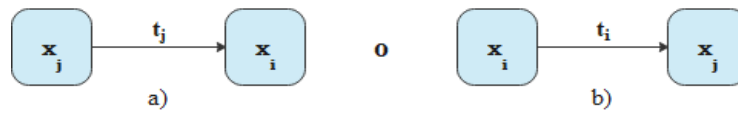
x_i : Task i start date

x_j : Task j start date

t_i : Task i execution time

t_j : Task j execution time

Figure 2. No task interference

Source: Prepared by the authors (2023)

Figure 2-a) shows that the task j is done before the task i, while Figure 2-b) shows the opposite; as a result, we obtain the relationships:

$$x_i - x_j \geq t_j \quad \vee \quad x_j - x_i \geq t_i \tag{1}$$

To determine the active constraint leaving the other inactive, an auxiliary binary variable needs to be defined y_{ij} (Medina et al., 2008).

$$y_{ij} = \begin{cases} 1, & \text{if } i \text{ before } j \\ 0, & \text{if } j \text{ before } i \end{cases} \tag{2}$$

Including (2) and M in (1), where M is a large enough number for the conditions of the problem, the non-interference restrictions are as follows:

$$x_i - x_j \geq t_j - My_{ij} \ \wedge \ x_j - x_i \geq t_i - M(1 - y_{ij}) \tag{3}$$

Precedence restrictions: Figure 3 shows that to start task j, task i must be performed first; that is, these are direct precedents.

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Figure 3. Tasks direct precedents
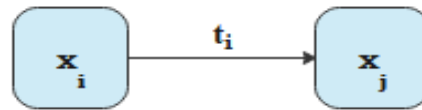
Source: Prepared by the authors (2023)

From Figure 3, the precedence constraints are as follows:

$$x_j - x_i \geq t_i \tag{4}$$

Lead time constraints: To analytically determine these constraints, it is essential to consider the following variables:

x_u : Start date of the last task u of the project P

t_u : Task execution time u

f_p : Project delivery deadline P

v_p : Deviation variable

m_p : Project delivery penalty per delay day P

Figure 4. Project delivery after deadline
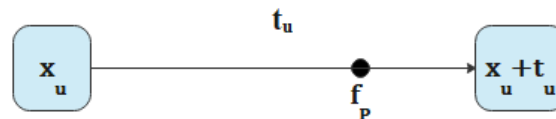
Source: Prepared by the authors (2023)

From Figure 4, the following relationship is deduced:

$$x_u + t_u > f_p \tag{5}$$

From (5) we get:

$$x_u + t_u + v_p = f_p \tag{6}$$

From (6), the deviation variable $v_p \leq 0$, in this case, a penalty is generated for delivering the project after the deadline.

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Figure 5. Project delivery before deadline

Source: Prepared by the authors (2023)

From Figure 5, the following relationship can be deduced:

$$x_u + t_u < f_p \tag{7}$$

From (7) we get:

$$x_u + t_u + v_p = f_p \tag{8}$$

From (8), the deviation variable $v_p \geq 0$, in this case the project is delivered before the deadline; that is, there is no penalty.

Substituting the deviation variable v_p:

$$v_p = s_{pa} - s_{pb} \; ; \; s_{pa} \; , \; s_{pb} \geq 0 \tag{9}$$
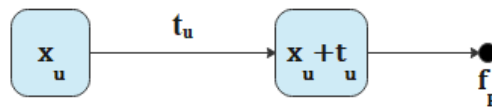
Where,

s_{pa} is the deviation variable that does not generate a penalty and s_{pb} is the deviation variable that generates a penalty in the project's delivery time. When $s_{pb} > 0$ and $s_{pa} = 0$, de (9) the variable $v_p < 0$, this generates a penalty per day of delay in the delivery of the project (see Figure 4). However, if $s_{pa} > 0$ and $s_{pb} = 0$, the variable $v_p > 0$, which indicates that there is no penalty in the delivery of the project (see Figure 5).

Replacing (9) in (8), the delivery time constraints are given as follows:

$$x_u + t_u + s_{pa} - s_{pb} = f_p \tag{10}$$

The model aims to determine a work plan for the machine that minimizes the total penalty, so the objective function is given by:

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$$\min Z = \sum_{p=1}^{n} (m_p \cdot s_{pb}) \tag{11}$$

Finally, we obtained optimization model that minimizes the penalty generated by delays in the delivery of projects operated by a machine (Arce et al., 2021):

$$min\, Z = \sum_{p=1}^{n} (m_p \cdot s_{pb})$$

s.a:

$$x_i - x_j \geq t_j - My_{ij}$$
$$x_j - x_i \geq t_i - M(1 - y_{ij})$$
$$x_j - x_i \geq t_i$$
$$x_u + t_u + s_{pa} - s_{pb} = f_p$$

Where:

$x_i, s_{pa}, s_{pb} \in Z_0^+$
$y_{ij} \in \{0,1\}$
M is a sufficiently large number

MATERIALS AND METHODS

Case study

The company JTZ S.R.L., with headquarters in the Province of Jaén of the Cajamarca Region in Peru, has a work plan to execute four construction projects simultaneously, having a single backhoe to perform 18 tasks. Due to high penalties for delays of previous deliveries, with the manager the machine operations network was designed, including non-interference restrictions, precedence, and delivery time, is shown in Figure 6. In addition, data was collected from expert members of the company regarding the variables: time used by the machine to execute each task (see Table 1), delivery deadline per project, and the penalty per delay day (see Table 2).

Figure 6. Machine task network

Source: Prepared by the authors (2023)
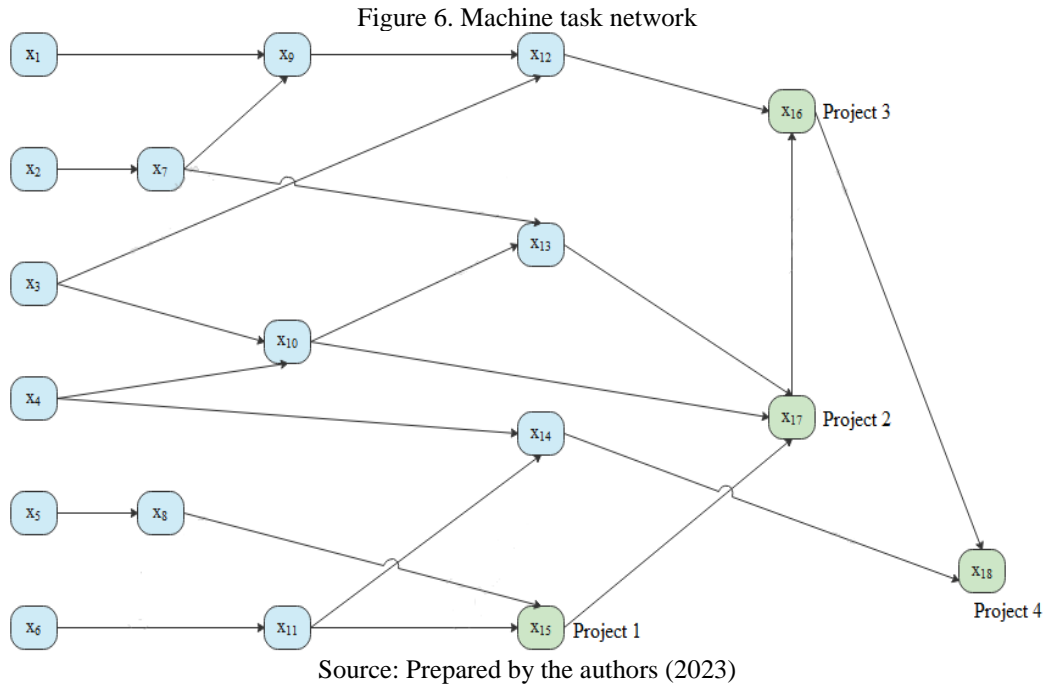
Table 1. Machine time to execute each task

Task	Runtime
	days
T1	10
T2	17
T3	13
T4	20
T5	14
T6	15
T7	21
T8	30
T9	25
T10	16
T11	18
T12	23
T13	12
T14	20
T15	26
T16	18
T17	14
T18	28

Source: Prepared by the authors (2023)

Table 2. Delivery date and penalty per delay day

Projects	Delivery Deadline	Penalty
	days	soles/day
P_1	95	1000
P_2	205	1600
P_3	280	2500
P_4	330	3000

Source: Prepared by the authors (2023)

Mathematical Modeling of the Case Study

To model the phenomenon under study, it is considered P a set of 4 construction projects and T a set of 18 tasks; for each $i \in T$, $j \in P$ we have:

x_i : Task i start date

s_{pa} : variable that does not generate penalty in the project j

s_{pb} : Variable that generates penalty in the project j

y_{ij} : Binary variable

$M = 5000$, exceeds all variable values.

Applying the optimization model, the objective function was represented as follows:

$$\min Z = 1000s_{1b} + 1600s_{2b} + 2500s_{3b} + 3000s_{4b} \tag{12}$$

Subject to non-interference, precedence, and delivery time restrictions, as follows:

Table 3. Contains all non-interference restrictions

Non-interference restrictions	
1. $x_1 - x_2 + 5000y_{1,2} \geq 17$	86. $x_{10} - x_5 - 5000y_{5,10} \geq -4986$
2. $x_2 - x_1 - 5000y_{1,2} \geq -4990$	87. $x_5 - x_{11} + 5000y_{5,11} \geq 18$
3. $x_1 - x_3 + 5000y_{1,3} \geq 13$	88. $x_{11} - x_5 - 5000y_{5,11} \geq -4986$
4. $x_3 - x_1 - 5000y_{1,3} \geq -4990$	89. $x_5 - x_{12} + 5000y_{5,12} \geq 23$
5. $x_1 - x_4 + 5000y_{1,4} \geq 20$	90. $x_{12} - x_5 - 5000y_{5,12} \geq -4986$
6. $x_4 - x_1 - 5000y_{1,4} \geq -4990$	91. $x_5 - x_{13} + 5000y_{5,13} \geq 12$
7. $x_1 - x_5 + 5000y_{1,5} \geq 14$	92. $x_{13} - x_5 - 5000y_{5,13} \geq -4986$
8. $x_5 - x_1 - 5000y_{1,5} \geq -4990$	93. $x_5 - x_{14} + 5000y_{5,14} \geq 20$
9. $x_1 - x_6 + 5000y_{1,6} \geq 15$	94. $x_{14} - x_5 - 5000y_{5,14} \geq -4986$
10. $x_6 - x_1 - 5000y_{1,6} \geq -4990$	95. $x_6 - x_7 + 5000y_{6,7} \geq 21$
11. $x_1 - x_7 + 5000y_{1,7} \geq 21$	96. $x_7 - x_6 - 5000y_{6,7} \geq -4985$
12. $x_7 - x_1 - 5000y_{1,7} \geq -4990$	97. $x_6 - x_8 + 5000y_{6,8} \geq 30$
13. $x_1 - x_8 + 5000y_{1,8} \geq 30$	98. $x_8 - x_6 - 5000y_{6,8} \geq -4985$
14. $x_8 - x_1 - 5000y_{1,8} \geq -4990$	99. $x_6 - x_9 + 5000y_{6,9} \geq 25$
15. $x_1 - x_{10} + 5000y_{1,10} \geq 16$	100. $x_9 - x_6 - 5000y_{6,9} \geq -4985$
16. $x_{10} - x_1 - 5000y_{1,10} \geq -4990$	101. $x_6 - x_{10} + 5000y_{6,10} \geq 16$
17. $x_1 - x_{11} + 5000y_{1,11} \geq 18$	102. $x_{10} - x_6 - 5000y_{6,10} \geq -4985$
18. $x_{11} - x_1 - 5000y_{1,11} \geq -4990$	103. $x_6 - x_{12} + 5000y_{6,12} \geq 23$

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19. $x_1 - x_{13} + 5000 y_{1,13} \geq 12$

20. $x_{13} - x_1 - 5000 y_{1,13} \geq -4990$

21. $x_1 - x_{14} + 5000 y_{1,14} \geq 20$

22. $x_{14} - x_1 - 5000 y_{1,14} \geq -4990$

23. $x_1 - x_{15} + 5000 y_{1,15} \geq 26$

24. $x_{15} - x_1 - 5000 y_{1,15} \geq -4990$

25. $x_1 - x_{17} + 5000 y_{1,17} \geq 14$

26. $x_{17} - x_1 - 5000 y_{1,17} \geq -4990$

27. $x_2 - x_3 + 5000 y_{2,3} \geq 13$

28. $x_3 - x_2 - 5000 y_{2,3} \geq -4983$

29. $x_2 - x_4 + 5000 y_{2,4} \geq 20$

30. $x_4 - x_2 - 5000 y_{2,4} \geq -4983$

31. $x_2 - x_5 + 5000 y_{2,5} \geq 14$

32. $x_5 - x_2 - 5000 y_{2,5} \geq -4983$

33. $x_2 - x_6 + 5000 y_{2,6} \geq 15$

34. $x_6 - x_2 - 5000 y_{2,6} \geq -4983$

35. $x_2 - x_8 + 5000 y_{2,8} \geq 30$

36. $x_8 - x_2 - 5000 y_{2,8} \geq -4983$

37. $x_2 - x_{10} + 5000 y_{2,10} \geq 16$

38. $x_{10} - x_2 - 5000 y_{2,10} \geq -4983$

39. $x_2 - x_{11} + 5000 y_{2,11} \geq 18$

40. $x_{11} - x_2 - 5000 y_{2,11} \geq -4983$

41. $x_2 - x_{14} + 5000 y_{2,14} \geq 20$

42. $x_{14} - x_2 - 5000 y_{2,14} \geq -4983$

43. $x_2 - x_{15} + 5000 y_{2,15} \geq 26$

44. $x_{15} - x_2 - 5000 y_{2,15} \geq -4983$

45. $x_3 - x_4 + 5000 y_{3,4} \geq 20$

46. $x_4 - x_3 - 5000 y_{3,4} \geq -4987$

47. $x_3 - x_5 + 5000 y_{3,5} \geq 14$

48. $x_5 - x_3 - 5000 y_{3,5} \geq -4987$

49. $x_3 - x_6 + 5000 y_{3,6} \geq 15$

50. $x_6 - x_3 - 5000 y_{3,6} \geq -4987$

51. $x_3 - x_7 + 5000 y_{3,7} \geq 21$

52. $x_7 - x_3 - 5000 y_{3,7} \geq -4987$

53. $x_3 - x_8 + 5000 y_{3,8} \geq 30$

54. $x_8 - x_3 - 5000 y_{3,8} \geq -4987$

104. $x_{12} - x_6 - 5000 y_{6,12} \geq -4985$

105. $x_6 - x_{13} + 5000 y_{6,13} \geq 12$

106. $x_{13} - x_6 - 5000 y_{6,13} \geq -4985$

107. $x_7 - x_8 + 5000 y_{7,8} \geq 30$

108. $x_8 - x_7 - 5000 y_{7,8} \geq -4979$

109. $x_7 - x_{10} + 5000 y_{7,10} \geq 16$

110. $x_{10} - x_7 - 5000 y_{7,10} \geq -4979$

111. $x_7 - x_{11} + 5000 y_{7,11} \geq 18$

112. $x_{11} - x_7 - 5000 y_{7,11} \geq -4979$

113. $x_7 - x_{14} + 5000 y_{7,14} \geq 20$

114. $x_{14} - x_7 - 5000 y_{7,14} \geq -4979$

115. $x_7 - x_{15} + 5000 y_{7,15} \geq 26$

116. $x_{15} - x_7 - 5000 y_{7,15} \geq -4979$

117. $x_8 - x_9 + 5000 y_{8,9} \geq 25$

118. $x_9 - x_8 - 5000 y_{8,9} \geq -4970$

119. $x_8 - x_{10} + 5000 y_{8,10} \geq 16$

120. $x_{10} - x_8 - 5000 y_{8,10} \geq -4970$

121. $x_8 - x_{11} + 5000 y_{8,11} \geq 18$

122. $x_{11} - x_8 - 5000 y_{8,11} \geq -4970$

123. $x_8 - x_{12} + 5000 y_{8,12} \geq 23$

124. $x_{12} - x_8 - 5000 y_{8,12} \geq -4970$

125. $x_8 - x_{13} + 5000 y_{8,13} \geq 12$

126. $x_{13} - x_8 - 5000 y_{8,13} \geq -4970$

127. $x_8 - x_{14} + 5000 y_{8,14} \geq 20$

128. $x_{14} - x_8 - 5000 y_{8,14} \geq -4970$

129. $x_9 - x_{10} + 5000 y_{9,10} \geq 16$

130. $x_{10} - x_9 - 5000 y_{9,10} \geq -4975$

131. $x_9 - x_{11} + 5000 y_{9,11} \geq 18$

132. $x_{11} - x_9 - 5000 y_{9,11} \geq -4975$

133. $x_9 - x_{13} + 5000 y_{9,13} \geq 12$

134. $x_{13} - x_9 - 5000 y_{9,13} \geq -4975$

135. $x_9 - x_{14} + 5000 y_{9,14} \geq 20$

136. $x_{14} - x_9 - 5000 y_{9,14} \geq -4975$

137. $x_9 - x_{15} + 5000 y_{9,15} \geq 26$

138. $x_{15} - x_9 - 5000 y_{9,15} \geq -4975$

139. $x_9 - x_{17} + 5000 y_{9,17} \geq 14$

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55. $x_3 - x_9 + 5000 y_{3,9} \geq 25$	140. $x_{17} - x_9 - 5000 y_{9,17} \geq -4975$
56. $x_9 - x_3 - 5000 y_{3,9} \geq -4987$	141. $x_{10} - x_{11} + 5000 y_{10,11} \geq 18$
57. $x_3 - x_{11} + 5000 y_{3,11} \geq 18$	142. $x_{11} - x_{10} - 5000 y_{10,11} \geq -4984$
58. $x_{11} - x_3 - 5000 y_{3,11} \geq -4987$	143. $x_{10} - x_{12} + 5000 y_{10,12} \geq 23$
59. $x_3 - x_{14} + 5000 y_{3,14} \geq 20$	144. $x_{12} - x_{10} - 5000 y_{10,12} \geq -4984$
60. $x_{14} - x_3 - 5000 y_{3,14} \geq -4987$	145. $x_{10} - x_{14} + 5000 y_{10,14} \geq 20$
61. $x_3 - x_{15} + 5000 y_{3,15} \geq 26$	146. $x_{14} - x_{10} - 5000 y_{10,14} \geq -4984$
62. $x_{15} - x_3 - 5000 y_{3,15} \geq -4987$	147. $x_{10} - x_{15} + 5000 y_{10,15} \geq 26$
63. $x_4 - x_5 + 5000 y_{4,5} \geq 14$	148. $x_{15} - x_{10} - 5000 y_{10,15} \geq -4984$
64. $x_5 - x_4 - 5000 y_{4,5} \geq -4980$	149. $x_{11} - x_{12} + 5000 y_{11,12} \geq 23$
65. $x_4 - x_6 + 5000 y_{4,6} \geq 15$	150. $x_{12} - x_{11} - 5000 y_{11,12} \geq -4982$
66. $x_6 - x_4 - 5000 y_{4,6} \geq -4980$	151. $x_{11} - x_{13} + 5000 y_{11,13} \geq 12$
67. $x_4 - x_7 + 5000 y_{4,7} \geq 21$	152. $x_{13} - x_{11} - 5000 y_{11,13} \geq -4982$
68. $x_7 - x_4 - 5000 y_{4,7} \geq -4980$	153. $x_{12} - x_{13} + 5000 y_{12,13} \geq 12$
69. $x_4 - x_8 + 5000 y_{4,8} \geq 30$	154. $x_{13} - x_{12} - 5000 y_{12,13} \geq -4977$
70. $x_8 - x_4 - 5000 y_{4,8} \geq -4980$	155. $x_{12} - x_{14} + 5000 y_{12,14} \geq 20$
71. $x_4 - x_9 + 5000 y_{4,9} \geq 25$	156. $x_{14} - x_{12} - 5000 y_{12,14} \geq -4977$
72. $x_9 - x_4 - 5000 y_{4,9} \geq -4980$	157. $x_{12} - x_{15} + 5000 y_{12,15} \geq 26$
73. $x_4 - x_{11} + 5000 y_{4,11} \geq 18$	158. $x_{15} - x_{12} - 5000 y_{12,15} \geq -4977$
74. $x_{11} - x_4 - 5000 y_{4,11} \geq -4980$	159. $x_{12} - x_{17} + 5000 y_{12,17} \geq 14$
75. $x_4 - x_{12} + 5000 y_{4,12} \geq 23$	160. $x_{17} - x_{12} - 5000 y_{12,17} \geq -4977$
76. $x_{12} - x_4 - 5000 y_{4,12} \geq -4980$	161. $x_{13} - x_{14} + 5000 y_{13,14} \geq 20$
77. $x_4 - x_{15} + 5000 y_{4,15} \geq 26$	162. $x_{14} - x_{13} - 5000 y_{13,14} \geq -4988$
78. $x_{15} - x_4 - 5000 y_{4,15} \geq -4980$	163. $x_{13} - x_{15} + 5000 y_{13,15} \geq 26$
79. $x_5 - x_6 + 5000 y_{5,6} \geq 15$	164. $x_{15} - x_{13} - 5000 y_{13,15} \geq -4988$
80. $x_6 - x_5 - 5000 y_{5,6} \geq -4986$	165. $x_{14} - x_{15} + 5000 y_{14,15} \geq 26$
81. $x_5 - x_7 + 5000 y_{5,7} \geq 21$	166. $x_{15} - x_{14} - 5000 y_{14,15} \geq -4980$
82. $x_7 - x_5 - 5000 y_{5,7} \geq -4986$	167. $x_{14} - x_{16} + 5000 y_{14,16} \geq 18$
83. $x_5 - x_9 + 5000 y_{5,9} \geq 25$	168. $x_{16} - x_{14} - 5000 y_{14,16} \geq -4980$
84. $x_9 - x_5 - 5000 y_{5,9} \geq -4986$	169. $x_{14} - x_{17} + 5000 y_{14,17} \geq 14$
85. $x_5 - x_{10} + 5000 y_{5,10} \geq 16$	170. $x_{17} - x_{14} - 5000 y_{14,17} \geq -4980$

Source: Prepared by the authors (2023)

Table 4. Contains all precedence constraints

Precedence restrictions		
1. $x_9 - x_1 \geq 10$	9. $x_9 - x_7 \geq 21$	17. $x_{16} - x_{12} \geq 23$
2. $x_7 - x_2 \geq 17$	10. $x_{13} - x_7 \geq 21$	18. $x_{17} - x_{13} \geq 12$

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3. $x_{10} - x_3 \geq 13$	11. $x_{15} - x_8 \geq 30$	19. $x_{18} - x_{14} \geq 20$
4. $x_{12} - x_3 \geq 13$	12. $x_{12} - x_9 \geq 25$	20. $x_{17} - x_{15} \geq 26$
5. $x_{10} - x_4 \geq 20$	13. $x_{13} - x_{10} \geq 16$	21. $x_{18} - x_{16} \geq 18$
6. $x_{14} - x_4 \geq 20$	14. $x_{17} - x_{10} \geq 16$	22. $x_{16} - x_{17} \geq 14$
7. $x_8 - x_5 \geq 14$	15. $x_{14} - x_{11} \geq 18$	
8. $x_{11} - x_6 \geq 15$	16. $x_{15} - x_{11} \geq 18$	

Source: Prepared by the authors (2023)

Table 5. Contains all delivery time restrictions

Delivery time restrictions
1. $x_{15} + s_{1a} - s_{1b} = 114$
2. $x_{17} + s_{2a} - s_{2b} = 186$
3. $x_{16} + s_{3a} - s_{3b} = 252$
4. $x_{18} + s_{4a} - s_{4b} = 322$

Source: Prepared by the authors (2023)

Where:

$x_i,\ s_{pa},\ s_{pb} \in z_0^+$
$y_{ij} \in \{0,1\}$

Solution of the Case Study

The model was implemented in IBM ILOG CPLEX Optimization Studio software, version 12.4, running on a personal computer based on Intel(R) Core (TM) i5–8250U CPU @ 1.60 GHz 1.80 GHz with 6.00 GB RAM. The optimal solution was obtained in a time of 9 seconds.

RESULTS AND DISCUSSION

Table 4 shows the solution of the mathematical model for the case study along with the start date of the eighteen tasks executed. These results confirm that the four projects were delivered with 8, 11, 12, and 10 days of delay, respectively, generating a total penalty of S /. 85600. In addition, the optimal work plan implemented for the machine is confirmed, with which it was possible to minimize the total penalty generated by the delays in the delivery of the four projects.

Table 6. Solution of the mathematical model with respect to the case study

Work Plan	Start Date (x_i)	Execution time (t_i)	Delivery Date (f_p)	Delay (s_{pb})	Penalty (m_p)
T6	0	15			
T11	15	18			
T5	33	14			
T8	47	30			
T15	77	26	95	8	S/. 8000
T4	103	20			
T3	123	13			
T2	136	17			
T10	153	16			
T7	169	21			
T13	190	12			
T17	202	14	205	11	S/. 17600
T1	216	10			
T9	226	25			
T12	251	23			
T16	274	18	280	12	S/. 30000
T14	292	20			
T18	312	28	330	10	S/. 30000
Optimal Penalty					**S/. 85600**

Source: Prepared by the authors (2023)

Figure 7 shows the optimal sequence of tasks performed by the machine that minimized delays in the delivery of the four construction projects.

Figure 7. Optimal task sequence
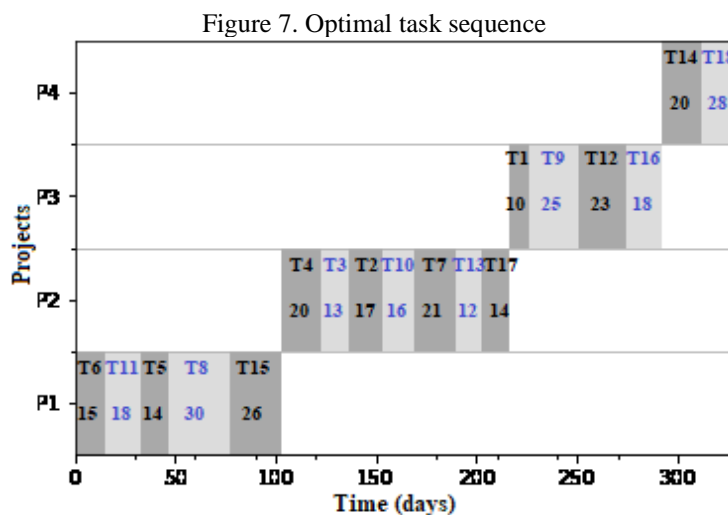
Source: Prepared by the authors (2023)

The numerical results obtained for the case study demonstrated the effectiveness and efficiency of the optimization model used, capable of minimizing the penalty generated by delays in the delivery of construction projects operated by a backhoe. Avolio y Fuduli (2022), developed an optimization problem of a single machine was developed, balancing the weighted

average completion times of two classes of jobs, using a model capable of calculating an optimal solution in all case studies. Low et al. (2016), they studied a problem of scheduling jobs with a single machine, managing to minimize the total delays on an expected expiration date; to accomplished that, they selected the dynamic programming method, which was the most efficient in obtaining optimal cost solutions. It is important to indicate that the researchers cited above got their expected results as was done in this work.

Hwang y Lin (2011), investigated about the programming of coupled tasks of a single machine for to minimize the waiting times subject to a given work sequence. Due to the complexity of the problem, they proposed an algorithm to build a feasible program that solves it; however, the state of the complexity of the problem studied remains open. In our work, it was not necessary to implement a metaheuristic as an alternative solution to the problem; the model considered for the case study provides a solution in 9 seconds.

Afshar-Nadjafi et al. (2017), investigate about project scheduling with rental resources and due date for activities, by finding the start time of activities they managed to minimize the cost per resource and late delivery penalties, their results revealed that an increase in the project deadline decreases the value of the objective function. In our work, the optimization model estimated the start date for the execution of the tasks; with this, we elaborated a work plan for the machine that helped to minimize the penalty generated by project delivery delays. The model studied in Khoshjahan et al. (2013) allows managers to schedule projects with limited resources and deadlines per activity, minimizing delay penalty costs. However, this model not applied to solve a real-life problem. According to Alzara et al. (2016), in Saudi Arabia 70% of public construction projects are delayed, there are nine important delay factors, one of these is the low budget, which has caused the delay of many projects; the researchers affirm that these delay factors are solved through phases with many filters, for example, using only selected and high-quality contractors and socializing risk assessment documents with suppliers. In Peru, the reality is much more worrying with respect to the delay in public construction projects.

In our work, it was possible to minimize delays in the delivery of projects; however, the results obtained can be more accurate if, in a future model, the uncertainty in the processing time of the tasks is considered.

CONCLUSION

The mathematical model we developed estimated the start date of the eighteen tasks that form the backhoe operations network. With this, we formulated a work plan considering the

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tasks' systematization to achieve an optimal sequence. The numerical results demonstrated the model's usefulness in minimizing the penalty generated by delays in the delivery of the four projects, with the optimal penalty estimated during execution equal to S /. 85600.

The results also indicate that this model could help construction companies to obtain an optimal work plan for their machinery (backhoe), thus reducing the delay in the delivery of their projects and avoiding excessive penalties. This study has social relevance, as it favors companies to continue operating, considering that the construction sector is among those that generate the largest workforce in the world. The scientific contribution provided by this study is significant for future work since it shows the application of optimization in a specific area of civil engineering.

The optimization model can be generalized to perform "n" tasks, but due to the high complexity of the problem it must be solved with metaheuristics that, although they do not give the optimal solution, they do return a good solution in a reasonable time.

A limitation of this study is that the uncertainty that may exist in the execution of tasks in a construction project, for example, in the beginning and/or completion of tasks, was not considered, which would imply that in the model, some parameters and/or decision variables are uncertain. In future work, this limitation can be addressed by incorporating uncertainty into the study, and the model can be expanded for a larger number of machines and projects.

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