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Conceptualising one mathematics teacher's process of becoming in relation to teaching mathematics and climate justice: The story of Karl

Conceptualizando el proceso de transformación de un profesor de matemáticas en relación con la enseñanza de las matemáticas y la justicia climática: la historia de Karl

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Abstract ∞ The story of Karl aims to show the complexity involved in navigating issues of climate justice as a mathematics teacher. We present a layered text as both the process and product of our analysis to demonstrate the way in which one secondary mathematics teacher, Karl, embraces contradictions and draws upon multiple forms of knowing during his process of becoming in relation to teaching mathematics and climate justice in the context of a small-scale professional development programme. We illustrate the different ways in which mathematics teaching and climate justice are conceptualised and enacted by Karl before presenting a framework that we call *Teaching Mathematics 4 Climate Justice* (*TM4CJ*), consisting of four dimensions. We show that *TM4CJ* is relevant to mathematics teaching beyond lessons where the content relates directly to issues of climate justice, towards a way of being a mathematics teacher. Implications for mathematics teacher education and professional development are discussed.

Keywords ∞ Climate Justice; Mathematics teaching; Mathematics teacher education and professional development; Layered text

Resumen ∞ La historia de Karl tiene como objetivo mostrar la complejidad que implica abordar cuestiones de justicia climática como profesor de matemáticas. Presentamos un texto en capas como proceso y producto de nuestro análisis para exponer la forma en que un profesor de matemáticas de secundaria, Karl, acepta las contradicciones y recurre a múltiples formas de conocimiento durante su proceso de transformación en relación con la enseñanza de las matemáticas y la justicia climática en el contexto de un programa de desarrollo profesional a pequeña escala. Ilustramos las diferentes formas en que Karl conceptualiza y pone en práctica la enseñanza de las matemáticas y la justicia climática antes de presentar un marco que llamamos *Teaching Mathematics 4 Climate Justice* (*TM4CJ*), que consta de cuatro dimensiones. Mostramos que *TM4CJ* es relevante para la enseñanza de las matemáticas más allá de las lecciones donde el contenido se relaciona directamente con cuestiones de justicia climática, hacia una forma de ser profesor de matemáticas. Se discuten las implicaciones para la formación del profesorado de matemáticas y el desarrollo profesional.

Palabras clave ∞ Justicia Climática; Enseñanza de las matemáticas; Formación y desarrollo profesional del profesorado de matemáticas; Texto en capas

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1. Climate justice

The final synthesis report by the *Intergovernmental Panel on Climate Change* (IPCC) paints a bleak picture of the effects of human driven climate change (IPCC, 2023). At the launch of the report, the United Nations Secretary-General António Guterres called on states to tackle the climate change "time bomb" (Guterres, 2023). This is a dire situation for the planet and most urgent for those most exposed to the impacts of climate change. Climate justice is concerned with the inequitable outcomes that arise as a direct result of a warming planet and the associated policy and practice responses that aim to address climate change, some of which can exacerbate existing inequalities. Many of the individuals and communities most vulnerable to the impacts of the climate crisis are already the most disadvantaged members of society and the least responsible for climate change. Climate justice therefore draws attention to the way in which the consequences of climate change are unevenly distributed and highlights the intersectionality of challenges relating to social, economic, racial, intergenerational, and environmental injustices. Such complex problems are referred to by some as "wicked problems" which are characterised as "unique and highly complex issues that defy complete definition, and for which there can be no clear solution since any resolution generates further issues" (Hawkey et al., 2019, p. 447) and by others as "post-normal situations" which have "a high level of conflict, a high level of uncertainty, and a high level of urgent risks" (Barwell and Hauge, 2021, p. 169). Science alone cannot tell us how to respond and political decisions must be taken, however, policies to prevent climate breakdown have implications for the lives and livelihoods of people. In response to this complex problem, education has been identified as a key pillar for transformation to a sustainable future (UNESCO and UNFCCC, 2016). However, the 'wicked' nature of the problem of climate change makes it difficult to approach in today's classrooms.

One barrier to effective climate justice education is the lack of focus on issues of sustainability within teacher education (TE) and professional development (PD) settings. In this study, we explore a small-scale PD project for secondary (students aged 11-18 years) mathematics teachers in the UK in the context of mathematics teaching and climate justice. Ten secondary mathematics teachers participated in the project consisting of three full-day workshops over the course of an academic year. We focus specifically on one mathematics teacher's (Karl, third author) process of becoming in relation to teaching mathematics and climate justice which we observe through his actions and articulations in relation to the PD project and capture by co-creating *The story of Karl* in the form of a layered text. We use the term *becoming* here to emphasise our focus on *process* (as opposed to product or outcome) which we view as dynamic, continuous, and ever-changing. We wish to avoid any suggestion that an end state (i.e., a state of having 'become') is either desirable or possible. Our findings are presented in the form of a conceptual framework that emerged from the process of creating Karl's story that we call *Teaching Mathematics 4 Climate Justice* (*TM4CJ*).

1.1. Mathematics education and climate justice

Despite a growing understanding of how mathematics is implicated in issues relating to climate change (Barwell, 2013; Skovsmose, 2021), there is still a lack of recognition at policy level regarding the intersection of climate change and mathematics education. In the UK, for example, the recent Department for Education (DfE, 2022) *Strategy on sustainability and climate change* completely overlooks the role of mathematics education (DfE, 2022). Over the past three decades, historically dominant paradigms within mathematics education have received challenges from emerging schools of thought, such as critical mathematics education (CME) (Frankenstein, 1983; Skovmose, 1994a, 1994b). CME in particular draws on ideas from critical theory and critical pedagogy to problematise previous conceptualisations of mathematics and mathematics education. Until more recently, CME has tended to concentrate on issues of *social* justice without explicit reference to environmental issues (Renert, 2011; Barwell & Hauge, 2021). Valero's (2004) socio-political perspective and Gutiérrez's (2011) framework for equity are both examples of work that draw on critical theory in conceptualising mathematics education from a social justice perspective and in creating a practitioner-research network called *Teaching Mathematics for Social Justice Network*, Wright (2021) demonstrates CME in action.

Nonetheless, scholars have begun the task of linking environmental issues and critical approaches to mathematics education (e.g., Barwell, 2013; Renert, 2011; Steffensen, 2021). Compelling arguments from a CME perspective have been formulated regarding the relationship between mathematics and climate change (e.g., Barwell, 2013), the roots of which often derive from Skovsmose's thesis that "*mathematics is formatting our society*" (Skovsmose, 1994a, p. 43). Formatting here refers both to the way mathematics can be used to present us with a (potentially distorted) view of an existing phenomenon which can become further "objectivised when situated in a reality of mathematics" (p. 43), as well as to the way in which mathematics shapes that phenomena, simultaneously changing reality itself. In the case of formatting climate change, Barwell (2022) explains, "the mathematical treatment of climate change *becomes* climate change, such that what we actually respond to are the descriptions, projections, and models" (p. 113, emphasis added). Mathematics and climate change are thus inextricably linked. Particular views of climate change, created by the use of mathematics, may not only conceal many of the social dimensions and associated injustices that arise as a result of the warming planet, they may exacerbate or even cause them.

1.2. Mathematics teacher education and climate justice

In relation to mathematics teacher education, there are more examples that relate mathematics to social justice than to the climate. Though not abundant, examples relating mathematics teacher education in the context of the climate do exist (e.g., Coles, 2022; Coles & Helliwell, 2023; Hauge et al, 2015; Helliwell and Ng, 2021). With this study, we contribute to this growing body of knowledge by exploring mathematics teaching and climate justice from the perspective of a mathematics teacher

(Karl) participating in PD. We chose to tell Karl's story as somebody who has a history, prior to the beginning of the PD project, of introducing climate related issues in his mathematics classroom. This basis allowed us to explore the ways in which Karl's particular orientation towards mathematics teaching and climate justice continued evolving over the course of the project.

We are guided within this particular study by the following two research questions:

> *1) What is one secondary mathematics teacher's process of becoming in relation to teaching mathematics and climate justice?*
>
> *2) How can teaching mathematics and climate justice be conceptualised?*

2. Towards a conceptual framework for Teaching Mathematics 4 Climate Justice

To investigate these questions, we first present our readings of existing and pertinent theoretical frameworks in order to develop a set of conceptual tools for analysing Karl's process of becoming. Since we have been unable to find an existing framework relating specifically to mathematics teaching and climate justice, we draw on a range of relevant frameworks from mathematics education and education for sustainability. Specifically, 1) the three traditions of CME (*Freirean*, *Foucauldian*, and *Nordic*) as summarised by Andersson and Barwell (2021) since these traditions have formed the basis of much mathematics education research in the context of social justice and climate change; 2) Sterling's (2011) three-stage model of educational responses to sustainability; and 3) Barwell and Hauge's (2021) principles for teaching mathematics in the context of climate change. The three frameworks vary in terms of focus and specificity. To operationalise the three traditions of CME (which are more like schools of thought) and Sterling's three-stage model (which relates to education and curriculum more broadly), we draw out implications for the teaching and learning of mathematics and use these implications alongside Barwell and Hauge's principles (already operational) as a set of conceptual tools to analyse Karl's process of becoming in relation to teaching mathematics and climate justice.

Andersson and Barwell (2021) describe three traditions of CME namely the *Freirean* tradition, the *Foucauldian* tradition and the *Nordic* tradition. Each tradition has evolved from different theoretical ancestors, which Andersson and Barwell organise as a way of "making sense of the different strands of thought" (p. 8) from CME. For each tradition, they present the theoretical roots and the way in which mathematics is viewed from each perspective, being careful to point out that they should not "be seen as creating discrete silos of work" (p. 8). In Table 1, we present a view of mathematics from each tradition, along with implications for the teaching and learning of mathematics from each perspective.

Table 1. Three traditions of CME (from Andersson and Barwell, 2021) and implications for mathematics teaching and learning

Views of mathematics from three traditions of CME (Andersson and Barwell, 2021, pp. 8-11)	**Implications for mathematics teaching and learning.**
Nordic tradition From the *Nordic* perspective mathematics is seen as "shap[ing] modern society, often through technology, in often invisible ways" (pp. 10-11).	- Emphasis on the exploration and critique of the mathematics being used. - Not sufficient to know how to do/use mathematics. - Children need to "learn about, and how to critique, the role of mathematics in their lives, in society and not least, as a crucial part of economic development" (p. 11). - Student-centred environment (i.e., students are given a voice to engage in "dialogue, reflection and critique" (p. 11)). - Mathematics classrooms are spaces where social issues and mathematics can come together directly.
Foucauldian tradition From the *Foucauldian* perspective, mathematics is viewed "as a discourse that plays a role in the organisation of human affairs" (p. 9).	- Teaching and learning of mathematics "is not simply a process of transmitting facts and procedures from teachers to students" (p. 9). - Teaching and learning involves challenging "assumptions about the nature of rationality, of students, of teachers and of society" (pp. 9-10) that underly and shape structures and practices in relation to the content of mathematics lessons as well as the classroom environment. - A focus on equitable practices that disrupt a view of mathematics classrooms where the teacher is the authority/arbiter of knowledge. - Democracy is valued and students take a highly agentic role.
Freirean tradition From the *Freirean* perspective mathematics is understood "as a way of reading and transforming the world" (p. 8), it is a "human activity and as such reflects human relations" (p. 9).	- Processes aim to effect change through empowering students to act. - Teaching addresses "the social basis of mathematical knowledge and its implication in the structure of society, in order to empower students to challenge oppression" (p. 9). - Focus on consciousness raising through learning "classical" mathematics (Gutstein, 2006) as well as how mathematics is used to understand (read) the world and to change (rewrite) it. - Normative structures are challenged (e.g., how students are grouped and the opportunities they are presented with). - Students use mathematics as a motivation and as a vehicle for change.

The next framework comes from the work of Sterling (2011), who developed a three-stage model of educational responses to sustainability. In this model we have education *about, for* and *as* sustainability to refer to the stages of *accommodation*, *reformation* and *transformation*. Sterling's model has already been utilised in mathematics education by Renert (2011) who considered how mathematics teachers might approach two particular examples (namely large numbers and chaos) from the perspective of each stage. From Renert's work and our own reading of Sterling's model, we offer (in Table 2) a distillation of Sterling's model along with a set of implications for the teaching and learning of mathematics.

The final framework that we present comes from Barwell and Hauge (2021), both of whom have made significant contributions to mathematics education in relation to climate change. The authors draw on Skovsmose's (1994b) CME perspective (i.e., from the *Nordic* tradition) as well as and that of post-normal science to develop a set of principles to guide mathematics teaching in the context of climate change. In Table 3, we present their 12 principles which they organise into

three groups: *forms of authenticity, forms of participation, and reflecting on and with mathematics* along with a description of each of the three groupings.

Table 2. Stage model of educational responses to sustainability (from Sterling, 2011) and implications for mathematics teaching and learning

Educational responses to sustainability (Sterling, 2011, pp. 60-61)	**Implications for mathematics teaching and learning**
Accommodation (education *about* sustainability) Focused on content/knowledge and is easy to assimilate within existing educational paradigms. Some sustainability concepts are included in the curriculum but it is assumed that knowledge *about* sustainability is uncontested.	- Focus on mathematics curriculum content and knowledge of sustainability concepts. - Issues relating to sustainability provide a context in which to do mathematics (but without critique or reflection on the use of mathematics within that context, or on the issues themselves).
Reformation (education *for* sustainability) Still primarily knowledge focussed but some attention is paid to values and skills. Emphasis is on learning *for* change. Includes some critique and reflection, although positioned largely within the existing paradigm.	- Focus on developing mathematical skills (e.g., reasoning, predicting, justifying) which are motivated through issues of sustainability. - Students use mathematics to better understand issues relating to sustainability. - Students reflect on the use of mathematics and its role in creating issues relating to sustainability.
Transformation (education *as* sustainability) Facilitates transformative learning experiences where the focus is on processes such as being creative and reflexive as well as imagining, participating and cooperating. Knowing is relational and provisional and emerges from continual exploration through practice.	- Focus on process as opposed to content (both in mathematical terms and sustainability terms but also in relations between the two). - The classroom is a democratic space where students work on meaningful problems *alongside* the teacher. - Students explore their own questions and seek creative and imaginative solutions as a community both in the classroom and beyond.

Table 3: Principles for teaching mathematics in the context of climate change (Barwell and Hauge, 2021, p. 177).

Principles for teaching mathematics in the context of climate change (from Barwell and Hauge, 2021, pp. 172-177)	
Groups of principles	**Specific principles**
Forms of authenticity Students engage with climate change in relevant and authentic ways, through engaging in meaningful debate, using data and information that relates to actual local concerns, or to students interests and values now or in the future.	Authenticity through... - relevant problems - real data - students' own ideas and values - classroom discussion
Forms of participation Participation is key to preparing students to contribute actively to their communities. This includes the posing of problems to be investigated through productive collaborations, including discussion about, reflection on and critique of one another's ideas. The results of these investigations could lead to changes in the students local communities.	Participating... - in mathematics - in classrooms - in community - in public debate
Reflecting on and with mathematics Reflection is key to students' understanding of the role of mathematics in society. Students develop a critical awareness of how mathematics is used to understand climate change and how it can be used and misused to respond in particular ways, to serve certain purposes and undermine others.	Reflecting... - on how mathematics is useful - on limits of mathematics - on values of mathematics - on uncertainty

The right-hand columns of Table 1, Table 2 and Table 3 together comprise the set of conceptual tools we utilise in our process of analysis.

3. Methodology

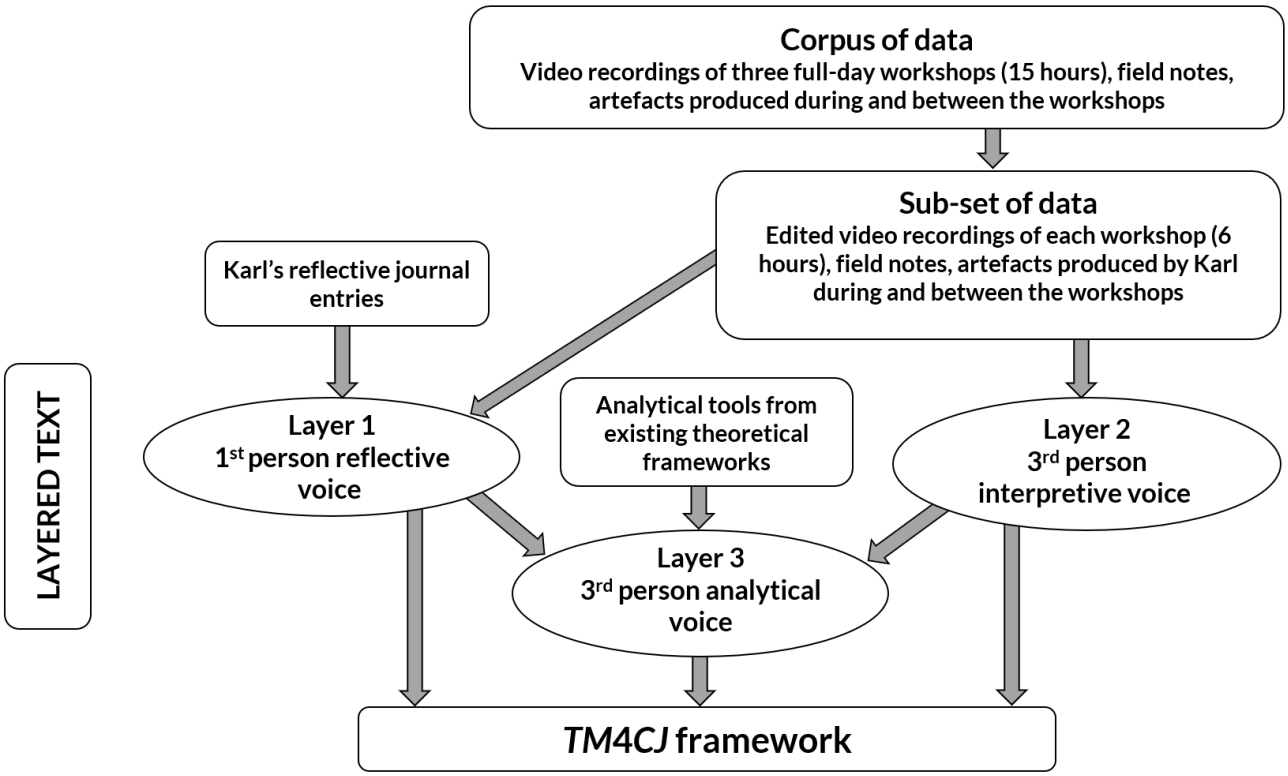
The workshops were designed to support participants through incorporating a range of arts-based and creative processes alongside quantitative modelling processes. Facilitators of the PD workshops comprised a multi-disciplinary team including two MTE researchers (Tracy and Lauren), a research mathematician, a practicing site responsive artist, a novelist and creative writing lecturer and a geographical scientist. Over the course of the project, data was collected in the form of video recordings of the three workshops, field notes, and artefacts produced during the workshops. Teachers were also invited to keep their own notes and ideas in their journals. We (Tracy and Lauren) reviewed the corpus of data which included approximately 15 hours of video, to locate all instances where the participants were making audible contributions and created an edited video of each workshop by stitching these instances together. To engage deeply with the data, we decided to create a "layered text" (Rath, 2012), that is, the creation of a narrative (Karl's process of becoming in relation to mathematics teacher and climate justice) through different lenses. The creation of the layered text is both the process and the product of our data analysis. Our decision to focus on one teacher's lived experiences over the course of the project reflects our conviction that much can be learned through deep exploration of individual cases.

3.1. Creating a layered text

The process of creating our layered text and the *TM4CJ* framework is summarised in Figure 1. Layer one (a first-person autobiographical voice, marked in bold) comprises Karl's retrospective reflections which he produced while watching the edited videos and revisiting artefacts and journal entries from the project. To create the second layer (a third-person interpretive voice, marked as underlined), we (Tracy and Lauren) transcribed the edited videos of each workshop, examined our field notes and had several discussions about what we saw as significant to capture in our interpretive accounts. We permeate layer two with extracts of transcript from the edited video recordings (marked in the text as "quotes") to anchor the accounts with Karl's words. We saw the process of writing these accounts as a significant phase of our process of analysing the data, and although we actively avoided using our conceptual tools at this stage, we acknowledge there is an element of theorising that takes place through a process of "aesthetic structuring" (Winter, 1988, p. 233). With the two layers constructed, we worked as a group of three to weave the layers into a coherent story, along with a leading narrative (in plain text) to provide context in relation to the workshops. For the third layer (a formal-analytical voice, yet still narrative in style, marked in plain text), we read and re-read the first two layers, identifying those concepts, ideas and principles that we viewed as corresponding with our conceptual tools (marked in *italics* throughout the third layer). Having produced the layered text, our last stage was to present the story as a series of episodes, using labels that emerged through the process of analysis that ultimately

became three of the four dimensions of our *TM4CJ* framework (teaching mathematics *about*, *with*, and *for* climate justice) inspired by Sterling's (2011) *about*, *for*, *as* labels from a sustainability context. We placed these labels (as subheadings) at points where we observed significant shifts in Karl's perspective to indicate the emergence of a dimension of *TM4CJ* (but not to suggest the episode relates solely to that category). This process led to a conceptualisation of *TM4CJ* that we present in section 5.

Figure 1: Process of creating the layered text and the *TM4CJ* framework



Karl's story commences from the beginning of the first workshop.

4. The story of Karl (the layered text)

Karl was one of ten local secondary mathematics teachers who chose to participate in the project. At the time of the initial workshop, Karl had been teaching mathematics for six years, and like other participants, he had already introduced his students to global issues relating to climate change as context for doing mathematics.

> **I wanted to show my students how they can use the mathematics they have learned to make sense of the changing world around them. We have calculated how much the sea levels would rise if the polar ice caps melted and investigated how long it would take for the world's oil supplies to run out, as well as the world's rainforests, should deforestation continue at the current rate it is today.**

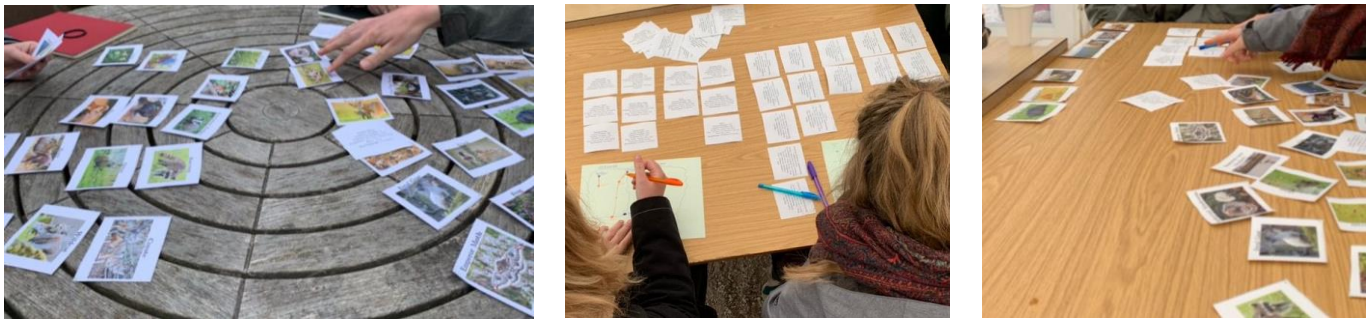
Episode 1: Teaching mathematics *about* climate justice

During the first workshop, teachers engaged in a creative writing activity led by Emma (the creative writer on the project) who presented a set of increasingly severe dystopian scenarios over a series of decades, followed by a set of increasingly utopic scenarios over the same series of decades. Teachers were invited to write in response to each of the scenarios, to imagine themselves living through each scenario retaining their identities as mathematics teachers throughout the writing process, and to consider what their teaching would look like. Having engaged in this activity, the teachers explored, with one another, the various assumptions unearthed during the writing process.

> Karl is noticing potential contrasts between mathematics relating to climate justice and mathematics "typically taught in schools". He reflects on the complexity of "capturing the 'real' world through mathematics", challenging an assumption that "there is a 'best' way to describe a problem mathematically", particularly in cases where there is so much uncertainty. He is becoming aware of several assumptions that he would like to challenge relating to mathematics and its applications, such as "models can accurately reflect and predict real-life phenomena" and "mathematics is free of ethical responsibility" as well as assumptions relating to students such as "there are some people that just can't engage with a problem mathematically".

Following a visit to the local Botanical Gardens, a place where rare species of local fauna are preserved, the teachers worked in groups on a task relating to native species of flora and fauna. The aim of the task which was led by Emilia, the mathematician on the project, was to find the largest network of species that could coexist based on various information (environment, predators, etc.), presented on a set of cards (one card per species, 50+ cards, see Figure 2).

Figure 2. Card activity and networking



> **Mathematical modelling is a feature of the advanced mathematics curriculum. Sometimes I tell my students to keep their initial models simple so we can quickly and easily approximate to a general system. I can imagine my students using computers to construct more complex models than is possible to do by hand, although I don't yet know how to do that myself. In the real-world, models can be adjusted to reflect certain actions and beliefs, and if we are not careful to challenge the basis of these models, it can become easy to accept**

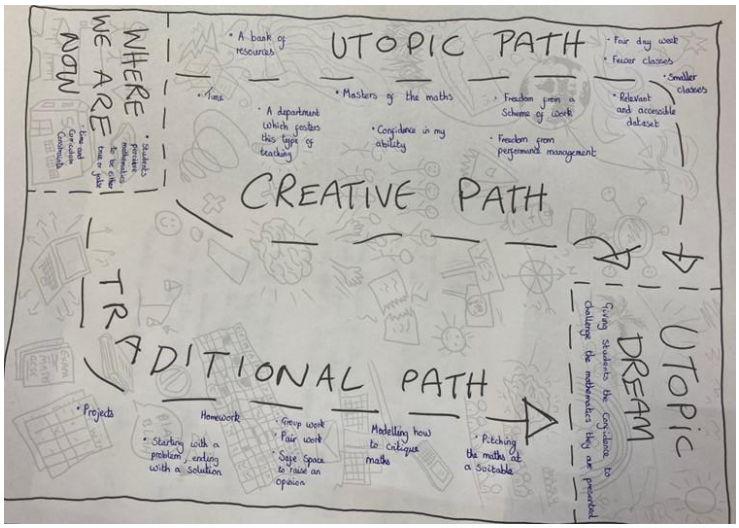
> **their conclusions as fact. I am concerned that mathematical systems are being tweaked in order to prioritise action in one particular part of the world, not because it is fair, but because it fits someone's agenda. Do I ever encourage my students to critique how mathematics is used and the impact it can have on society, or the environment? Perhaps I wasn't aware of it myself. I would like my students examining whether models accurately reflect and predict real-life phenomena, whether certain mathematical choices have social, political, or environmental implications. What examples are there of mathematics being embedded in technologies that have real social or ecological effects?**

Karl is contrasting different views of mathematics, suggesting how the mathematics of mathematics and climate justice is complex and uncertain, and requires multiple approaches and methods. He is beginning to interrogate the role, use, value and consequences of mathematical models, and mathematics itself. This focus on values and critical reflection suggests a move to Sterling's *reformation* stage. There are hints of Karl beginning to question assumptions about students of mathematics, the beginning of a *Foucauldian* framing although Karl is still positioning himself as the knower ("I don't yet know how to do that myself"). The *justice* of climate justice is emerging for Karl which alongside his growing awareness of the role of mathematics leads him to consider specific practices relating to *reflecting on and with mathematics*, and specifically, *on how mathematics is useful* and *on values of mathematics*. His primary focus seems to align with concerns from the *Nordic* perspective.

Episode 2: Teaching mathematics with climate justice

Teachers engaged in a creative mapping activity led by Emma. For this task the teachers were asked to formulate their 'utopic goals' in relation to teaching mathematics and climate justice, to which they were encouraged to map three different pathways (see Figure 3): a 'traditional' pathway (made up of steps they might usually take as mathematics teachers), a 'utopic' pathway (where *anything* is possible) and a 'creative' pathway (a middle way between the first two). Each teacher was invited to complete their own mapping, and to use this mapping to transform their ideas and goals into concrete (but creative) actions which they would then try out in school before the final workshop.

Figure 3: Karl's partially completed utopic map



> Karl wants to "challenge traditional modes of teaching mathematics" and offer students new ways of seeing the subject. He speaks of students as becoming "meta-mathematicians" who, through "engaging in critical discussion", can be prompted to "ask their own questions" and to develop a critical awareness in relation to mathematics. Karl imagines a classroom where students can plan and follow their own lines of inquiry. He would like "students to challenge the mathematics they are presented", to have "belief in themselves and their own abilities". He has been working on building his student's confidence through inviting advanced mathematics students into his classroom when he teaches younger students. He believes that "the younger students come away feeling inspired". The introduction of role-models challenges a view of Karl as the arbiter of knowledge by his students, a perception that Karl is keen to disrupt through a process of democratisation, where he provides his students with "a platform" on which to share views, and to "vote on something they care about".

As a more experienced teacher, I recognise my initial approach to teaching mathematics disregarded the actual challenges my students might face outside of the classroom and spoke nothing of our shared concern for environmental sustainability. I now want to support my students in becoming reflective, democratic, and active members of society by equipping them with knowledge to navigate the discourse of a mathematically written world. I envisage transforming my practice to operate more flexibly within the constraints of the curriculum. I imagine my students discussing what constitutes a proof and being empowered to investigate their own conjectures.

> Karl intends to ask his students "what assumptions they might have about mathematics" before sharing with them a set of visual prompts that relate to issues of climate justice as a way to invite questions from the students. He wants this to be an "organic" experience, without a tight plan, open for the students to determine the direction of travel. He sees himself more as a "subject specialist that guides them by answering questions they might have" as opposed to being the person who makes all of the decisions, who knows what issues the students should care about, and plans tightly around those constraints.

Karl's focus is steering more towards pedagogical questions which we relate to Sterling's *reformation* stage. He describes *forms of authenticity* such as utilising the *students' own ideas and values* and critical *classroom discussion.* He imagines his students *reflecting on and with mathematics* do develop a critical awareness in relation to mathematics. He explores notions of power in a *Foucauldian* sense which includes a repositioning of himself in relation to his students, his students in relation to one another, and his students in relation to mathematics. Although this repositioning is a condition for what Sterling describes as the *transformation* stage, Karl's sense of staying within the constraints of the curriculum, suggests his perspective remains more aligned to *reformation*. He is beginning to envisage himself letting go of being the knower, utilising *forms of participation in mathematics*, such as students selecting problems to work on. Karl is starting to consider how to empower his students, both in the *Foucauldian* sense of democracy and in the *Freirean*

sense of empowerment through developing mathematical skills that allow students to read the world.

Episode 3: Teaching mathematics for climate justice

I do not have access to scientific models or, in some cases, understand the mathematics used to create them. I wonder whether I have the right to dictate which issues of climate justice we investigate. On the one hand, I have a responsibility as a teacher to choose activities I deem beneficial to the learning of my students. On the other, it is my responsibility to ensure I am not prioritising one issue over another. When I am planning particular activities, I could consider the following questions: Does the issue raised relate to my students' personal experience? Does it inspire action?

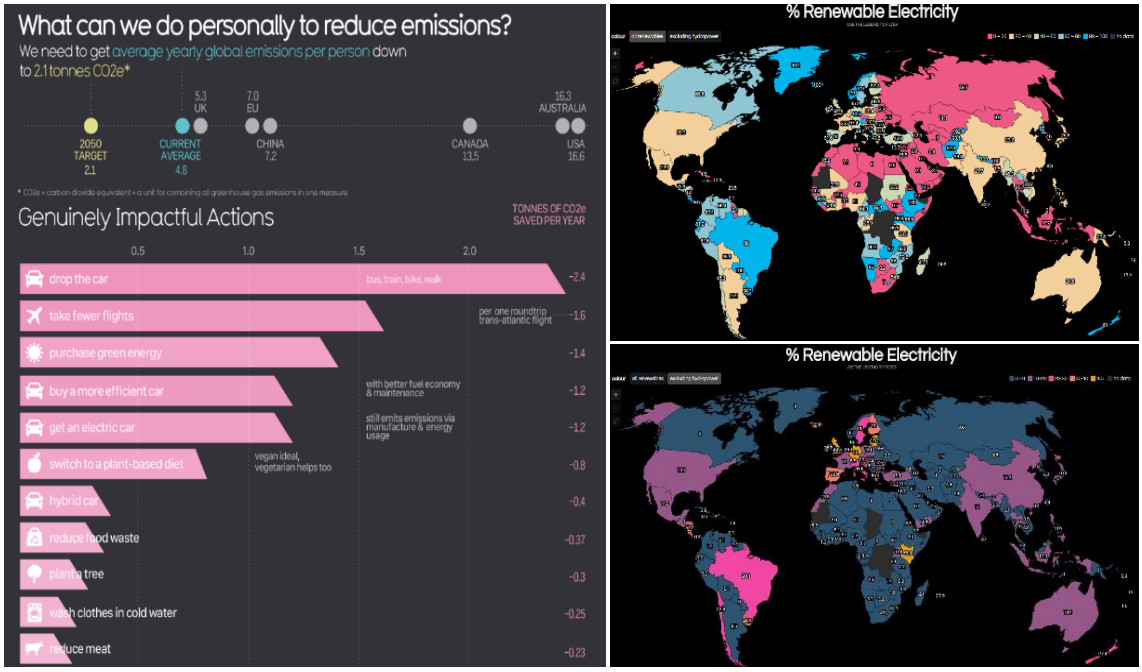
Karl continues to value democracy as he imagines offering his students choice, realising that issues of climate justice can be localised and have the potential to inspire action (in the *Freirean* sense). From the process of utopic mapping, teachers begin to enact some of their 'creative' pathways in their schools and classrooms. Over the course of a few months, the teachers were invited to realise some of what they had committed themselves to. In coming back together, the group shared their experiences and rehearsed their ideas with one another.

> Karl has come to recognise the potential of language, and its relationship with power. He extends language to include all forms of information, including statistics and worries that those in powerful positions take advantage of mathematical language to "push their own agendas". If mathematics is perceived as a form of truth, or you lack the confidence to question what is being presented, then "conclusions might be simply accepted". Karl wants his students to leave school with the "confidence to challenge the mathematics that they might be presented" but he worries whether questioning models of climate change could suggest questioning climate change itself.

Having begun framing mathematics teaching and climate justice using notions more associated with a *Foucauldian* perspective (e.g., power, discourse), Karl's focus moves towards those notions more associated with the *Freirean* tradition. He takes the perspective of empowerment which includes the need for mathematical literacy (i.e., to *read* the world) and the associated confidence this brings. With this confidence to read the world mathematically, Karl imagines his students could be empowered to make changes. He is thinking to the future, in more *transformational* terms.

> Karl has been offering his students "activities where [he] gives [his] students information based on some mathematics" using publicly available infographics [Figure 4]. He has been explicit with his students that there are no 'right' answers to the questions posed, and stresses the focus is on being "critical in relation to the sources of information". He designs questions [Figure 5] that invite students to "pause, take a step back, and ask where the information has come from". Over time, he wants his students "to come up with their own questions", to develop a critical stance of their own.

Figure 4. Infographics used by Karl (Image credited to: David McCandless, www.informationisbeautiful.net)



He has been asking his students to share their ideas and to raise any points of interest. So far he has limited these kinds of activities and is exploring how he can develop these further whilst still covering the necessary mathematics curriculum content.

Figure 5. Question sheet used by Karl alongside the emissions infographic

How might the *average yearly global emissions per person* for each of the given countries be calculated? Similarly, how might the *tonnes of CO_2 saved per year* be calculated for each action?

Do you think the model for calculating these averages would be fair? Consider here the balance between simplifying the model and including as many parameters as possible.

Does it matter who is measuring this data?

Why might the average for the UK, EU, China, Canada, Australia and the USA be included? Should we pay attention to the emission levels of other countries?

How does this infographic make you feel? Does it persuade you to act differently?

Karl values *forms of authenticity and participation* such as using *relevant data* which allows his students to engage in *meaningful and public debate.* In the future he imagines students *participating in mathematics* by coming up with their own questions, a move further towards Sterling's *transformation* stage. He has been offering his students a range of questions, providing spaces for his students to *reflect on and with mathematics* in the *Nordic* sense of critiquing mathematical models, in

the *Foucauldian* sense of considering notions relating to power, and in the *Freirean* sense of empowerment. Karl's vision of mathematics teaching in relation to climate justice is beginning to push the limits of *reformation* although still framed within perceived constraints of the mathematics curriculum.

> Karl is developing his sense of what it means to be critical in relation to climate justice. It involves "considering a range of perspectives… thinking about other people's situations, putting yourself in their shoes… seeing the bigger picture and considering the implications of any one particular action". He is moving away from a focus on "individual action" to those of the "collective", a change he recognises in himself through his use of "we" when addressing his students. He has presented his youngest students with an online "world average temperature" counter, displaying to 10 decimal places, asking them to "imagine they were government advisors" and "to decide on which accuracy to report on". He wants his students to understand the importance of context in relation to orders of magnitude, and the how mathematical choices made can give significantly different views of the world. He wants to retain an emphasis on mathematics and will continue exploring how he can achieve this whilst building in time for his students to reflect on the wider implications.

> **Through group discussions, I have learned how I can take further the idea of collective action when relating mathematical activities with issues of climate justice. Instead of asking my students what action *they* might take, I reframe that by asking how *we* might take action. I have begun focussing on nurturing mathematical behaviours such as predicting and representing as well as looking at things on a smaller scale, closer to home, where it feels more meaningful. Students bring prompts, and we respond to them, asking where is the mathematics? And what mathematics can we do to help us understand the situation? If it is more meaningful, and comes from the students, I think it is more likely to inspire change.**

Karl is enacting processes associated with Sterling's *transformation* stage as his understanding of climate justice is evolving through *continual exploration through practice*, and he is moving towards this image within his own classroom, placing himself alongside his students, no longer always the one who knows. He has designed activities where mathematics can be used to understand real situations and real situations to understand mathematics and he envisages making more space for his students to *reflect on and with mathematics.* His framing of teaching and learning has shifted in relation to his evolving conception of climate justice so that as a teacher he sees himself as supporting his students through making collective change, in the *Freirean* sense, utilising all *forms of authenticity and participation.*

5. Discussion and emergent framework: *TM4CJ*

Throughout the process of analysis we identified concepts, ideas and principles from across the existing three theoretical frameworks summarised in section 2, not in clearly demarcated sections, but woven throughout the story. We recognised points at which the story where Karl's perspective seemed to shift significantly and

labelled each episode to reflect these shifts. In Table 4, we present our conceptualisation of *TM4CJ* that emerged through creating the layered text which includes existing ideas and principles from our conceptual tools along with new concepts, ideas and principles that emerged from capturing Karl's process of becoming.

The four dimensions of *TM4CJ* are not intended to suggest a hierarchy or a series of stages. We see the dimensions as offering a range of possibilities, each with different emphases and the potential to be enacted simultaneously. The last dimension was not an episode that featured explicitly in Karl's story. We see this dimension as requiring a paradigm shift in the same way as Sterling's (2011) educational *as* sustainability requires and we consider this kind of transformation to involve a paradigm shift at an institutional/policy level. Thus, our description of teaching mathematics *as* climate justice was more speculative, inspired by Sterling's work as well as Karl's utopic imaginings articulated during the workshops and in his retrospective reflections. For this reason, we do not include examples from Karl's story, as we do for the other three dimensions.

Table 4: The four dimensions of Teaching Mathematics 4 Climate Justice (*TM4CJ*)

	Description of dimension	Examples from Karl's story
Teaching mathematics ***about*** climate justice	Climate justice is the context in which to do mathematics. Mathematics is viewed as complex and uncertain, requiring multiple methods when approaching problems. The teacher is largely responsible for the design of the tasks, the questions and prompts and much focus is given to practices such as reflecting on and with mathematics. There is a focus on interrogating the role, use, value and consequences of mathematical models, and mathematics itself. This dimension links most closely with the concepts and ideas associated with the Nordic tradition of critical mathematics education.	- Using computers to construct more complex models than is possible to do by hand so focus can be on challenging the assumptions these models are based on. - Encouraging students to critique how mathematics is used and the impact it can have on society, or the environment. - Examining whether proposed models accurately reflect and predict real-life phenomena. - Asking if certain mathematical choices have social, political, economic or environmental implications. - Using examples where mathematics is embedded in technologies that have real social or ecological effects.
Teaching mathematics ***with*** climate justice	Emphasis is on the justice of climate justice, which is enacted through the creation of a democratic classroom where students' views are valued, discussed and critiqued. The teacher acts more as a guide, commenting on the process of mathematical thinking and reflection, as opposed to evaluating correctness. There is a focus on developing mathematical thinking skills such reasoning, predicting, analysing, as well as critical thinking skills that include an explicit aim of uncovering assumptions. Authenticity is a key principle, that is realised through students' choice of climate related problems and a process of collaboration and classroom discussion. This dimension links most closely with the concepts and ideas associated with the Foucauldian tradition of critical mathematics education.	- Positioning students as "meta-mathematicians". - Students asking their own questions, planning and following their own lines of inquiry, investigating their own conjectures. - Students challenging the mathematics they are presented. - Students of varying ages working together and supporting one another. - Providing a platform for students sharing their views and collectively deciding on what they care about. - Using visual prompts relating to issues of climate justice as a way of inviting questions from students. - Allowing for "organic" experiences, without tight planning and being open for students determining the direction of travel.

	Description of dimension	Examples from Karl's story
Teaching mathematics ***for*** climate justice	Climate justice relates directly to the lives of the students and their environments. Mathematics is viewed as a vehicle for positive change. The emphasis is on using all forms of mathematics for empowerment, both individually and collectively, to effect change at a level that is local and/or of relevance to the students. The teacher is situated *alongside* the students and the classroom is a framed as a community space. Forms of participation are central, allowing students to engage in meaningful and public debate which can lead to concrete changes in climate-related issues of local concern. This dimension links most closely with the concepts and ideas associated with the Freirean tradition of critical mathematics education.	- In planning, considering the following questions: Does the issue raised relate to my students' personal experience? Does it inspire action? - Using publicly available infographics on climate-related issues that are based on mathematical models, and are relatable and relevant to the students. - Exploring implications and emotional responses of students, e.g., how does this information make you feel? Does it persuade you to act differently? - Being explicit that there are no 'right' answers and referring to the classroom as a community. - Framing possibilities for action in terms of what action 'we' could be taking. - Inviting students to bring their own prompts and supporting students to engage in critical debate, coming up with their own questions, sharing their ideas, and raising points of interest or concern.
Teaching mathematics ***as*** climate justice	Climate justice is embodied in the structures, practices and norms of mathematics education. Classrooms stretch beyond boundaries, both physically and interdisciplinarily and consist of communities of diverse groups of students and teachers who plan alongside one another on the basis of their collective concerns. Learning by all members of the community is relational and emergent and exploration is the key mode of being. Climate related issues are considered from a range of disciplinary perspectives, including mathematics, and mathematics is critiqued in relation to issues in focus through processes of embedded reflection. Authenticity and participation are norms. Within this dimension, all notions from a critical mathematics education perspective are applicable but are themselves held up for constant examination and scrutiny.	

By prioritising the future of the planet we need not abandon our endeavour to work with teachers of mathematics to create classrooms where mathematical thinking, exploration and curiosity are central. Teaching mathematics *with* climate justice, with its pedagogical focus, is as important and relevant to *TM4CJ* as those dimensions where the content of lessons links directly to issues of climate justice. Through telling Karl's story, we have come to realise that *TM4CJ* can be as much about a way of being with students, a process of democratisation, giving students a voice, and listening. It can be through inquiry-based learning that students learn to become self-aware, critical and questioning, all kinds of mathematical behaviours promoted across the mathematics education community, including those who do not consider themselves to be informed by a critical perspective.

The work we do as academic members of the mathematics education community is essential. We need theories that help us understand the complexities of teaching and learning mathematics, especially in a time of crisis. These theories are most likely to be enacted if we develop them alongside the community of mathematics teachers. Thus, one role of mathematics teacher education is to provide spaces where MTEs work *alongside* teachers of mathematics to engage together in a collaborative process of exploring what *TM4CJ* means for them, to continuously develop their own (embodied and articulated) frameworks. If we had chosen to follow a different teacher's process of becoming, we suspect a different framework would have emerged. It is likely there would be many overlapping features, but we would also expect to see differences. This does not mean we see our framework as

being of less use, in fact, we imagine it could be a starting point for mathematics teachers or MTEs wanting to introduce aspects of climate justice into their classrooms, TE or PD programmes. For us, the process of developing the framework was a process of becoming, for us as teachers, MTEs and researchers. In terms of next steps, we would like to observe Karl (and other participating teachers) in action, to consider questions relating to what and how students learn when the concepts and principles from our framework are enacted in the classroom and how these observations and experiences further inform our conceptualisation of *TM4CJ*.

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Conceptualizando el proceso de transformación de un profesor de matemáticas en relación con la enseñanza de las matemáticas y la justicia climática: la historia de Karl

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En este estudio, exploramos un proyecto de desarrollo profesional a pequeña escala para profesores de matemáticas de secundaria (estudiantes de 11 a 18 años) en el Reino Unido en el contexto de la enseñanza de las matemáticas y la justicia climática. Diez profesores de matemáticas de secundaria participaron en el proyecto que consiste en tres talleres de un día completo en el transcurso de un año académico, facilitado por un equipo multidisciplinar formado por dos investigadores formadores de profesores de matemáticas; un matemático investigador; un artista en activo, un novelista, un profesor de escritura creativa; y un científico geógrafo. En este artículo, nos enfocamos específicamente en el proceso de transformación de un profesor de matemáticas en relación con la enseñanza de las matemáticas y la justicia climática que observamos a través de sus acciones y articulaciones en relación con las actividades de desarrollo profesional y captamos mediante la co-creación de *la historia de Karl* en forma de un texto en capas. Presentamos este texto en capas como el proceso y el producto de nuestro análisis para exponer la forma en que Karl acepta las contradicciones y se basa en múltiples formas de conocimiento a lo largo del proyecto. Nos basamos en marcos teóricos existentes y pertinentes de la educación matemática crítica y la educación para la sostenibilidad para desarrollar un conjunto de herramientas conceptuales que identificamos a lo largo de la historia de Karl y, al hacerlo, ilustramos las diferentes formas en que Karl conceptualiza y promulga la enseñanza de las matemáticas y la justicia climática. Finalmente, presentamos un marco que llamamos *Teaching Mathematics 4 Climate Justice* (*TM4CJ*) que consta de cuatro dimensiones (enseñanza de las matemáticas *acerca de* la justicia climática; enseñanza de las matemáticas *con* justicia climática; enseñanza de las matemáticas *para* la justicia climática; y enseñanza de las matemáticas *como* justicia climática) que surgieron a través del proceso de análisis. Mostramos que *TM4CJ* es relevante para la enseñanza de las matemáticas más allá de las lecciones donde el contenido se relaciona directamente con cuestiones de justicia climática, hacia una forma de ser profesor de matemáticas. Se discuten las implicaciones para la formación y el desarrollo profesional del profesorado de matemáticas a la luz del marco del *TM4CJ,* así como el proceso por el cual surgió el marco.