RHV, 2023, No 22, 105-130 eISSN 0719-4242
DOI: https://doi.org/10.22370/rhv2023iss22pp105-130
Sección Artículos-Miscelánea / Articles Section

Interactive Hypotheses: Towards a Dialogical Foundation of Surrogate Reasoning[1]

Hipótesis interactivas: hacia un fundamento dialógico del razonamiento sustitutivo

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Abstract

The aim of this paper is to propose a logical justification for the process of surrogate reasoning in modeling practice in science. To this end, we understand hypothesis generation as the creation of an interactive, formal dialogue between the model and the target system. In order to describe this idea from a logical point of view, we will rely on the pragmatic approach of Dialogic as the ideal framework to illustrate logical interactions.

Keywords: hypothesis, interaction, scientific modeling, dialogic, surrogate reasoning.

Resumen

El objetivo de este artículo es proponer una justificación lógica del proceso de razonamiento sustitutivo en la práctica del modelado en ciencia. Para ello, definimos la generación de

[1] This work was made possible thanks to the support of ANID: FONDECYT Regular No. 1221132 and FONDECYT Postdoctoral No. 3210531.

Received: 01/11/2022. Final version: 23/05/2023

hipótesis como la creación de un diálogo interactivo y formal entre el modelo y el sistema objetivo. Para describir esta idea desde un punto de vista lógico, nos basaremos en el enfoque pragmático de la Dialógica como marco ideal para ilustrar las interacciones lógicas.

Palabras claves: hipótesis, interacción, modelización científica, diálogo, razonamiento sustitutivo.

1. Introduction

> La recherche scientifique est donc un dialogue entre l'esprit et la nature. La nature éveille notre curiosité; nous lui posons des questions; ses réponses donnent à l'entretien une tournure imprévue, provoquant des questions nouvelles auxquelles la nature réplique en suggérant de nouvelles idées, et ainsi de suite indéfiniment.
>
> (Bergson, 1934, p. 258)

This paper aims to propose a logical-dialogical approach to the generation of hypotheses in the practice of modeling in science. Our proposal should be considered as a response to Contessa's statement about surrogate reasoning: "an activity as mysterious and unfathomable as soothsaying or divination" (2007, p. 61). In effect, Contessa refers to the 'obscure' relation between epistemic representation and valid surrogative reasoning. Nevertheless, the aim of the present article is not to clarify said relation, as we distance ourselves from the notion of representation. On the contrary, our objective is to fill the void described by Contessa with a proposal which connects surrogate reasoning to a logical foundation. Nor do we argue against the idea of grounding surrogate reasoning in the notion of representation. Therefore, our proposal of a dialogical approach to hypothesis generation, which does not require the above argumentation, should be considered on a stand-alone basis.

When we discuss the relation between representation and surrogate reasoning, we refer to the understanding of surrogate reasoning as, according to Frigg & Nguyen (2017), a "kind of representational-based thinking". The idea, presented by Swoyer (1991, p. 449), has been defined as a mode of thinking based on representation, considering the relation between *Models* (hereinafter *M*) and their *Target Systems* (hereinafter *TS*) as a relation of *structural representation*:

> Structural representation allows us to reason directly about a representation to draw conclusions about the things it represents. By examining the behavior of a scale model of an aircraft in a wind tunnel, we can draw conclusions about the response of a newly designed wing to wind shear, rather than testing it on a Boeing 747 over Denver. By using numbers to represent the lengths of physical objects, we can represent facts about objects numerically, perform calculations of various types, and then translate the results

> into a conclusion about the original objects. In these cases, we use one type of object as a substitute in our thinking about another, so I'll call it surrogative reasoning. (Swoyer, 1991, p. 449)

Although our proposal is refractory to the notion of representation, we will incorporate two notions from Swoyer into our approach, albeit with different implications: *hypothesis* and *substitution*. Thus, our proposal considers, on the one hand, that surrogate reasoning is the generation of *hypotheses* and, on the other, that we generate hypotheses about *TS* by reasoning from a *substitute* object: the *M*-model. The real challenge is to arrive at an explanation in purely logical terms.

2. Surrogate Reasoning and Mystery

We propose that the mysterious character referred to above by Contessa alludes, in general, to the very peculiar act of thinking using a substitute. We can certainly think 'as if' we were generating conclusions directly in *TS* (cf. Vaihinger, 1927; Fine, 1993). But the latter, from our point of view, only highlights the need to clarify the subrogating character of the model–especially from an inferential point of view. Let us try to represent this peculiarity using an example. An agent can draw conclusions regarding a size 5 soccer ball: 'it is made of leather', 'it is a bit deflated', and so on. But he could also see the ball as representing Planet Earth and from there the statements he creates are the true challenge of surrogate reasoning: they are statements of the ball but about the Earth, however they are also about the ball, although not exactly about the ball. For example, a slightly deflated ball can inspire the agent conclude–and rightly so–that the Earth is not a perfect sphere and that it has a non-homogeneous distribution of matter in its volume, from which it is inferred that the center of mass is located in a place which must generate certain particularities in its rotational motion on itself, and even predict an oscillation in the Earth's axis of rotation. Nonetheless, all the above statements could also be considered statements about the ball, otherwise the ball would not be the model of the Earth, right?

There appears to be no conclusion obtained in *M* that is not also a conclusion about *M*, despite the small adjustments we make to consider it in *TS*. For example, a map of subway stations prioritizing distances proportionally allows us to infer that getting from $subway_1$ to $subway_2$ will take us twice as long as getting from $subway_2$ to $subway_3$. This conclusion alluding to real distances and times (intended to be evaluated), however, is an adjustment of the conclusion with regards to the relation of the distance between points on a map. In a way, it is the same statement, but with a different character on each side.

In regards to the above statement, it is necessary to clarify that our approach does not attempt to propose a notion of representation. Nor a criterion for the elaboration of suitable models according to different types of *TS*. To some extent, our contribution presupposes that the nexus between an *M* and its respective *TS* has already been established (correctly or incorrectly), and we aim to justify the implications of generating a hypothesis between *M* and *TS*, i.e. surrogate reasoning, from a logical point of view. In other words, one should justify why conclusions obtained in *M* are considered in *TS*, even if they are all later falsified or negatively evaluated in *TS*. From this perspective, the current proposal is not limited to the justification of surrogate reasoning in the reconstruction of successful modeling cases.

Furthermore, it is necessary to clarify the difficulty in contrasting our approach with other perspectives which justify surrogative reasoning. Indeed, specialized literature on the subject which indicates–from our point of view–clear and systematic justification of what it means to generate a hypothesis from a model, has not yet been presented. It is from this standpoint that Contessa's statement can be understood. The existing, varied approaches to reasoning with models have not as yet offered a definition, and have offered a mere mention of the notion of representation as a solution. For example, within the semanticist approach (Suppes, 1960, 1970; Stegmüller, 1970; Balzer *et al*., 1987; Suppe, 1989), we could interpret and discuss the problem from the notion of *empirical assertion*, which describes the relation between real empirical systems and theoretically-defined models. However, its aim is only the theoretical and mathematical reconstruction of scientific theories in terms of a class or set of models–defined as adequate representations of phenomena (Díez and Moulines, 2016, p. 348). According to this approach, these representations, when effectively applied to the world, allow theories to describe the characteristics of the world's phenomena. As a demonstration, semanticists construct quite efficient axiomatic characterizations (in set theory terms) of the relation between a model and the system of phenomena, defined in terms of a morphism (homomorphism, isomorphism, among others). This assumes the existence of mathematical and phenomenal structures, which they are placed in relation to. That is to say, this relation is understood as a dyadic relation of correspondence (or identity, approximation, subsumption, etc.) between the representative structures and the phenomenal structures to which they are directed. The entire discussion is then centered on finding the best characterizations of how it is that a scientific model *M* represents its target *TS*. But the latter occurs if and only if *TS* is isomorphic to *M*.

In addition, we have the case of the *similarity* perspective (or cognitivist perspective) of Ronald Giere (1988). This approach is part of the structuralist discussion on representation, but now emphasizes the various uses that scientists make of the term–no longer in logical-mathematical terms. For Giere, models, rather than mathematical structures or entities (conjunctive entities), are entities of a broader nature and without a given form (diagrams, drawings, maps, organisms, etc.), which are used based on a different connection compared to that of mathematical connections or logical connections (in conjunctive terms): the idea of similarity. The structuralist notion of isomorphism between models and the world intends to capture only one of the ways scientists use models. Models would only be similar in certain aspects and in sufficient degrees depending on their use and according to a specific epistemic context (Giere, 1988, p. 81). Likewise, the relations established between a model and the real system to which they are directed would also be relations of similarity: "A real system is identified as being similar to one of the models [of the theory]" (Giere, 1988, p. 86). Giere proposes that the representational function of a model understood as similarity leads to thinking of models in cognitive and pragmatic terms: "they are 'internal maps' of the external world" (Giere, 1988, p. 6). Rather, this would imply taking into consideration cognitive factors involved in scientists' strategies of constructing world maps in order to map, and thus solve, problems that emerge from the phenomena they are studying. On the whole, Giere does not attempt to provide a precise definition of similarity: the success of our representations of the world is underpinned by our cognitive capacities (language, attention, perception, imagination, explanation, etc.). Thus, in order to study the success of a representation, one must study the results of cognitive sciences, thereby allowing us to demonstrate and explain the success of the endeavor of scientific knowledge on a broad scale. His cognitivist approach to science thus shows that the problem of representation is connected to natural capacities of knowledge agents in the context of the utilization of available resources to represent the world. Therefore, the knowledge agent would remain at the center of a *cognitive theory of science*, a term he uses to refer to his approach. "That humans (and animals) create internal representations of their environment (as well as of themselves) is probably the central notion in the cognitive sciences" (Giere, 1988, p. 6). Despite this effort, however, Giere adds nothing regarding the way in which surrogate reasoning operates or how hypotheses are generated with a model.

With the inferentialism of Mauricio Suárez (2004), the notion of representation is displaced, and the focus turns to substitutive inferences about phenomena from models. Models are then defined as tools which allow us to target and generate plausible hypotheses about their systems of phenomena. This idea suggests a deflationary definition of representation. Thus, the primary function of a model is its inferential function, creating the possibility to gain knowledge without directly examining the *TS*, but instead by looking at *M* straight on, provided that *M* is "coherent" or "addresses" the target system in appropriate respects and to sufficient degrees. The above affirmation assumes that inferences bridge the gap between the "model world" and the real world, whether deductive, inductive, or abductive (López Orellana, 2020). Scientific modeling proceeds in this manner and depends only on a scientist's ability -or agency- to point to and make inferences from models. Scientific representation is then defined as follows: *M* represents *TS* only if the representational force of *M* means towards *TS*, and *M* allows competent and informed agents to draw specific inferences regarding *TS* (Suárez, 2004, p. 773). In that case, what is the generation of hypotheses from a model? How is a hypothesis set up for testing? What are the implications of making inferences with a model? What does it mean that conclusions made in a model are taken to their target system as hypotheses to be tested in it? How is such a step justified? Mauricio Suárez does not elaborate on this question. Our paper will attempt to answer these questions.

3. Modeling and Generation of Hypotheses

From our point of view, the term surrogate does not allude to a type of reasoning but rather the intended relation between *M* and *TS*. That is, it is not a type of reasoning (as one would differentiate between induction and deduction), but a way of pointing out that an inference holds in two places at the same time, *M* and *TS*. The latter, from our point of view, must be logically justified. We therefore assume that, on the one hand, we define surrogative reasoning in such a way; and, on the other hand, that we are dealing with modeling practices oriented to generate hypotheses over *TS* (since not all modeling practices have that same objective). Likewise our proposed contribution contemplates two important restrictions: (1) firstly, we are analyzing modeling cases in which hypotheses are generated from *M* and in order to be evaluated on *TS*. (2) Secondly, we will refer to the latter as 'surrogate reasoning' and a definition must be generated regarding the place it occupies in the modeling practice (i.e. its initial point and end point).

Regarding 1, said restriction does not prevent us from extending our considerations to other cases. Nevertheless, cases in which modeling is done for other purposes will be excluded. Allow us to present an example: in archaeology, we can take a small number of pottery fragments dating around 5000 years old, uncovered for the sole purpose of discovering their original form, and model the original object they came from: an amphora, a lamp, etc. On the other hand, it would likewise be possible to create a model which generates hypotheses and evaluates them. For example, according to the theory of material degradation, the fragments could indicate not being broken in a natural way, but rather through impact. We could then widen the search area and, if we find fragments at a greater distance, our hypothesis is confirmed.

In both cases there are models, but, according to our point of view, we only have a case of hypothesis generation, from the *M* and on the *TS*, in the second case. The first case is close to what some authors identify as an 'explanatory model,' as opposed to a 'predictive model'. We will focus on the second case outlined above, given that the aim of our paper is to propose a logical basis for the attribution to *TS* of the conclusions obtained in *M*.

With respect to the second restriction (2), it is necessary to account for the limits within which surrogate reasoning operates. Let us consider the following question: what element of reasoning is present in surrogate reasoning? We believe three options exist: (i) we refer to the generation of assertions in *M*; (ii) we refer to the tenability in *TS* of the assertions obtained in *M*: (iii) both. The first option alone, from our point of view, does not contain the problem we are trying to solve, but it is the source of the assertions that we will then, according to option (ii), sustain in *TS* to be evaluated. In other words, for the moment we will assert that surrogate reasoning corresponds to option (ii). Moreover, our logical proposal of justification does not indicate how such a statement was obtained in *M*, but rather the justification of considering it at the same time in *TS*. However, we will also put forth that the assertions were obtained by logical methods in *M*, such as deduction or induction. What's more, we must relax the restriction here considering the fact that, we believe, we cannot exclude abductions that take place in the model. In other words, cases where the hypotheses evaluated in *TS* are similarly hypotheses in *M*. Therefore, we consider the entire conceptual weight of reasoning to point to its nature as a logical process and, as such, it must be justified logically.

In other words, the previous statements are conclusions of reasoning whose premises (a) gather information from *M*, while, in turn (b) has been elaborated from information retrieved from the *TS*. In the present paper we will not explore either (a) or (b), as we do not consider either as part of surrogate reasoning. However, we would be willing to include that which follows as part of surrogate reasoning. We are referring to the consideration in *M*, from an inferential point of view, of the result of the evaluation of the hypotheses in *TS*. Evaluating a hypothesis implies evaluating *A* in *TS* directly or indirectly. A possible result of such an evaluation would be *A* or Not*A*. This result, according to our point of view, should be part of the database in *M* which would generate the subsequent statements *B*, *C*, etc. Especially interesting is the case of No*A*: we have argued elsewhere that such a phenomenon is comparable to what Belief Revision scholars call 'epistemic hell' (Olsson & Enqvist, 2011, p. vi; Redmond, 2020). The most important aspect for our proposal, however, is its autonomy from the latter, given it is independent from logical justifications we provide for the hypotheses generated between *M* and *TS*.

Finally, we will address sustainability in an inferential sense. Indeed, we say that the conclusions obtained in *M* are hypothetically sustained in *TS*, while the proposed contribution of the present article aims at logically justifying that sustainability. In line with the above, we believe that it becomes necessary to make a distinction between the assertion as a logical conclusion obtained in *M* and that same assertion as conjecture intended to be evaluated in *TS*. Firstly, from our point of view, we believe that the assertion is presented as having a different status on each side. Nonetheless in the present article we do not argue in favor of this thesis. What is important for the present article is that our dialogical contribution does not aim to explain or to justify this change of status. As previously stated our aim is to justify from a logical (dialogical) point of view, how a conclusion in *M* is simultaneously maintained in *TS*. Nor does it explain the refutable character of this relation. Indeed, if there is a remarkable feature of this process, it is that the hypothesis can be refuted, i.e. falsified. This does not seem to be defeasible reasoning because if I increase the number of premises I will not succeed in 'validating' or 'invalidating' these hypotheses. What I will achieve, by transforming the database, is to generate new hypotheses.

Let us now move on to elaborating our approach. We will take an idea elaborated by Aristotle himself in his *Analytica Priora* as a starting point.

4. Hypotheses, Agreement, and Dialogues

As noted above, we propose developing a logical view of the foundations underpinning surrogate reasoning in modeling practice in science. In doing so, we will consider the notions of *hypothesis* and *substitution* for the development of an inferential point of view. By 'giving the foundations' we refer to providing the necessary, sufficient conditions for surrogate reasoning from a purely logical conception, i.e., outlining an explanation of surrogate reasoning in exclusively logical terms. We believe both the notion of hypothesis and the notion of substitution have an inferential, or logical, aspect and will serve as a basis for our article. To achieve this last objective, we will take an idea Aristotle presents in *Prior Analytics* concerning 'demonstrations by hypothesis' as inspiration; the perspective within which we will develop our approach will be that of dialogical pragmatism.

4.1 Aristotle and hypothesis as agreement

Our current proposal puts forth that generating a hypothesis implies generating an agreement between agents, which can be represented as a *formal dialogue*[2]. That is to say, a hypothesis is not a propositional content–often identified with the antecedent of a conditional–but rather the interaction itself. This idea, which we will detail from a dialogical point of view, was inspired by Aristotle's explanation of demonstrations by hypothesis in *Prior Analytics*. Indeed, in *Prior Analytics* (50a19-24) he states the following:

> E.g., suppose that, after assuming that unless there is someone potentiality for contraries there cannot be one science of them, you should then argue that not every potentiality is for contraries, e.g., for the healthy and for the diseased, for if there is, the same thing will be at the same time healthy and diseased: then it has been shown that there is not one potentiality for all contraries, but it has not been shown that there is not one science. It is true that the latter must necessarily be admitted, but only *ex hypothesi* and not as the result of syllogistic proof. (Aristotle, 1962, p. 387)

According to Aristotle, an agreement is established in demonstration by hypothesis with an interlocutor. This agreement, from our point of view, can be understood as a logical agreement. Indeed, for 'p is not given: there is no one potentiality of the contraries' and 'q is not given: there is no one science', we would have an agreement in the following sense: that if we prove that 'p is not given', 'q is not given either'. This can be understood as a relation between two proofs, but, most importantly and according to Aristotle himself, the proof of the former is not the proof of the latter. Indeed, we prove that 'p is not given' by reduction to the impossible: for if p were given, then 'the same thing will be at the same time healthy

[2] This agreement could be represented as a conditional as well, although suitable semantic conditions must be established, given it cannot be understood as a material conditional (veritative-functional).

and diseased' (first proof). However, the latter is not proof that 'q is not given'. Indeed, by deductively proving that 'p is not given', it has been proved that 'q is not given' only by what has been agreed on by hypothesis with our interlocutor[3].

The same idea appears in the following (41a39):

> The same is true of all other hypothetical proofs;[4] for in every case the syllogism is effected with reference to the substituted proposition, and the required conclusion is reached by means of a concession or some other hypothesis. (Aristotle, 1962, p. 323)

Here again Aristotle refers to the two moments in a demonstration by a hypothesis: a first moment that is deductive and a second moment of the proof that corresponds to an agreement: the hypothesis[5]. From our point of view, and following Ross (1957, p. 31)[6] and Crubellier (Aristotle, 2014, p. 279) on this point, we consider that the notion of hypothesis as agreement that Aristotle is proposing here is a clear dialogical interaction that could be represented as a formal dialogue. Thus, it concerns a dialogical interaction between the arguer and the interlocutor: something agreed with the interlocutor or something that we ask the interlocutor to accept (Aristotle's two definitions of hypothesis in *Second Analytics* 72a18-24 and 76b27-34). For example, in the quote above, we would have the following: we agree with the interlocutor that if we succeed in proving the former (*if there is no unique power of opposites*), he/she will concede that the latter is proven (*there is no unique science either*).

We will consider this idea as crucial for our proposal: the hypothesis is not a kind of proposition but rather a type of interaction between propositions, or proofs, and agreed upon by two interlocutors. Below we will argue that Dialogic is the ideal theoretical framework to capture the latter.

The idea of *substitution* mentioned by Aristotle in 41a39 helps to complete the idea of hypothesis generation as a relation between propositions or proofs. Indeed, according to Crubellier, substitution would allude to an agreement between interlocutors. That is, the interlocutors agree that if *p* is true, then q is true; '*p* is true' is proved deductively and '*q* is true' is proved by the agreement (hypothesis): we say then that '*p*' substitutes '*q*'. The following reference in Ross alludes to the same issue:

[3] A simpler example to understand this, according to our point of view, would be if we had a sugar detector for liquids with a minimum 'five spoonfuls' unit. Upon detecting at least five spoonfuls in a cup of coffee, it would be proven at the same time (if someone agrees with us) that it has three spoonfuls.

[4] Smith (1989, p. 38) translates this sentence as "And likewise also all the other kinds of deduction that are from an assumption (ἐξ ὑποθέσεως)".

[5] We can observe that, in these cases, Aristotle does not call the starting point a hypothesis, unlike Plato, who in Menon 87b examines whether virtue is teachable or not from a hypothesis as a starting point.

[6] If a certain proposition A is to be proved, it is first agreed by the parties to the argument that A must be true if another proposition B can be proved. This agreement, and the use made of it, are the non-syllogistic element of the argument; the syllogistic element is the proof of the substituted proposition (41a39, 45b18). Once B has been proved, A follows in virtue of the agreement (41a40, 50a18, 25).

> The general nature of such proof is that, desiring to prove a certain proposition, we first extract from our opponent the admission that if a certain other proposition can be proved, the original proposition follows, and then we proceed to prove the substituted proposition (*to metalambanómenon*, 41a39). The substituted proposition is said to be proved syllogistically, the other not syllogistically but *ex hypozéseos*. (1957, p. 371)

Finally, we would like to conclude with the following assertion: Aristotle's demonstrations by hypothesis can be understood as surrogate demonstrations. That is to say, paraphrasing Swoyer, we are using a proposition "as a substitute in our thinking about another" (Swoyer, 1991, p. 449). What is decisive for our approach in this idea of Aristotle's, according to our point of view, is that these generated hypotheses find their foundation in agreements between interlocutors, as part of an entirely logical demonstrative process.

Summary: Inspired by Aristotle's idea of demonstrations by hypothesis we support the idea that hypothesis generation implies establishing an agreement which is established in the framework of an interactive dialogue. This idea of hypothesis as agreement, we believe, is adequate to represent the inferential agreement between *M* and *TS* from a logical point of view, which we refer to as surrogate reasoning. What's more, we propose the pragmatic and dynamic approach of the Dialogic is the ideal framework to capture hypothesis generation as an agreement.

This last reflection leads us to the second development as the basis of our work.

4.2 The pragmatic and dynamic approach of dialogical logic

The second development on which our work relies is the pragmatic and dynamic approach of Dialogic. In effect, we claim that the ludic-dialogical approach to logic is an optimal "frame" to capture the inferential process engaged in modeling in science, allowing it to formally reflect the relation between the model and the target system as a dynamic and pragmatic interaction. Dialogic is a pragmatic approach elaborated from the notions of *use* and *agency* and therefore we consider it to be optimal for capturing the fluidity of the exchanges between the model and the target system as a formal dialogue. And particularly for the purpose of the current article, we consider Dialogic as a suitable framework for giving an account of the inferential process where a model can be generated from hypotheses (surrogate reasoning) about the target system.

Dialogues are mathematically-defined language games that establish the interface between the concrete linguistic activity and the formal notion of demonstration (for a general overview, see Appendix I). Two interlocutors (Proponent and Opponent) exchange movements that are concretely linguistic acts. The Proponent enunciates a thesis–the thesis of the dialogue–and undertakes its defense by responding to the opponent's criticisms. The criticism permitted is defined in terms of the structure of the statements affirmed in the dialogue. For example, if a player affirms conjunction *A and B*, at the same time he gives the opponent the possibility to choose one of the two and demand he affirm it. The very notion of asserting is defined by the context of critical interaction: asserting means committing oneself to provide *justification* to a critical interlocutor. But in dialogues there is no general theory of justification: only insofar as they are logically complex statements whose justification can be found in simple statements. In turn, simple statements are justified in reciprocal action with the critical interlocutor. That is, as the rule exhorts, the Proponent may consider an elementary statement justified, *if and only if* the Opponent has granted that justification (*copycat* rule). This rule confirms the formality of the dialogue: the Proponent wins without presupposing justifications for any particular statement. Of course, there is the possibility to assign presuppositions to the Opponent (material dialogues) as is the case we will consider in the present article (see below).

Finally, we want to point out that using dialogic as a basic inferential frame does not mean committing to a specific type of inferential relation. The dialogical frame allows for the construction of different inferential relations: classical, intuitionistic, paraconsistent, etc.

5. Generation of hypotheses and modelization: a logical foundation of surrogate reasoning

In general terms, the modeling process is described as follows:

i. Two parts: M and TS. M assembles (a) target system information + (b) general theoretical information (e.g., mathematical, geometrical, physical, chemical, biological approaches).

ii. We infer in M: we obtain conclusions C_n in M from (a) and (b) and following different types of inferential relations (inductive, deductive, statistical, etc.).

iii. The conclusions C_n inferred in M are considered for evaluation in TS.

iv. Evaluating C_n in TS.[7]

We distance ourselves from this notion and propose a logical justification of the process starting at point ii. To this end, we consider that to generate a hypothesis is to agree with an interlocutor that what is inferred in M has sustainability in TS, i.e., it is destined to be evaluated in TS. The process could be described in two steps:

- First step: a set of conclusions $C_n = \{C_1, C_2, \ldots C_i\}$ is obtained from the model and according to different types of 'inferential relations' (deduction, induction, abduction, etc.).
- Second step: the conclusions C_n are evaluated in TS.

In formal language: C_n being the set $\{C_1, C_2, \ldots C_i\}$ of conclusions we draw in M, the following holds for all C_i:

[7] Both (i) and (iv) are controversial issues on which there is no unanimous agreement and which require extensive discussion. In this paper we consider that neither of them is part of surrogate reasoning.

$$M \vdash C_i \rightarrow_{[\mathrm{Hip}]} TS \vdash C_i \qquad (1)$$

The expressions $M \vdash C_i$ and $TS \vdash C_i$ mean, in our view, that the conclusions C_i are logically obtained in *M* and intended to be evaluated in *TS*, respectively. The expression "$\rightarrow_{[\mathrm{Hip}]}$" bears our idea that we have generated a hypothesis. In effect, when (1) is established, our claim is that a formal dialogue between *M* and *TS* has been generated. Thus, from our point of view, the hypotheses generation is an inferential interaction that must be justified in strictly logical terms.

In the following, we propose two ways of logically grounding the process of surrogate reasoning, from the pragmatic, dynamic, and interactive perspective of Dialogic: I. Dialogical point of view and II. Dialogical and Modal approach. In general, the point is to identify the general features of dialogical generation of hypothesis following these two main questions:

1. How can the generation of hypothesis as hypothetical-agreement be characterized from a logical point of view?
2. How can the surrogative function of the model be characterized from a logical point of view?

5.1 Dialogical point of view

According to the pragmatic and dynamic approach of Dialogic that we would like to defend, generating a hypothesis would consist in establishing a formal dialogue between *M* and *TS*. Therefore, it is not a kind of assertion but the agreed interaction between *M* and *TS* as cognitive agents. It is for this last reason that we consider Dialogic as an adequate frame to capture the generation of hypotheses as dynamic and agential interactions according to intentions, purposes, etc.[8] That *M* subrogates *TS* means, according to our view, that there is a dialogue established between the two to seek a winning strategy for a thesis asserted by *M* over *TS*.

We will say that the subrogating character of *M* in relation to *TS*, from an inferential point of view, is formalized as the dialogical interaction between *M* and *TS* to reach Winning Strategies for the theses. More precisely, *M* subrogates *TS* when:

1. There is an interactive relation established between *M* and *TS*: a dialogue scheme $\Delta_{(M,\, TS)}$ for a thesis φ.

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[8] In this sense, we believe that the generation of hypotheses cannot be reduced to a material conditional (a relation between assertions), since it would be a relation between proofs (as Plato and Aristotle point out) and/or between contexts. Therefore, we believe it is important not to confuse 'demonstrations by hypothesis' with hypothetical syllogisms.

2. There is a winning strategy for the thesis φ. In effect, the dialogue scheme $\Delta_{(M,\ TS)}$ allows the identification of Winning Strategies for the thesis φ defended by *M* in the interaction with *TS*.

One of the most significant contributions of this 'purely dialogical' approach is that the proofs (or winning strategies) are constructed in the dialogical interaction between *M* and *TS*. According to this first way of formalizing the process, the dialogue Δ is the result of the commitment of *M* and *TS* for testing a thesis (generation of a hypothesis). In the latter, we do not lose sight of the fact that in the dialogical framework the very notion of proof is conceived in terms of interactions between agents.

In other words, justifying from a logical point of view that which is inferred in *M* is also valid in *TS* (for evaluation) means that, from a dialogical point of view, we have constructed the proofs, or winning strategies, of the theses with *M* and *TS* as interacting agents in a dialogical frame.

Consider the following dialogue scheme:

T			*M*		
c	P_1, P_2, P_3, R_1, R_2, etc.				
				Thesis φ	0

From a dialogical point of view, as mentioned above, *T* (target) and *M* assume the roles of the Opponent and Proponent, respectively. And the thesis affirmed in '0' is an affirmation in *M* that will be defended from attributes (properties and conditions of the Model) assigned in *T* (in 'c'). Thus, the interaction between *M* and *T* is presented as an interactive game. This information in 'c' corresponds, in dialogic, to the information that O concedes to P to play. And this corresponds in modeling, according to our point of view, to the information that *TS* grants for the elaboration of *M*.

For this approach to the justification of surrogative reasoning, we will consider as a basis the standard dialogic but with an enriched version of the particle rules from Constructive Type Theory (Martin-Löf, 1984; Rahman *et al*., 2018). The new element to consider in these rules is the expression 'a:A' which means that player X possesses a proof 'a' of A. The justification for using this basic notion of Constructive Type Theory derives from the need to think in terms of proofs. Thus, we say in modeling that the proof of A in *M* (a:A) is proof of A in *TS*, i.e., that we are justified to evaluate A in *TS*[9]. The particle rules will be as follows:

[9] We recall that A is not a hypothesis, A is a proposition that 'by hypothesis' is considered to be evaluated in *TS*.

	X	Y	X
Conjunction	$A \wedge B$	$?\wedge_L$ $?\wedge_R$	a:A b:B
Disjunction	$A \vee B$	$?\vee$	a:A or b:B
Implication	$A \rightarrow B$	a:A	b:B
Negation	$\neg A$	A	--

After X plays '$A \wedge B$', Y attacks ($?_L/?_R$) requesting the proof of the conjunction components (left/right, respectively). As we saw above, she/he can request only one of the two or both. X responds to this challenge with the requested proof: a:A/b:B, respectively.

Rule of atomic justification

If the Proponent (*M*) plays an atomic, the Opponent (in our case *TS*) has the right to ask the Proponent (*M*) how she/he justifies the proof of that atomic. If it has been played according to the standard structural rules, P justifies her/his play with a similar previous play of the Opponent (copy cat/mirror play).

P	O	P
a:A	?a Where did you get that proof?	P(n)=a:A You gave it in n

Let's now observe a more detailed example that presupposes the rules of classical propositional dialogic and a Model that combines two properties: A y B[10]:

[10] We have considered this case which appears simple, although usually atomic statements are evaluated in *TS*, not logically complex structures. An assertion can be the conclusion of a logically complex structure, but what leads to *TS* is simply the conclusion.

TS				***M***	
c	$A \wedge B$				
				$A \vee B$	0
1	$?\vee$			a:A	4
3	a:A			$?\wedge_L$	2
5	?a			*TS*(3)=a:A ☺	6

The dialogue starts at 0 with the thesis and presupposes the concession $A \wedge B$ in c. This concession is the dialogical way of indicating *M*'s previous resources (the data with which the model has been constructed). *TS* attacks the disjunction in 1 by asking it to prove at least one of the two members of the disjunction. *M* attacks the concession in 2 by asking for the proof of the left member. *TS* answers the previous challenge in 3. *M* uses the previous answer to answer the challenge of 1. In 5, according to the atomic justification rule, *TS* asks for the justification of the proof played in 4. *M* answers in 6 the same proof *TS* gave in 3.

Summary: generating a hypothesis, according to this approach, means establishing a dialogical interaction between *M* and *TS* which justifies the evaluation of theses with winning strategies. The hypothesis is not the proven thesis with a conclusion on one side of reasoning and an element to be evaluated on the other, but instead the agreement, or interaction, that justifies from a dialogical point of view that the theses with winning strategies in a formal dialogue between *M* and *TS* can be evaluated in the phenomena (*TS*) corresponding to the Model.

5.2 Modal point of view

Our modal point of view emphasizes the following surrogate reasoning: the proof of A in the Model 'is also' the proof of A in the target system.[11] To be the proof [also] in the target system means that what is inferred in *M* is considered for evaluation in *TS*. And we say that we are entitled to consider it to be so because the Model 'inferentially subrogates' the *TS* and our article aims to substantiate this subrogation in purely logical terms. In order to achieve the latter, we will consider in this second point a *modal* approach.

Our dialogical and modal view of hypothesis generation is inspired by two previous developments:

1. The modal logic of fiction which considers 'According to the story...' as a *modal operator* which sends to a possible world (Woods, 1974; Lewis, 1978)

[11] Unlike Plato and Aristotle's considerations that the proof of one assertion is the proof of another assertion.

2. The dialogical and modal approach to abduction developed in Barés Gomez & Fontaine (2017), proposes a new explanation of abduction in terms of concession-problem and subjunctive knowledge (by relying on ideas of the Gabbay-Woods (2006) schema of abduction (GW) and Aliseda's approach (2006).

On the basis of these previous developments, the questions guiding our own dialogical and modal approach are the following:

1. How can we characterize the generation of hypotheses from a dialogical and modal point of view?
2. How can we characterize the subrogative function of the model from a dialogical and modal point of view?

The answer to the first question is the following: our modal proposal consists in identifying the generation of hypotheses with the construction of an *accessibility* between two contexts: the Model and the Target System. For the latter it is necessary to introduce a modal operator that we will call the 'hypothesis generation' box operator $\boxed{HG}$. The construction of accessibility corresponds to the choice of a new context on the part of O when it attacks the thesis followed by $\boxed{HG}$ in the initial context.

Static and dynamic accessibilities

The above allows us to propose a distinction between two types of accessibilities: a *dynamic* and a *static* one. Static accessibility is an accessibility that gives rights to the proponent, by an agreement before starting, for example reflexivity, transitivity, symmetry, etc. Whereas dynamics give rights to the proponent, only if the opponent built it before. In this sense, we can say that dynamics is an interactive accessibility and also that statics could be seen as the result of a previous interaction.

We have then that generating a hypothesis, from a dialogical point of view, means establishing a dialogue where the thesis that P asserts in context d_1 (the Model) is defended in context $d_{1.1}$ (*TS*), introduced by O. We call 'dynamic accessibility' the result of this last interaction that is dialogically generated at the *play level* (Rahman *et al.*, 2018, Ch. 3).

In this sense, we say that the model inferentially subrogates the target system (Surrogate Reasoning) when a modal dialogue is established where the agents (O and P) construct a dynamic accessibility between *M* and *TS* for the thesis asserted by P. To construct dynamic accessibility is to dialogically and modally formalize the generation of hypotheses as a dynamic interaction between agents.

In order to elaborate a modal dialogic to formalize hypothesis generation, the following definitions and rules are necessary.

Contexts and choices

The dialogical approach to modal logic requires a context-dependent notion of choice, that is to say, one in which the choices of the players are relative to different contexts in which the players perform their moves (attacks and defenses). The elements involved are the following:

thesis + attacks + defenses + contexts

In a dialogue, there is an initial context (represented by the level c_0) where the thesis has been uttered. As explained below, in the course of a dialogue other contexts, such as c_1, $c_{1.1}$ $c_{1.2}$ $c_{1.1.1}$, etc., might occur.

From a modal dialogical point of view, when P asserts a thesis, he commits himself to its defense in all the contexts O is willing to allow. The modal operator that captures the latter is the box (which we will represent as $\Box$), and thus every dialogue for a thesis φ starts with the assertion $\Box\varphi$.

Similar to the case of quantifiers, attacking or defending modal operators will concern choices. In the case of modal operators, it will affect choices of contexts. The choices of contexts are specified in the particle rule of the box $\Box$:

Rule for the box $\Box$

Formula Uttered	Attack	Defense
X-!-$\Box\varphi_{[ck]}$	Y-?-$\Box_{[ci]}$	X-!-$\varphi_{[ci]}$
X utters the formula $\Box\varphi$ at the context [c*k*] and must be defended (!) Uttering $\Box\varphi$, the player X commits itself to uttering φ in any context.	Y attacks by demanding that X performs the utterance φ in a context c*i* chosen by Y. Y chooses the context	The defense consists in uttering φ within the context c*i* chosen by Y.

Choices and Contexts

i. Players X and Y choose a context when attacking the box $\Box$.

ii. To *introduce* a new context implies choosing a new one when attacking a box $\Box$.

iii. Only O can *introduce* new contexts.

Structural Rules

Given that in modal logic all moves are performed in contexts, it is necessary to replace the rule "(SR-3) (Formal use of atomic formulas)" with the following:

(*SR-3-Modal Logic*) (Formal use of atomic formulas in contexts)

Atomic formulas may be uttered for the first time only by O. The proponent P can play an atomic formula only if the same formula was already uttered by O and in the same context. In other words, P can utter an atomic formula in a context ci only if the same atomic formula was already granted in the same context ci by O. Atomic formulas cannot be attacked.

Consider the following example:

		O			P		
φ: en *M* y *TS*		$A_T \wedge B_T$					
					$\Box$ $(B_T \vee C_T)$	0	*M*
M	1	?-$\Box_{TS}$	0		$B_T \vee C_T$	2	*TS*
TS	3	?-v	2		B_T ☺	6	*TS*
TS	5	B_T		φ	$?\wedge_2$	4	*TS*

Notes: The dialogue starts at 0 with P's assertion of the thesis $B_T \vee C_T$ preceded by the modal operator $\Box$ and in *M* (context d_1). O attacks according to the box operator rule and introduces a new context: *TS* (context $d_{1.1}$). The proof continues normally according to the rules of classical logic (cf. Rules above) and P obtains a Winning Strategy for the thesis. We may then conclude that, from a dialogical and modal point of view, the thesis asserted in *M* has gained a Winning Strategy in *TS*. For the latter, dynamic accessibility was introduced between the two. We call the action of introducing this dynamic accessibility 'generating a hypothesis'. With the latter, we claim to have shown that surrogate reasoning is not a type of representational-based thinking but rather, according to our point of view, intentional and logically-based dynamic thinking.

6. Final remarks

In this paper, we engaged in a logical justification for the process of surrogate reasoning in modeling practice. For this purpose, we advanced a pragmatic view of the generation of hypotheses in science. Indeed, we proposed two logical routes for this justification: a dialogical one and a modal-dialogical one. For both, we base our work, on the one hand, on an interpretation of 'demonstrations by hypotheses' in Aristotle, which we consider to be quite prolific in order to explain the logical character of hypothesis production in science.

And, on the other hand, we rely on the pragmatic approach of Dialogic as the ideal framework to capture interactions in logic. For the former, we understood hypothesis generation as the creation of an interactive, formal dialogue between the model and the target system. For the modal approach, we introduced a modal operator which allowed us to think of the model and the target system as two contexts (or possible worlds) with dynamic accessibilities.

Acknowledgements

This paper has been financially supported by the Agencia Nacional de Investigación y Desarrollo de Chile (ANID), with the following FONDECYT projects: No. 3210531 (2021-2024) 'Por un enfoque pragmático-filosófico de los sistemas-objetivo en la modelización de sistemas biológicos', and No. 1221132 (2022-2024) 'La generación inferencial de hipótesis en los procesos de modelización en ciencia desde un enfoque dinámico e interactivo.'

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Appendix I: Standard Dialogical Logic[12]

Let L be a first-order language built as usual upon the propositional connectives, the quantifiers, a denumerable set of individual variables, a denumerable set of individual constants and a denumerable set of predicate symbols (each with a fixed arity).

We extend the language L with two labels O and P, standing for the players of the game, and the question mark '?'. When the identity of the player does not matter, we use variables X or Y (with X≠Y). A *move* is an expression of the form 'X-*e*', where *e* is either a formula φ of L or the form '?[φ_1,..., φ_n]'.

We now present the rules of dialogical games. There are two distinct kinds of rules named particle (or local) rules and structural rules. We start with the particle rules.

Previous move	X-φ∧ψ	X-φ∨ψ	X-φ→ψ	X-¬φ
Challenge	Y-?[φ] or Y-?[ψ]	Y-?[φ,ψ]	Y-φ	Y-φ
Defence	X-φ resp. X-ψ	X-φ or X-ψ	X-ψ	– –

Previous move	X-∀xφ	X-∃xφ
Challenge	Y-?[φ(a/x)]	Y-?[φ(a_1/x),...,φ(a_n/x)]
Defence	X-φ(a/x)	X-φ(a_i/x) with 1≤ i ≤ n

In this table, the a_is are individual constants and φ(a_i/x) denotes the formula obtained by replacing every occurrence of x in φ by a_i. When a move consists in a question of the form '?[j_1,...,j_n]', the other player chooses one formula among j_1,...,j_n and plays it. We can thus distinguish between conjunction and disjunction on the one hand, and universal and existential quantification on the other hand, in terms of which player has a choice. In the cases of conjunction and universal quantification, the challenger chooses which formula he

[12] The following brief presentation of standard dialogical logic is due to Nicolas Clerbout. The main original papers on dialogical logic are collected in Lorenzen and Lorenz (1978). For an historical overview see Lorenz (2001). Other papers have been collected more recently in Lorenz (2008, 2010a, 2010b). A detailed account of recent developments since, say, Rahman (1993) and Felscher (1994), can be found in Rahman and Keiff (2005) and Clerbout & McConaughey (2022). For the underlying metalogic see Clerbout (2013a; 2013b). For a textbook presentation: Rückert (2011). For the key role of dialogic in regaining the link between dialectics and logic, see Rahman and Keiff (2010). Fiutek *et al.* (2010) study the dialogical approach to belief revision. Clerbout *et al.* (2011) studied Jain Logic in the dialogical framework. Popek (2012, pp. 223-244) develops a dialogical reconstruction of medieval *obligationes*. See also Magnier (2013) —on dynamic epistemic logic and legal reasoning in a dialogical framework. Rahman *et al.* (2018) studied Immanent reasoning or equality in action.

asks for. Conversely, in the cases of disjunction and existential quantification, the defender is the one who can choose between various formulas. Notice that there is no defence in the particle rule for negation.

Particle rules provide an abstract description of how the game can proceed locally: they specify the way a formula can be challenged and defended according to its main logical constant. In this way we say that these rules govern the local level of meaning. Strictly speaking, the expressions occurring in the table above are not actual moves because they feature formulas schemata and the players are not specified. Moreover, these rules are indifferent to any particular situations that might occur during the game. For these reasons we say that the description provided by the particle rules is abstract. The words "challenge" and "defence" are convenient to name certain moves according to their relationship with other moves. Such relationships can be precisely defined in the following way. Let Σ be a sequence of moves. The function p_Σ assigns a position to each move in Σ, starting with 0. The function F_Σ assigns a pair $[m,Z]$ to certain moves N in Σ, where m denotes a position smaller than $p_\Sigma(N)$ and Z is either C or D, standing respectively for "challenge" and "defence". That is, the function F_Σ keeps track of the relations of challenge and defence as they are given by the particle rules. A *play* (or dialogue) is a legal sequence of moves, i.e., a sequence of moves which observes the game rules. The rules of the second kind that we mentioned, the structural rules, give the precise conditions under which a given sentence is a play. The *dialogical game* for φ, written D(φ), is the set of all plays with φ as the thesis (see the Starting rule below). The structural rules are the following:

SR0 (Starting rule)

Let φ be a complex formula of L. For every $\pi \in D(\varphi)$ we have:

- $p_\pi(\text{P-}\varphi)=0$,
- $p_\pi(\text{O-n}{:=}i)=1$,
- $p_\pi(\text{P-m}{:=}j)=2$

In other words, any play π in D(φ) starts with P-φ. We call φ the *thesis* of the play and of the dialogical game. After that, the Opponent and the Proponent successively choose a positive integer called *repetition rank*. The role of these integers is to ensure that every play ends after finitely many moves, in a way specified by the next structural rule.

SR1 (Classical game-playing rule)

- Let $\pi \in D(\varphi)$. For every M in π with $p_\pi(M)>2$ we have $F_\pi(M)=[m',Z]$ with $m'<p_\pi(M)$ and $Z \in \{C,D\}$
- Let r be the repetition rank of player X and $\pi \in D(\varphi)$ such that,
    - » the last member of π is a Y move,

» M_0 is a Y move of position m_0 in π,

» $M_1,\ldots,M_n$ are X moves in π such that $F_\pi(M_1)=\ldots=F_\pi(M_n)=[m_0,Z]$.

Consider the sequence[13] π' =π*N where N is an X move such that $F_{\pi'}(N)=[m_0,Z]$. We have π' ∈ D(φ) only if n < r.

The first part of the rule states that every move after the choice of repetition ranks is either a challenge or a defence. The second part ensures finiteness of plays by setting the player's repetition rank as the maximum number of times he can challenge or defend against a given move of the other player.

SR2 (Formal rule)

Let ψ be an elementary sentence, *N* be the move P-ψ and *M* be the move O-ψ . A sequence π of moves is a play only if we have: if *N* ∈ π then *M* ∈ π and $p_\pi(M)<p_\pi(N)$.

A play is called *terminal* when it cannot be extended by further moves in compliance with the rules. We say it is X terminal when the last move in the play is an X move.

SR3 (Winning rule)

Player X wins the play π only if it is X terminal.

Consider for example the following sequences of moves: P-Qa→Qa, O-n:=1, P-m:=12, O-Qa, P-Qa.

We often use a convenient table notation for plays. For example, we can write this play as follows:

	O		P	
			Qa→Qa	0
1	n:=1		m:=12	2
3	Qa	(0)	Qa	4

The numbers in the external columns are the positions of the moves in the play. When a move is a challenge, the position of the challenged move is indicated in the internal columns, as with move 3 in this example. Notice that such tables carry the information given by the functions *p* and *F* in addition to represent the play itself.

However, when we want to consider several plays together —or example when building a strategy— such tables are not that perspicuous. So we do not use them to deal with dialogical games for which we prefer another perspective. The *extensive form* of the dialogical game D(φ) is simply the tree representation of it, also often called the game-tree. More precisely, the extensive form E_φ of D(φ) is the tree (*T*, *l*, *S*) such that:

i. Every node *t* in *T* is labelled with a move occurring in D(φ)

[13] We use π*N to denote the sequence obtained by adding move *N* to the play π.

ii. $l : T \to N$

iii. $S \subseteq T^2$ with:

- There is a unique t_0 (the root) in T such that $l(t_0)=0$, and t_0 is labelled with the thesis of the game.
- For every $t \neq t_0$ there is a unique t' such that $t'St$.
- For every t and t' in T, if tSt' then $l(t')=l(t)+1$.
- Given a play π in D(φ) such that $p_\pi(M')=p_\pi(M)+1$ and t, t' respectively labelled with M and M', then tSt'.

A *strategy* for Player X in D(φ) is a function which assigns an X move M to every non terminal play π with a Y move as last member such that extending π with M results in a play. An X strategy is *winning* if playing according to it leads to X's victory no matter how Y plays.

A strategy can be considered from the viewpoint of extensive forms: the extensive form of an X strategy σ in D(φ) is the tree-fragment $E_{\varphi,\sigma}=(T_\sigma, l_\sigma, S_\sigma)$ of E_σ such that:

i. The root of $E_{\varphi,\sigma}$ is the root of E_φ.

ii. Given a node t in E_φ labelled with an X move, we have that $tS_\sigma t'$ whenever tSt'.

iii. Given a node t in E_φ labelled with a Y move and with at least one t' such that tSt', then there is a unique $\sigma(t)$ in T_σ where $tS_\sigma\sigma(t)$ and $\sigma(t)$ is labelled with the X move prescribed by σ.

Here are some examples of results which pertain to the level of strategies[14].

Winning P strategies and leaves. *Let* w *be a winning* P *strategy in* D(φ). *Then every leaf in* $E_{\varphi,w}$ *is labelled with a* P *signed atomic sentence.*

Determinacy. *There is a winning* X *strategy in* D(φ) *if and only if there is no winning* Y *strategy in* D(φ).

Soundness and Completeness of Tableaux. *Consider first-order tableaux and first-order dialogical games. There is a tableau proof for* φ *if and only if there is a winning* P *strategy in* D(φ).

By soundness and completeness of the tableau method with respect to model-theoretical semantics, it follows that the existence of a winning P strategy coincides with validity: *There is a winning* P *strategy in* D(φ) *if and only if* φ *is valid.*

[14] These results are proven, together with others, in Clerbout (2013b).