Journal of Regional Research
Investigaciones Regionales

ISSN: 1695-7253 e-ISSN: 2340-2717
investig.regionales@aecr.org
AECR - Asociación Española de Ciencia Regional
www.aecr.org
España – Spain

Efficiency in the estimation of technical coefficients and interregional multipliers: the Jahn methodology versus the GRAS and Gravity-RAS methodologies

José Daniel Buendía Azorín, Rubén Martínez Alpañez, María del Mar Sánchez de la Vega
Efficiency in the estimation of technical coefficients and interregional multipliers: the Jahn methodology versus the GRAS and Gravity-RAS methodologies
Investigaciones Regionales - Journal of Regional Research, 58, 2024/1
Asociación Española de Ciencia Regional, España

Available on the website: https://investigacionesregionales.org/numeros-y-articulos/consulta-de-articulos

Additional information:

To cite this article: Buendía Azorín, J. D., Martínez Alpañez, R., & Sánchez de la Vega, M.M. (2024). Efficiency in the estimation of technical coefficients and interregional multipliers: the Jahn methodology versus the GRAS and Gravity-RAS methodologies. Investigaciones Regionales – Journal of Regional Research, 2024/1(58), 179-207. https://doi.org/10.38191/iirr-jorr.24.008

Investigaciones Regionales – Journal of Regional Research (2024/1) 58: 179-207
https://doi.org/10.38191/iirr-jorr.24.008

Articles

Efficiency in the estimation of technical coefficients and inter-regional multipliers: the Jahn methodology versus the GRAS and Gravity-RAS methodologies

José Daniel Buendía Azorín, Rubén Martínez Alpañez**, María del Mar Sánchez de la Vega****

Recieved: 03 December 2021
Accepted: 06 July 2023

Abstract:

The use of location quotients for the estimation of regional input-output tables has been found to be a useful and efficient tool to estimate intra-regional production multipliers. Building on this tool, more complex procedures have been developed that simultaneously estimate inter-regional coefficients. This paper assesses the capacity of this extended methodology (called the Jahn methodology) to obtain both intra-regional and inter-regional multipliers for the Spanish case, using the Input-Output Table (IOT) of Spain 2015 to estimate those corresponding to the Spanish regions of Andalusia, the Basque Country and Navarra for the same year and whose results are available via survey. In order to contrast their reliability, efficiency and accuracy, the results obtained with this procedure are then compared with other methodologies widely used for their recognised efficiency, the GRAS and Gravity-RAS methodologies.

Keywords: Location quotients; FLQ; non-survey method; regional input-output tables; RAS; output multipliers.
JEL Classification: C13; C67; R15; R59.

Eficiencia en la estimación de coeficientes técnicos y multiplicadores interregionales: la metodología Jahn versus las metodologías GRAS y Gravity-RAS

Resumen:

El uso de cocientes de localización para la estimación de tablas input output regionales se ha considerado como una herramienta útil y eficiente en la estimación de multiplicadores de producción intrarregionales. A partir de esta herramienta, se han desarrollado procedimientos más complejos que estiman simultáneamente coeficientes interregionales. En este trabajo se evalúa la capacidad de esta metodología ampliada (que denominamos metodología Jahn) para la obtención de multiplicadores tanto intrarregionales como interregionales para el caso español, estimando a partir de la Tabla Input-Output (TIO)de España 2015 las correspondientes a las de las regiones españolas de Andalucía; País Vasco y Navarra para el mismo año y para las que disponemos de sus resultados mediante encuesta. Para contrastar su fiabilidad, eficiencia y precision, los resultados obtenidos con el procedimiento anterior se comparan con otras metodologías ampliamente utilizadas por su reconocida eficiencia, las metodologías GRAS y Gravity-RAS.

[*] Department of Applied Economics. University of Murcia. España. jdbuen@um.es
[**] Department of Applied Economics. University of Murcia. España. ruben.ma@um.es
[***] Department of Quantitative Methods for Economy and Business. University of Murcia. España. marvega@um.es
Corresponding Author: jdbuen@um.es

Palabras clave: Cocientes de localización; FLQ; métodos indirectos; tablas input-output regionales; RAS; multiplicadores.
Clasificación JEL: C13; C67; R15; R59.

1. Introduction

The development of input-output analysis at regional level has been established as a suitable statistical tool for territorial economic analysis, both to support structural analysis and to assess economic impacts based on the evaluation of the so-called Leontief multipliers.

Whilst regional input-output tables obtained through survey-based methodologies are becoming increasingly available, the fact remains that such tables require a high concentration of both financial and time resources. This handicap has led to the development and consolidation of estimation or regionalisation methods which, based on the corresponding national input-output table and under certain assumptions, make it possible to obtain a suitable approximation to the true regional input-output table.

Within the wide range of existing regionalisation methodologies above-mentioned, those based on the use of location quotients are now consolidated as suitable due to their relatively straightforward and efficient implementation in terms of precision. Among these, there is widespread consensus on the goodness of fit achieved with the FLQ (Flegg et al., 1995; Flegg et al., 1997) or the augmented version AFLQ methodologies (Flegg & Webber, 2000), which shall serve as a basis to obtain a regional input-output table provided that the result obtained by applying this methodology is complemented with available survey information (obtained from surveys) or combined with other techniques to improve the precision of the estimation (Flegg & Tohmo, 2013; Flegg & Tohmo, 2019).

On the other hand, bi-proportional techniques such as the RAS (Stone, 1961) or some of its variants are widely used, such as the Generalized-RAS or GRAS, whose advantage lies in being implementable when the table contains both positive and negative values (Günlük-Senesen & Bates, 1988 and Junius & Oosterhaven, 2003); the Cell-corrected RAS (Mínguez et al., 2009) , which uses cell variation distributions computed from multiple matrices of different periods or different regions, to modify the RAS solution by solving an additional optimisation problem that produces the most likely cell corrections; or PATH-RAS (Pereira-López et al., 2013), that can be applied to rectangular matrices and has minimal information requirements. Likewise, several works have provided improvements to the GRAS methodology: correcting the objective function (Huang et al., 2008; Lemelin, 2009; Lenzen et al., 2007); ensuring the fulfilment of some constraints infeasible by other RAS methods, through an iterative method that allows changing the sign in successive iterations (Lenzen, Moran, et al., 2014; Temurshoev et al., 2013); working with multidimensional tables (Valderas-Jaramillo & Rueda-Cantuche, 2021; Holý& Šafr, 2022) ; or by incorporating partial information and allowing a compromise solution to be found between inconsistent constraints (Lenzen et al., 2006, 2009; Paelinck & Waelbroeck, 1963).

One of the key problems associated with the estimation of a regional table using the national table is the estimation of inter-regional trade, which cannot be derived directly from the national table taken as a reference, making it a decisive element in obtaining an appropriate regionalised table (Miller & Blair, 2009).

Using various goodness-of-fit statistics with complementary characteristics, this paper undertakes a comparative assessment of three regionalisation methodologies or techniques that can be used to estimate the input-output table including the estimation of inter-regional trade: The Jahn methodology, involving the extension of location quotients for the estimation of inter-regional trade (Jahn, 2017; Jahn et al., 2020); the GRAS methodology; and the so-called Gravity RAS (Cai, 2022, 2020, Fournier, 2020; Sargento, 2009; Sargento et al., 2012; Sargento, 2007), which combines gravity models with the RAS method.

Following this introduction, the second section describes the regionalisation techniques compared in this paper. The third section outlines the statistics that were used to compare the selected methodologies as well as the result of such comparison after applying the input-output tables of the Andalusia, Basque Country and Navarra regions for the year 2015 to the corresponding national input-output table for the same year. This comparison is made taking into account the capacity of each of the methodologies to estimate all the elements of the input-output table, assuming that exclusively the production values by homogeneous branches of activity are known. Finally, conclusions are drawn in section 4.

2. Regionalisation Methodologies

2.1. Location Quotients

The use of location quotients as a technique to regionalise input-output tables has evolved significantly since the first text proposing it back in the 1950s (Isard, 1951). At present, the number of available papers applying this technique is practically unlimited and, from the theoretical development of the technique to its applications in terms of results, it is undoubtedly useful in economic terms.

The use of location quotients to create regional input-output tables from a national table is an effective tool when survey-based tables are not available (Flegg et al., 1995). In a relatively simple way and with reasonable and available information needs, the regional table can be estimated in the absence of survey data, although the result obtained must be reviewed by the analyst and refined in order to obtain the best possible approximation to the inter-industrial economic reality of the estimated territory in question (Flegg & Webber, 2000).

In the absence of specific information, the location quotient technique starts, roughly speaking, from the hypothesis that the production structure and production technology is similar in the region to that shown in the national table to which the region belongs. Thereafter, corrections based on regional size are made.

2.1.1. Simple Location Quotient (SLQ)

Let x_i^r and x^r be the total output of sector i in region r and the total output of region r respectively, and let x_i^n and x^n be the respective totals referred to the national level, the simple location quotient (SLQ) for sector i in region r can be defined as:

$$SLQ_i = LQ_i^r = \frac{x_i^r/x^r}{x_i^n/x^n} = \frac{x_i^r/x_i^n}{x^r/x^n}. \quad (1)$$

Where, in the final expression, the numerator presents the share of total national output of product i produced in region r and the denominator represents the share of total regional output in the national total.

Alternatively, given the actual availability of data on regional sectors, in practice, non-output data are often used to establish the proportionality sought, such as employment data (Kowalewski, 2015; Miller & Blair, 2009; Sargento et al., 2012), sectoral value added, income, and others showing such proportionality (Flegg et al., 2014; Jahn, 2017).

The domestic coefficient a_{ij}^{rr}, which is the difference between the regional technical coefficient a_{ij}^r and the regional import coefficient a_{ij}^{sr}, will be derived from the adjustment by the location quotient of the national coefficient a_{ij}^n for each industry. Thus:

$$a_{ij}^r = \begin{cases} (SLQ_i)a_{ij}^n & \text{if } SLQ_i < 1 \\ a_{ij}^n & \text{if } SLQ_i \geq 1 \end{cases}. \quad (2)$$

The regional total technical coefficient a_{ij}^r will coincide with the national coefficient if the location quotient is greater than unity, due to the assumption of coincidence of productive structure between the region and the higher national level. On the other hand, the coefficient will be rectified when the location quotient is less than unity, on the understanding (not as in the previous case) that the difference will be derived from imports.

2.1.2. Cross-Industry Location Quotient (CILQ)

The Cross-Industry Location Quotient (CILQ), attributed to Charles Leven by Tiebout in 1966 (Schaffer & Chu, 1969), introduces an alternative to the SLQ that adjusts the matrix cell by cell, considering the relative size of both the selling sectors, i, and the buying sectors, j (Ramos, 1998).

The cross-industry location quotient CILQ is:

$$CILQ_{ij} = \frac{x_i^r / x_i^n}{x_j^r / x_j^n}. \quad (3)$$

Then:

$$a_{ij}^r = \begin{cases} (CILQ_{ij})a_{ij}^n & \text{if } CILQ_{ij} < 1 \\ a_{ij}^n & \text{if } CILQ_{ij} \geq 1 \end{cases}. \quad (4)$$

Comparing the relative sizes of sectors i and j, it will be assumed that, if $CILQ_{ij}$ is less than unity, the relative size of sector i is smaller than the relative size of sector j in the region under analysis, so that it will need to import product to satisfy the demand of j.

As can be deduced (Miller & Blair, 2009), $CILQ_{ij} = LQ_i^r / LQ_j^r$, so that the elements where i = j will be equal to unity. In this case, a rectification is necessary (Flegg et al., 1995), completing the evaluation of the quotient for the determination of the coefficient a_{ij}^r as follows:

$$a_{ij}^r = \begin{cases} (CILQ_{ij})a_{ij}^n & \text{if } CILQ_{ij} < 1 \\ a_{ij}^n & \text{if } CILQ_{ij} \geq 1 \end{cases} \quad \text{for } i \neq j,$$

$$a_{ij}^r = \begin{cases} (SLQ_i)a_{ij}^n & \text{if } SLQ_i < 1 \\ a_{ij}^n & \text{if } SLQ_i \geq 1 \end{cases} \quad \text{for } i = j. \quad (5)$$

Bakhtiari and Dehghanizadeh (2012) offer an alternative, called the adjusted inter-industry location quotient (ACILQ), which consists of adjusting the CILQ quotient based on the size of the region whose table is to be estimated. $ACILQ = CILQ * K$ where $K = \frac{e^m - e^{-m}}{e^m + e^{-m}}$ y $m = 10 \left[\frac{x^r}{x^n}\right]$.

2.1.3. Flegg Location Quotient (FLQ) and Augmented FLQ (AFLQ)

The quotients seen so far, namely SLQ and CILQ, have certain limitations such as the overestimation of intra-regional trade – underestimating inter-regional trade – or, for instance, the fact that the productive structure of a given territory has a higher or lower share of procurement relative to the national average (Flegg et al., 1995; McCann & Dewhurst, 1998; Miller & Blair, 2009). In an attempt to improve them, the Flegg Location Quotient (FLQ) (Flegg & Webber, 1997) has been implemented:

$$FLQ_{ij} = \begin{cases} (\lambda)CILQ_{ij} & \text{if } i \neq j \\ (\lambda)SLQ_i & \text{if } i = j \end{cases},$$

$$\text{where } \lambda = \left(log_2\left(1 + \frac{x^r}{x^n}\right)\right)^{\delta}, 0 \leq \delta \leq 1. \quad (6)$$

Then:

$$a_{ij}^r = \begin{cases} (FLQ_{ij})a_{ij}^n & \text{if } FLQ_{ij} < 1 \\ a_{ij}^n & \text{if } FLQ_{ij} \geq 1 \end{cases}. \quad (6)$$

In order to properly capture the possible regional specialisation that would lead a given region to be more specialised than what the national coefficient indicates, the methodological proposal of Flegg was modified with a new proposal (Flegg & Webber, 2000), namely:

$$AFLQ_{ij} = \begin{cases} [\log_2(1 + SLQ_j)]FLQ_{ij} & \text{if } SLQ_j > 1 \\ FLQ_{ij} & \text{if } SLQ_j \leq 1 \end{cases} \tag{7}$$

And in this way:

$$a_{ij}^r = \begin{cases} (AFLQ_{ij})a_{ij}^n & \text{if } SLQ_j > 1 \\ (FLQ_{ij})a_{ij}^n & \text{if } SLQ_j \leq 1 \end{cases} \tag{8}$$

A significant number of research papers expressly mention the Flegg location quotient, FLQ (Flegg et al., 1995; Flegg & Webber, 1997), in the comparison of methodologies, either directly acknowledging it shows better goodness-of-fit compared to other techniques (Jahn, 2017; Lampiris et al., 2019), or indirectly using it as a comparative reference, regardless of whether it was determined to be the best estimator or not (Kowalewski, 2015; Lamonica & Chelli, 2018; Zhao & Choi, 2015; Mastronardi & Romero, 2012; Lamonica & Chelli, 2018; Lampiris et al., 2019; Flegg & Tohmo, 2013; Flegg et al., 2014).

It is worth noting that, in practice, this augmented alternative (AFLQ) does not generally perform better than the simple version (FLQ) (Bonfiglio, 2009), hence the latter remains the most suitable for undertaking regionalisation processes.

Thus, the alternative proposed in the FLQ and AFLQ methodologies entails a highly significant dependence on the value given to the parameter δ (Flegg & Tohmo, 2019a; Kowalewski, 2015; Lamonica & Chelli, 2018; Lampiris et al., 2019), the determination of which is complex.

2.1.4. Two-dimensional location quotient (2D-LQ)

The two-dimensional location quotient (Pereira-López et al., 2020) is based on the premise that the adjustment needed in the regionalisation process for the cost structure of a given industry does not necessarily have to be related to the adjustment needed in the sales structure, allowing a different adjustment parameter to be chosen for each of the two cases.

The characteristic elements of the matrix of intermediate coefficients, $A^r = (a_{ij}^r)_{i,j=1,2,\dots,m}$, are to be defined from the following expression:

$$A^r = R(\alpha)A^nS(\beta), \tag{10}$$

where $A^n = (a_{ij}^n)_{i,j=1,2,\dots,m}$ is the matrix of the national coefficients.

As follows:

$$a_{ij}^r = r_i(\alpha)a_{ij}^n s_j(\beta) \quad i,j = 1,2,\dots,n \tag{11}$$

Where $R(\alpha)$ and $S(\beta)$ are going to be diagonal matrices whose elements will be null except for those of the main diagonal which will take the following values:

$$r_i(\alpha) = (SLQ_i)^\alpha \tag{12}$$

$$s_j(\beta) = (wx_j^r)^\beta \quad \text{with} \quad wx_j^r = x_j^r / x_j^n \tag{13}$$

Thus, both regional specialization and regional size are corrected by the values of the matrices $R(\alpha)$ and $S(\beta)$ respectively.

Depending on the value of the simple location coefficient SLQ this methodology will cause the elements of the regionalized matrix to take the following values

$$a_{ij}^{r} = \begin{cases} (SLQ_i^r)^{\alpha} a_{ij}^{n} \left(wx_j^r\right)^{\beta} & if\ SLQ_i^r \le 1 \\ \left[\frac{1}{2}\tanh(SLQ_i^r - 1) + 1\right]^{\alpha} a_{ij}^{n}\left(wx_j^r\right)^{\beta} & if\ SLQ_i^r > 1 \end{cases} \tag{14}$$

2.2. The estimation of inter-regional trade

The relative importance of trade relations between regional territories is much more decisive than that of international trade (Thissen et al., 2014). Therefore, in order to be able to undertake the appropriate impact and structural change analysis at the regional level based on input-output tables and using indirect methods, i.e., regionalising a higher order table, the endogenous information in the estimated table needs to be expanded considering the trade transactions between the analysed region and the other regions with which it has commercial relations within the same country. This information is not available in the higher order table and must be estimated, usually by gravity models. However, in the absence of a regional table, it is often advisable (Boomsma & Oosterhaven, 1992; Isard et al., 2017; Round, 1983) to estimate a bi-regional (inter-regional or multi-regional) model in which both the region under study and the relations with a second region covering the rest of the country are considered. While it is true that the problems associated with the estimation of input-output frameworks of two or more regions is not a recent topic, addressed for example since Isard (1960), Round (1983), or Oosterhaven et al. (1986), it is equally true that the current economic reality – the economic interconnection between different territories and fully globalised economies – has led to the deepening and development of estimation techniques that include this type of table. For instance, in Wang and Canning (2004), Kratena et al. (2013), Többen and Kronenberg (2015), Jahn (2017), Boero et al. (2018), Temursho et al. (2021) or Krebs (2020).

2.3. Extending FLQ to the Inter-Regional Framework

The use of location quotients in regionalisation processes through inter-regional modelling has not been very widespread in the literature, though seen in Hermannsson (2016) or Jahn (2017).

Thus, the procedure proposed by Jahn (2017) – which was tested for its goodness-of-fit in Jahn et al. (2020) but without comparing it with other methodologies – is based on the estimation of the domestic regional IOT using FLQ location quotients, thereby obtaining a first estimate of the regional IOT.

When estimating intra-regional trade using quotients, it makes sense to consider the generation of a residual resulting from the difference between the corresponding value of each element of the national IOT and the value obtained by applying the FLQ methodology.

Let z_{ij} be the element of the national IOT purchased by the industry j from the industry i, and let $\bar{z}_{ij}^{rr}$ be the element ij estimated from the domestic matrix of the region r, this residue can be defined as:

$$\epsilon_{ij}^{FLQ} = z_{ij} - \textstyle\sum_r \bar{z}_{ij}^{rr} \geq 0 \tag{15}$$ [1]

Since the main focus of this paper is on the estimation of the input-output table for a particular region, an estimation of bi-regional trade between the region of interest and the "rest of the country" region is enough in order to establish the importance of the estimation of inter-regional trade. Thus, using a straightforward trade model (Jahn et al., 2020) and relative production data by industry (Flegg & Tohmo, 2019) – without the need to resort to gravity models for estimating inter-regional trade – the value of inter-regional trade between industries i, j and between regions r, s is established as follows:

$$h_{ij}^{sr} = \begin{cases} \bar{x}_i^s \bar{x}_j^r & \text{for } s \neq r \\ 0 & \text{for } s = r \end{cases}, \tag{16}$$

[1] It must be noted that the graphs depicted in Pereira-López et al. (2022), which display the elements of the different matrices of the EA-19 territory and 4 of the countries in it, show that errors in the estimation mean that in some cases the sign of this residual may be negative.

where $\bar{x}_i^s$ is the (estimated) production of the industry i in the region s.

Scaling this result to ensure that the sum of h_{ij}^{sr} is equal to unity, the value of inter-regional trade can then be obtained by applying it to the residual:

$$\bar{z}_{ij}^{sr} = \frac{h_{ij}^{sr}}{\sum_{r',s'} h_{ij}^{sr}} \epsilon_{ij}^{FLQ} = \frac{h_{ij}^{sr}}{\sum_{r',s'} h_{ij}^{sr}} \left(z_{ij} - \sum_{r} \bar{z}_{ij}^{rr} \right) \text{ for } s \neq r \tag{17}$$

At this point in the estimation process, the intra-regional trade matrix of internal transactions is already available, referring both to the specific region (in this study for the specific cases of the Andalusia, Basque Country and Navarra) and to the so-called "rest of the country" region, as well as an estimate that includes the cross-industry transactions between these two regions, having to estimate the part corresponding to the final demand of both.

The procedure followed proposes the estimation of external trade branch by branch (or product by product) based on the weight of total exports, on the one hand, and imports, on the other, for the region of interest, and weighting this by the corresponding variable in the reference input-output table.

This regionalisation method, unlike the one presented by Wang and Canning (2004) on which it is based, offers an estimate of Total Final Demand (TFD), whose origin lies in each of the differentiated regions, regardless of the destination where consumption actually takes place. Thus, for example, household final consumption expenditure will not only include the value of such expenditure of residents of the region of interest but will also include that of households of non-residents of the region who consume goods and services produced in the region.

To calculate the DFT, taking into account the typology of the table to be estimated – domestic – as well as the procedure proposed and given the data that are normally available, there are grounds for proposing a subsequent alternative for its proper estimation, differentiating between domestic final demand and that which comes from the rest of the country. This is a decisive factor for being able to make estimates of the closed model from which the induced effects are derived in the event of final demand shocks.

The first estimate of the bi-regional table must be optimised by minimising the square of the distances, which is equivalent to maximising entropy, by solving the following optimisation problem (Jahn, 2017):

$$Min\, S = \sum_{i,j,s,r} \frac{\left(z_{ij}^{sr} - \bar{z}_{ij}^{sr}\right)^2}{w_{ij}^{sr} z_{ij}^{sr}} + \sum_{i,r} \frac{(x_i^r - \bar{x}_i^r)^2}{x_i^r} + \sum_{i,r} \frac{(v_i^r - \bar{v}_i^r)^2}{v_i^r} + \sum_{i,r} \frac{(y_i^r - \bar{y}_i^r)^2}{y_i^r} + \sum_{i,r} \frac{(m_i^r - \bar{m}_i^r)^2}{m_i^r} + \sum_{i,r} \frac{(e_i^r - \bar{e}_i^r)^2}{e_i^r}.$$

subject to:

$\sum_{j,s} z_{ij}^{sr} + m_i^r + v_i^r = x_i^r.$ (18.1)

$\sum_{j,s} z_{ij}^{rs} + y_i^r + e_i^r = x_i^r.$ (18.2)

$\sum_{s,r} z_{ij}^{rs} = z_{ij}.$ (18.3)

$\sum_r x_i^r = x_i.$ (18.4)

$\sum_r v_i^r = v_i.$ (18.5)

$\sum_r y_i^r = y_i.$ (18.6)

$\sum_r m_i^r = m_i.$ (18.7)

$\sum_r e_i^r = e_i.$ (18.8)

(18)

Where z_{ij}^{sr} is the value of intermediate inputs from sector i in region s to sector j in region r; m_i^r is the imported inputs (from abroad); e_i^r is the exports (from abroad); v_i^r is value added; x_i^r is production; and y_i^r is the domestic final demand for the goods produced by sector i in region r. The corresponding variables without superscripts z_{ij}, m_i, e_i, v_i, x_i, y_i denote national aggregate values. And variables with an upper bar correspond to the respective initial estimates.

The imposed restrictions ensure, on the one hand, that the IO tables are consistent within each region and, on the other hand, that the regional values are consistent with the national aggregates. Thus, as far as the first question is concerned, constraints (18.1) and (18.2), respectively state that the sum of all types of input and the sum of all types of output are equal to the production of each sector in each region. As regards the second question, the constraint (18.3) ensures that the sum of the regional values of the intermediate inputs of sector i for sector j is equal to the corresponding national value. Furthermore, constraints (18.4) to (18.8) impose, respectively, that the sum of the regional values of regional production, value added, domestic final demand for the goods produced, imported inputs (from abroad) and exports (to outside the nation), must coincide, for each sector, with their respective national aggregate values.

In the same way, and bearing in mind that the main focus is on obtaining as accurate an estimate as possible of the intermediate inputs matrix, the above-mentioned expression can be simplified to ensure that the matrix is optimised. (Jahn et al., 2020). Such estimation is achieved by solving the following problem:

$$Min\, S = \sum_{i,j,s,r} \frac{\left(z_{ij}^{sr} - \bar{z}_{ij}^{sr}\right)^2}{w_{ij}^{sr} z_{ij}^{sr}} \text{ subject to:}$$

$$\sum_{i,s} z_{ij}^{sr} \leq x_j^r \tag{19}$$

$$\sum_{s,r} z_{ij}^{sr} = z_j^r.$$

The proof of the estimation accuracy of the procedure described in Jahn (2017) is performed on the Korean multi-regional input-output table for the year 2005 (Jahn et al., 2020). This latter paper evaluates, when estimating inter-regional trade, the parameterisation of a different δ value for each region as opposed to the use of sectoral δ parameters, sectoral δ for each region or a single δ for all regions equally. Flegg and Tohmo (2019) argue that the alternative of using a sectoral parameter δ_j in each region (Kowalewski, 2015) gives optimal results for some regions, but not so good results for others, and they recommend the use of a different δ for each region.

With regard to inter-regional trade, while recognising that the use of gravity models offers optimal solutions, they conclude that the mere use of a simple trade model is adequate given the result obtained by the estimation. Considering that the final result of the process is expected to be a regionalised table of a single region, based on the estimation of the bi-regional table, this procedure was deemed to be the most appropriate.

The procedure entails solving equation (13) by minimising the square of the distances and obtaining the final estimate of the components of the input-output table. However, there is the possibility of assessing another procedure which involves using the estimation of inter-regional trade using equation (12) combined with the GRAS methodology.

2.4. GRAS Methodology

The simple RAS technique (Stone, 1961) is limited partly by the impossibility of adequately estimating negative elements in the projected matrix. This is addressed by the GRAS methodology (Junius & Oosterhaven, 2003; Temurshoev et al., 2013), which is able to project matrices that combine both positive and negative elements. Thus, given that it is indifferent to carry out the estimation process with either the transaction matrix or the matrix of technical coefficients (Dietzenbacher & Miller, 2009; Miller & Blair, 2009, p. 327), the authors propose a mathematical approach to the idea applied years ago by Günlük-Senesen & Bates (1988), which was to conduct the projection on two sub-matrices, splitting the matrix to be projected into one that contains all the positive elements and another one that contains all the negative elements.

The advantages of biproportional methodologies are many, both in terms of their ease of understanding and in their application, even though it is understandably not free of criticisms such as its economic justification beyond constituting an adequate mathematical solution to an estimation problem (Jackson & Murray, 2004). In any event, the RAS technique and its generalisation GRAS is, a priori, an adequate solution to the projection problem in question.

The GRAS algorithm, then, subdivides the technical coefficient matrix A into two matrices, one containing all the positive elements of the matrix A and the other containing all the negative elements of that matrix A. Below are the details of the problem:

- Let $A = (a_{ij})_{mxn}$ be a known matrix with m rows and n columns. And let $u_0 = Ai$ be the vector of m elements containing the sums of each row, and $v_0 = iA$ be the vector of n elements containing the sums of the n columns, where i is a vector of appropriate dimension consisting entirely of ones.
- A has both negative and positive elements.
- Let $u = (u_i)_{i=1,2,\ldots,m}$ y $v = (v_j)_{j=1,2,\ldots,n}$ be given vectors and let $u \neq u_0$ and $v \neq v_0$ be representing the "new" sums of rows and columns, respectively, where $iu = iv$.

The goal of the problem is to obtain an mxn order matrix$X$ whose difference with A is minimal and which satisfies $u = Xi$ and $v = iX$.

To solve it, the matrix A is divided in $A = P - N$, where $P = (p_{ij})_{mxn}$ is the matrix formed by the positive elements of A and $N = (n_{ij})_{mxn}$ is the matrix whose elements are the absolute values of the negative elements of A.

Applying the GRAS method, the objective matrix $X = (x_{ij})_{mxn}$ is obtained by:

$$x_{ij} = \begin{cases} r_i a_{ij} s_j \ \text{if}\ a_{ij} \geq 0 \\ r_i^{-1} a_{ij} s_j^{-1} \ \text{if}\ a_{ij} < 0' \end{cases} \qquad (20)$$

where $r_i > 0$ and $s_j > 0$ are solutions of the system of equations

$$p_i(s) r_i^2 - u_i r_i - n_i(s) = 0 \qquad (21.1)$$
$$p_j(r) s_j^2 - v_j s_j - n_j(r) = 0 \qquad (21.2) \qquad (21)$$

where

$$p_i(s) = \sum_j p_{ij} s_j \,, p_j(r) = \sum_i p_{ij} r_i, n_i(s) = \sum_j \frac{n_{ij}}{s_j}$$
$$y\, n_j(r) = \sum_i \frac{n_{ij}}{r_i}. \qquad (22)$$

The solutions of the second-degree equations (21.1) and (21.2) are obtained in the usual way, being

$$r_i = \frac{u_i + \sqrt{u_i^2 + 4p_i(s)n_j(s)}}{2p_i(s)}$$
$$y \quad s_j = \frac{v_j + \sqrt{v_j^2 + 4p_j(r)n_j(r)}}{2p_j(r)}, \tag{23}$$

which are calculated using the following iterative algorithm:

1. The process starts with a vector r with all its components equal to 1.
2. Then: $p_j(r) = \sum_i r_i p_{ij}$ and $n_j(r) = \sum_i \frac{n_{ij}}{r_i}$.
3. At each iteration k, $s_j(k)$ and $r_i(k)$ are calculated according to (23).
4. The algorithm stops at the iteration M, in which $|s_j(M) - s_j(M-1)|$ is smaller than a certain value (e.g., 10^{-8}) for all elements.
5. Once convergence is secured, the projected table is generated using the following formula with the values obtained in the *M* iteration:

$$a_{ij} = r_i(M)p_{ij}s_j(M) - \frac{n_{ij}}{r_i(M)s_j(M)}. \tag{24}$$

When implementing this method, the following precaution should be taken into consideration (Temurshoev & Timmer, 2010):

$$r_i = \begin{cases} \frac{u_i + \sqrt{u_i^2 + 4p_i(s)n_j(s)}}{2p_i(s)} & for\ p_i(s) > 0 \\ -\frac{n_i(s)}{u_i} & for\ p_i(s) = 0 \end{cases},$$
$$s_j = \begin{cases} \frac{v_j + \sqrt{v_j^2 + 4p_j(r)n_j(r)}}{2p_j(r)} & for\ p_j(r) > 0 \\ -\frac{n_j(r)}{u_j} & for\ p_j(r) = 0 \end{cases}. \tag{25}$$

In this case, an equivalent alternative for the projection of supply and use tables (SUT) is not the GRAS methodology, but the SUT-RAS methodology (Temurshoev & Timmer, 2011;Valderas-Jaramillo et al., 2019) which was developed for the application of the RAS methodology to SUT with great versatility, not requiring knowledge of the production totals by product for its projection. The SUT-RAS methodology uses the same objective function as the GRAS methodology (Lenzen et al., 2009). This methodology can be easily adapted according to the needs and availability of information of the analyst, both in relation to the assessment, either at basic prices or at purchase prices, and to the distinction of the origin of the destination flows, whether domestic or imported. A generalisation of this methodology to be used in the estimation of a sub-regional framework consistent with the regional total is presented in Valderas-Jaramillo et al. (2019). The difference with the RAS or GRAS methods lies in the fact that the source table is not projected separately from the destination table, but rather an integrated framework is provided from which the entire framework is projected.

2.5. Gravity Models

Based on the Newton gravity model, there are several formulations of gravity models used to estimate inter-regional flows (Miller & Blair, 2009, Chapter 8.6). The basic idea of this typology of models is that the flow of a commodity from one region to another depends on the amount of the commodity in the region of origin, the number of purchases of the commodity in the region of destination and the distance between the two regions.

Thus, the gravity model for a good i purchased by the industry j between the regions r and s will be given by (Holt, 2017):

$$Z_{ij}^{rs} = G\frac{P_r^{\alpha_1}P_s^{\alpha_2}}{d_{rs}^{\alpha_3}}. \tag{26}$$

Where α_1, α_2 and α_3 are parameters to be estimated; G is a proportionality constant that will depend either on j, on r, or on s. P_r and P_s collect information on the supply of the good i in the region r and the demand for the good i in the industry j in the region s, respectively. Conceptually speaking, the element d_{rs} refers to the distance between the region r and the region s, which can be defined in multiple ways (Greaney & Kiyota, 2020; Isard et al., 2017; Riddington et al., 2006; Sargento et al., 2012).

Regarding their use as part of the regionalisation process of input-output tables based on individual region estimation models, special attention ought to be paid to a set of questions which, a priori, cannot be automatically solved (Miller & Blair, 2009, p. 76). Issues such as technological implementation, level of professional qualification and industrial development must be taken into account insofar as they may differ substantially depending on which territory we are considering in terms of the reference country (Sargento, 2009) or, alternatively, attention must be paid to the fact that the region in question is not isolated from the environment (Sargento et al., 2012). Similarly, the size of the modelled region will be crucial in assessing the trade interconnection between the region under analysis and those around it. Regional size has been shown to affect the evolution of trade between regions (inter-regional) more significantly than intra-regional or international trade in the face of certain shocks (Jackson & Murray, 2004) and hence must be duly taken into account to optimise the projection process, insofar as an alteration in the production conditions of a given region will lead to an alteration in its volume of purchases and sales among the regions of the country, which can indirectly generate new variations in the levels of production of the region in question.

Riddington et al. (2006) evaluate how well location quotients estimate both coefficients and multipliers in comparison with gravity models using the DREAM methodology applied to Scotland and the United Kingdom. They combine the gravity model with the RAS methodology, recommending this methodology over those based on location quotients with which it compares it, given the goodness of fit in the multipliers obtained.

Sargento et al. (2009; 2012) assess the ability of different methodologies to estimate inter-regional trade on supply and use tables for 14 EU countries to conclude that, on the one hand, the starting values in the process of estimating inter-regional trade are determinant and, on the other hand, that the gravity model that performs most accurately is the one that alternatively estimates the distance decay parameter, known as β, by minimising the following error indicator (Sargento, 2007):

$$I = \sum_s |\sum_r e^{rs} - \sum_r \tilde{e}_0^{rs}| / \sum_r \sum_s e^{rs}, \tag{27}$$

Where e^{rs} and $\tilde{e}_0^{rs}$ are the export flow from region r to region s and the estimated initial export flow, respectively.

In the specific case at hand, attention must be paid to the estimation scenario that assumes a total lack of knowledge about inter-regional flows (Sargento, 2009, p. 2014). Based on prior knowledge of the total sum values of rows and columns of the matrix, the gravity model from (26) is used to approximate the assessment of intermediate transactions:

	Region 1	**Region 2**	**…**	**Region k**	**Sum**
Region 1	**0**	z_j^{12}	…	z_j^{1k}	$d_j^{1\,roc}$
Region 2	z_j^{21}	**0**		z_j^{2k}	$d_j^{2\,roc}$
…	…	…	**0**	…	…
Region k	z_j^{k1}	z_j^{k2}	…	**0**	$d_j^{k\,roc}$
Sum	$m_j^{roc\,1}$	$m_j^{roc\,2}$	…	$m_j^{roc\,k}$	$d_j = m_j$

Assuming that the margins of the intermediate consumption matrix between the regions of interest *r* and *s* are known, the application of the RAS technique on the gravity equation applied to trade (26) can be considered in the following form:

$$(\tilde{x}^{rs})_{RAS} = J^r \tilde{z}^{rs} L^s = J^r G \frac{(P^r)^{\alpha_1}(P^s)^{\alpha_2}}{(\delta^{rs})^{\alpha_3}} L^s. \tag{28}$$

Where J^r and L^s are the vectors that will guarantee the closest possible fit to the initial matrix.

Following Sargento (2009), the impossibility of knowing the value of certain parameters leads to the following assumptions:

- The parameters α_1, α_2 and α_3 are assumed to be unitary.
- The degree of specialisation of origin, DS^r, needs to be defined, taking the following form on the basis of export data by product e^k:

$$DS_r^k = \frac{x_r^k / \sum_1^k x_r^k}{\sum_1^r x_r^k / \sum_1^r \sum_1^k x_r^k}. \tag{29}$$

- The scalar G must ensure that the constraint $\sum_s \tilde{z}^{rs} = e^r$ is satisfied, so it will take the value

$$G^r = e^r \left(\sum_s \frac{P^r P^s DS^r}{\delta^{rs}}\right)^{-1}. \tag{30}$$

The gravitational equation will be:

$$\tilde{z}^{rs} = G^r \frac{P^r P^s DS^r}{\delta^{rs}}. \tag{31}$$

As we are dealing exclusively with the bi-regional level, assessing the precision in the estimation of our region of interest and the rest of the country, we can assume the distance to be unitary[2], i.e., $\delta^{rs} = 1$.

The RAS methodology combined with gravity modelling, applied to inter-regional frameworks, has been applied in different works such as Cai (2022, 2020), Fournier (2020), Sargento et al. (2012), Temursho et al. (2021) or Yamada (2015) by defining as a starting point the set of inter-regional trade variables based on the implementation of a given gravity model and then adjusting these variables to the necessary production and employment margins to obtain an adjusted solution.

Once the three methodologies under study have been introduced, a set of goodness-of-fit statistics – obtained from Valderas et al. (2015) and capable of providing information that allows for adequate discrimination between regionalisation methodologies – is used to compare them.

In the first instance, the goodness-of-fit analysis will be conducted by assessing the value obtained from the implementation of the Weighted Absolute Percentage Error (WAPE) statistic. This statistic is very often used in the input-output field. In order to avoid biases that may occur in the measurement of goodness-of-fit – derived from giving the same weight to all variables without taking into account the weight of each of the evaluated coefficients – this statistic will be able to measure the absolute error percentages, on average, weighted by the weight of each element with respect to the one they were calculated from (Valderas, 2015). It is expressed as follows:

[2] The recent work by Cai (2022) implements an application of the Gravity-RAS model in which he obtains a significant improvement in the estimation of the distance elasticity of inter-regional trade applied to the case of Italian regions. In particular, it establishes an econometric framework for estimating the distance elasticity of trade between regions within a country using data on trade between different countries.

$$\text{WAPE} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{|x_{ij} - \tilde{x}_{ij}|}{\sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|} \right). \tag{32}$$

Secondary and complementary to the previous analysis, a set of statistics will be used to enable the confirmation of the information obtained from the result of applying the main statistic, i.e. , the WAPE (27).

Thus, for example, considering that the significance of a coefficient does not derive exclusively from its size, it is considered appropriate to provide a goodness-of-fit measure that allows for the scaling of each variable. Thus, the Weighted Absolute Scaled Error (WASE) statistic will provide a lower sensitivity to anomalous elements, as it is not affected by changes in the scale or size of coefficients (Valderas, 2015). It is defined as:

$$WASE = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{|x_{ij}|}{\sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|} \right) \left| \frac{|x_{ij} - \tilde{x}_{ij}|}{\sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij} - \bar{x}_{ij}| / mn} \right|. \tag{33}$$

The $\rho - \text{SWAPE}$ statistic (29), based on the work developed by Arto et al (2014), can be used in a straightforward way for cross-sectional and inter-method comparisons (Valderas, 2015). Its interpretation is similar to a coefficient of determination – it takes unit value in case of a perfect fit and zero otherwise – and it is defined as:

$$\rho - SWAPE = 100 \left(1 - \frac{SWAPE}{200} \right).$$
$$\text{Where } SWAPE = 200 \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{|x_{ij}|}{\sum_{i=1}^{m} \sum_{j=1}^{n} |x_{ij}|} \right) \left| \frac{x_{ij} - \tilde{x}_{ij}}{x_{ij} + \tilde{x}_{ij}} \right|. \tag{34}$$

With regard to these statistics, which are referred to as subsidiary statistics for the sake of convenience, an analysis is made as to whether they follow the same direction in their interpretation as that obtained by using the main statistic. The results of all statistics are incorporated in Appendix 1.

In order to compare the extended location quotient methodology (Jahn, 2017) with the generalised version of the RAS methodology, the reference used is the input-output table for Spain corresponding to 2015 and that of three Spanish regions differentiated by their relative size in terms of Gross Domestic Product (large, medium and small), Andalusia, the Basque Country and Navarra, which present the symmetrical table for the same year.

3. Comparison of the Proposed Methodologies

In order to comparatively assess the accuracy of the estimation between the extended location quotient methodology (Jahn, 2017) and the GRAS methodology (Junius & Oosterhaven, 2003) location quotient-based methodologies must be used first. Thus, while Jahn (2017) resorts to the use of the FLQ methodology (Flegg et al., 1995) giving the delta parameter a value of 0.3, it is considered that such a parameterisation could cause a high bias in the estimation, so it is decided to make the comparison with those delta values that minimise the WAPE statistic, considering this estimation as optimal FLQ. In addition, it is considered appropriate to undertake a prior comparison between different methodologies based on location quotients, hence the goodness of fit of the FLQ methodology is compared with that obtained from the use of the 2D-LQ methodology. Regarding this latter methodology, whose value depends on two parameters that smooth the rectification by rows, α, and by columns, β, a similar procedure is followed, selecting those values that minimise the WAPE statistic taken as a reference.

For the estimation of the optimal values of δ, in the case of Spain, the largest possible number of rows/columns is maintained independently in each of them instead of homogenising all the regional tables to the same number of rows and columns. The reason for this is that excessive aggregation into branches can distort the regionalisation process (Flegg and Tohmo, 2013) and may lead to erroneous conclusions, as can be seen in Riddington (2006).

Of the seventeen existing Spanish regions, thirteen of them have an input-output table available, which allows for the application of the procedure in this context.

Spain does not have a multi-regional table and the availability of regional tables is neither standardised nor homogenised across regions. For this reason, the largest number of available regional input-output tables are homogenised in relation to the national tables taken as a reference, between 2005 and 2015 (further information is available in the appendix). First, for these regions, the different location quotient methods analysed are compared using the WAPE statistic.

Table 1 presents the results obtained for the different Spanish Regional Input-Output Tables, indicating the reference year. In the case of the 2D-LQ ratio, all possible combinations of the parameters α and β are tested by giving the parameter α values from 0 to 2 in 0.1 increments, evaluating all possible combinations with the parameter β, which takes all possible values from 0 to 1 in 0.01 increments. It is observed that the cases of Andalusia and the Balearic Islands constitute an exception in the evaluation, insofar as the minimum values of the statistic are obtained with the maximum possible value of α, namely $\alpha = 2$, so the range of values for these two cases is extended, taking values from 0 to 3. Andalusia obtains the minimum WAPE with an $\alpha = 2.7$ while, in the case of the Balearic Islands, the minimum WAPE is obtained with $\alpha = 3$.

As can be seen, the results indicate that it is the 2D-LQ method that obtains greater precision in most Spanish regions. However, the ACILQ ratio does not improve the rest of the ratios, in any case, so it can be stated that the smoothing performed on the CILQ ratio is not enough to improve the estimation when compared to the rest of the ratios.

The absence of a multi-regional input-output table for Spain may be considered a limitation in performing the proposed exercise. The application of the augmented location quotient methodology requires the estimation by means of location quotients of the domestic input-output table of the corresponding region called, in our case, the rest of the country (ROC). Since no such table is available, neither the optimal value of the parameters δ, in the case of FLQ, nor β and β, in the case of 2D-LQ, that minimise the reference statistic can be determined.

Buendía et al. (2022) propose an estimation equation for the parameter δ in the case of FLQ that offers the best estimate of this parameter, understood as the one that minimises the value of the WAPE statistic obtained in the estimation on the reference matrix. In the case of the Spanish regions, the regional δ values are estimated from the following regression equation:

$$ln\delta = \alpha\ lnIROW + \beta\ lnIRFT + e \tag{35}$$

where $IROW$ is the ratio of the region's propensity to imports from the rest of the world with respect to that of the nation, $IRFT$ is the weight of road freight transport flow from other regions in the total freight transport flow, and e is the residual.

Table 1.
Goodness of fit, according to WAPE, between different types of location quotients and differences with respect to the global optimum value

	WAPE				
Region	**FLQ**	**AFLQ**	**2D-LQ**	**ACILQ**	**MINIMUM**
Andalusia 2010	0.607	0.668	0.557	0.629	*2D-LQ*
Aragon 2005	0.881	0.961	0.922	1.234	*FLQ*
Principality of Asturias, 2015	0.772	0.902	0.738	0.788	*2D-LQ*
Balearic Islands 2004	0.759	0.910	0.737	0.759	*2D-LQ*
Canary Islands 2005	0.849	0.927	0.765	0.890	*2D-LQ*
Cantabria 2015	0.741	0.795	0.695	0.775	*2D-LQ*
Castilla-La Mancha 2005	0.731	0.819	0.693	0.760	*2D-LQ*
Catalonia 2011	0.933	1.294	1.089	0.982	*FLQ*
Galicia 2011	0.694	0.730	0.632	0.712	*2D-LQ*
Community of Madrid, 2010	0.779	0.840	0.769	0.894	*2D-LQ*
Community of Navarra, 2010	0.737	0.869	0.689	0.755	*2D-LQ*
Basque Country 2015	0.632	0.722	0.632	0.679	*FLQ*
La Rioja, 2008	0.839	0.864	0.801	0.978	*2D-LQ*

Source: Authors' calculations for regional IOTs in Spain.

Proposed Estimation of Parameter Values α, β

Similarly to what happens in the case of the δ parameter for the FLQ and AFLQ ratios, giving values for the parameters that modify the national coefficient in the case of 2D-LQ in regionalisation processes is problematic when no prior regional reference table is available.

The parameters α and β that smooth the rectification applied to the national coefficient matrix, according to the authors, are not associated with each other (López, Incera, & Fernández, 2013; Pereira-Lopez et al., 2020), although both papers establish –in their practical application – ranges of combined optimality between values of α for a given β and, alternatively, a range of values of β for a given α.

Whether the superiority shown in the accuracy of the estimation of the 2D-LQ ratio (table 1) can be considered generalisable or happens on an ad hoc basis needs to be assessed. Therefore, the construction of the 2D-LQ ratio should be reviewed in relation to the procedure established to obtain the values of the parameters α and β, incorporating an assignment of optimal values based on criteria established by economic theory. In this sense, this is a proposal for the estimation of the value of the parameter β which, combined with the range of values of the parameter α, provides a more accurate estimate of the values of the regional coefficients. Furthermore, given that there is greater sensitivity associated with changes in the β parameter compared to changes in the α parameter (Pereira-López et al., 2021), estimating the β parameter is considered crucial.

Therefore, to estimate the parameter β , a regression equation is proposed in which the explanatory variables are road freight transport (origin and destination) and regional size

$$\hat{\beta} = 1.78RS + 0.47FIT + e. \tag{36}$$

where, as noted above, RS represents the regional size measured in terms of GDP, FIT corresponds to the weight of freight transport flow from other regions (inter-regional transport) measured in tonnes over the total freight transport flow, and e is the residual. The two regressors are statistically significant at 1% and the regression equation has a value of R^2 = 0.704

In the case of the parameter estimation equation (smoothing the row rectification from the simple location quotient SLQ), the best specification is achieved in logarithmic terms:

$$\ln \hat{\alpha} = 0.5681 \ln RE - 0.4228 \ln FET + e. \quad (37)$$

where, now, the RE variable represents the relative regional size measured in terms of employment, while the FET variable corresponds to the weight of the transport flow of goods destined for other regions (inter-regional transport) measured in tonnes over the total transport flow of goods, including both the inter-regional transport flow and the transport flow generated within the same region (intra-regional) also measured in tonnes, and e is the residual. The RE and FET variables are statistically significant at 1% and 5%, respectively, and the model has a value of R^2 = 0.572. Table 2 presents the estimated α and β values, the WAPE statistic and the relative difference with regard to the optimal WAPE.

As shown, the values of the parameters α and β obtained with the proposed estimation (36) and (37) provide values of the fit statistic whose deviation from the optimum is quite acceptable and, in general, still obtain better values of the statistic than the rest of the methodologies based on location quotients. The largest differences in terms of fit to the optimum are found in the case of the Spanish regions that offer the best estimates with the FLQ methodology.

Once again, the superiority of the 2D-LQ methodology is evident when using the regression procedure proposed in this paper to estimate the values of the parameters α and β. Thus, in 72.7% of the cases the modified 2D-LQ methodology outperforms the best possible estimate obtained from the FLQ methodology using the optimal value of δ in goodness of fit. The only cases in which the results are worse than those obtained using the optimal value of δ are the two archipelagos (Balearic and Canary Islands) and Catalonia.

TABLE 2.
Value of the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$, and value of the WAPE statistic

Region	$\hat{\alpha}$	$\hat{\beta}$	WAPE	Dev. s/optimum 2D-LQ
Andalusia 2010	0.8200	0.3076	56.8420	2.00%
Aragon 2005	0.2145	0.2264	108.1597	22.78%
Principality of Asturias, 2015	0.1933	0.1354	74.7481	1.25%
Balearic Islands 2004	1.2118	0.0453	84.0523	14.06%
Canary Islands 2005	1.1505	0.0683	93.8604	22.63%
Cantabria 2015	0.1217	0.1761	69.9141	0.66%
Castilla-La Mancha 2005	0.2264	0.2395	69.5357	0.41%
Catalonia 2011	0.7939	0.4115	126.6419	35.70%
Galicia 2011	0.4485	0.3010	68.0631	7.70%
Community of Madrid, 2010	0.5362	0.5192	77.1477	0.31%
Community of Navarra, 2010	0.1333	0.0950	72.4648	5.16%
Basque Country 2015	0.3001	0.3568	65.1524	3.13%
La Rioja, 2008	0.0784	0.1859	82.2648	2.72%

Source: Authors' calculations for regional IOTs in Spain.

Nonetheless, insofar as the intention is to undertake a comparison of the global estimation capacity of the Jahn, GRAS and Gravity-RAS methodologies, the comparison should be made on the basis of the result obtained by both location quotient methodologies in order to observe whether the goodness of fit obtained by the FLQ and 2D-LQ methodologies is sustained in successive stages or whether there may be some kind of bias that results in a worse fit depending on the location quotient method used. To this end, three Spanish regions differentiated by their size in terms of GDP will be taken as a reference: Andalusia (13.47%), Basque Country (6.06%) and Navarra (1.68%); as well as the table for Spain (ESA 2010) year 2015[3] of the type of product by product which presents 64 homogeneous branches at basic prices. The identification and correspondence of the different branches of activity between the different territories is included in appendix 2.

The proposal put forward to evaluate the different estimation techniques is made under the assumption (compatible with the current statistical reality) that the values corresponding to added value, foreign trade and even final consumption expenditure and gross capital formation[4] are known beforehand. In this way the analysis is limited to the precision in the estimation of the intermediate consumption matrix. Based on this assumption, the procedure shall be as follows:

Initially, a first estimate of inter-regional trade between each region of interest and the rest of the country is obtained by calculating the residual (15), and both the GRAS technique and the optimisation proposal of Jahn (2017) are assessed.

Alternatively, inter-regional trade is estimated by means of the Gravity-RAS procedure detailed by Sargento (2009) with the implementation of equation (31).

The application proposed in the augmented location quotient procedure (Jahn, 2017) to obtain a first estimate of the bi-regional table (Region of interest - Rest of Spain) starts from the estimate of the added value by branch of activity (obtained by assuming the same ratio with respect to the value of production at both national and regional level) and the initial estimate of the final demand (measured by the relative importance of the added value of each regional product over its corresponding national product). Such a ratio will rectify the corresponding magnitude of the national input-output table. In other words, take

$$\bar{y}_i^r = y_i \frac{v_i^r}{v_i}. \tag{38}$$

Knowing the regional share of the national total both in exports es^r and imports ms^r, then

$$\bar{e}_i^r = x_i \times es^r, \tag{39}$$

$$\bar{m}_i^r = m_i \times ms^r. \tag{40}$$

In this paper's analysis, while aiming to determine the bi-regional inter-industrial trade matrix between each region of interest and the region known as the rest of Spain, the rest of the variables included in the input-output model are assumed to be known, as has already been mentioned. In this case, in relation to the GRAS procedure, it involves the reduction of the matrix to be estimated by incorporating new target margins that coincide with the margins of the matrix of expected intermediate consumption. As for the estimation technique proposed by Jahn (2017), both the optimisation objective function and the constraints are limited to what is stated in problem (14).

---

[3] https://www.ine.es/index.htm

[4] Certainly, the values corresponding to Final Consumption Expenditure, both for households and NPISHs and Public Administrations, as well as those corresponding to Gross Fixed Capital Formation and change in inventories, are values that are difficult to estimate in scenarios where no prior input-output framework is available. Nevertheless, this assumption has been forced in order to assess both the adjustment capacity of the cross-industry trade matrices of the bi-regional table and the importance, in terms of estimation precision, of having an adequate approximation to the true values of such items.

3.1. Goodness of Fit on Technical Coefficients

Following the process previously described and taking the WAPE statistic as the primary reference, the augmented location quotient methodology (Jahn, 2017), known as the Jahn methodology, the GRAS methodology, and the Gravity-RAS methodology developed by Sargento (2008) have been compared for the regions Basque Country, Andalusia, and Navarra.

Once a first version has been calculated by applying the location quotient methodologies and obtaining the residual, it is interesting to know whether the result obtained is relevant when applying one or the other location quotient methodology. And then, whether the adjustment obtained through the optimisation proposed by Jahn (2017) can be improved, either through a better adjustment in the domestic matrix or through a better estimation of the inter-regional import matrix after using a different methodology (Gravity-RAS) with respect to the estimation of imports.

It should be noted that the location quotient methodology used is relevant to obtain a better result. Thus, table 3 shows that the 2D-LQ methodology generally obtains better results than the FLQ methodology, taking into account that both methodologies have been implemented by applying the so-called optimal parameters with respect to the domestic coefficient matrices.

Table 3.
WAPE statistic values by region and applied methodologies distinguishing between domestic coefficients (Z11) and inter-regional import coefficients (Z21)

Region	Methodology	Z11		Z21	
		FLQ	2DLQ	FLQ	2DLQ
Andalusia	JAHN	0.607	0.558	1.001	0.872
	GRAS	0.685	0.598	1.172	0.894
	GRAVITY - RAS	0.618	0.592	1.064	0.944
Community of Navarra	JAHN	0.716	0.682	0.958	0.784
	GRAS	0.694	0.674	0.838	0.776
	GRAVITY - RAS	0.699	0.675	2.019	0.992
Basque Country	JAHN	0.634	0.608	1.344	0.908
	GRAS	0.809	0.609	0.813	0.876
	GRAVITY - RAS	0.822	0.666	0.867	0.824

Best estimates are shaded in the table.
Source: Authors' calculations for regional IOTs in Spain.

When using bi-dimensional 2D-LQ location quotients in two of the estimated regions, it is the Jahn methodology that obtains the best fit in the matrix of household coefficients. Only in the case of Navarra, a small region, does the GRAS methodology obtain a result that is 1.15% better than that obtained using the Jahn technique.

The use of the Gravity - RAS technique does not, in any case, provide the best fit with respect to the matrix of domestic technical coefficients.

Regarding the matrix of inter-regional coefficients, it cannot be concluded that one technique is superior to another. The 2D-LQ methodology combined with the Jahn methodology gives the best result for the large region, while the GRAS methodology gives a better fit in the other two regions, giving a better fit combined with 2D-LQ in the small region (Navarra) and combined with FLQ in the medium-sized region (Basque Country). It can be seen that a higher accuracy obtained in the application of the location quotient technique implies an improvement in the accuracy of the secondary adjustment technique with respect to the result. Thus, it is evident that the application of the Gravity-RAS technique to the matrices obtained from FLQ does not, in any case, provide the best possible adjustment either in the domestic matrix or in the inter-regional imports matrix, while using the more precise 2D-LQ quotient satisfactory results are obtained through the Gravity-RAS technique.

3.2 Goodness of Fit over Multipliers

Table 4 shows the goodness of fit on the Leontief multipliers obtained by applying each of the proposed methodologies, comparing the application of the different location quotient methodologies in parallel.

Table 4.
Values of the WAPE statistic obtained on the Leontief multipliers

Region	Methodology	FLQ	2DLQ
Andalusia	JAHN	0.177	0.166
	GRAS	0.205	0.197
	GRAVITY - RAS	0.198	0.194
Community of Navarra	JAHN	0.194	0.192
	GRAS	0.199	0.192
	GRAVITY - RAS	0.192	0.188
Basque Country	JAHN	0.182	0.181
	GRAS	0.261	0.185
	GRAVITY - RAS	0.264	0.217

Best estimates are shaded in the table.
Source: Authors' calculations for regional IOTs in Spain.

This shows that, except in the case of the small region of Navarra, the best combination always occurs with the use of the 2D-LQ methodology and the application of the procedure established by Jahn. In the case of Navarra, the Gravity-RAS methodology combined with the use of the 2D-LQ methodology is 2.43% more accurate than the Jahn methodology.

4. Conclusions

This paper compares the goodness of fit in regionalisation processes of different methodologies: a novel one that was called the Jahn methodology (Jahn , 2017), based on the use of augmented location quotients; a second one that is the GRAS methodology (Junius & Oosterhaven, 2003), widely used for its comparative accuracy; and the Gravity-RAS methodology proposed by Sargento (2009), that combines the gravity model and the RAS technique.

The first prescription derived from this research is that to make a first approximation to the estimation of the domestic intermediate input matrix by applying the FLQ and 2D-LQ location quotients – in which the optimal value of parameters δ, in the case of FLQ, and α and β, in the case of 2D-LQ must be determined – the most efficient proposal (that minimises the error) is the one that estimates the value of these parameters from a regression equation. In this, the regressors are, in the case of δ, the ratio of the region's propensity to imports from the rest of the world with respect to that of the nation and the weight of road freight transport flow from other regions in the total freight transport flow, and in the case of the parameters α y β, the regional size measured in terms of GDP and the weight of freight transport flow from other regions (inter-regional transport) over the total freight transport flow. Concerning the determination of the optimal δ the minimisation problem of the WAPE statistic is posed and solved with the General Algebraic Modeling System (GAMS) software. The calculation of these values improves by obtaining a lower value of the statistic than those obtained by minimising the statistic on a set of different values of δ in the interval [0,1] (generally 99 values in increments of 0.01).

A second recommendation that emerges from this paper is the superiority of the 2D-LQ bi-dimensional location quotient estimation over the FLQ methodology, systematically achieving better results.

The results of the comparative analysis of the different methodologies show that the implementation of a first approximation to inter-regional trade based on the residue obtained by applying the Jahn methodology offers better results than the GRAS methodology with respect to the estimation of the domestic intermediate input matrix. More specifically, the Jahn methodology obtains the most accurate results in the estimation of domestic coefficients in two of the three regions analyzed.

On the other hand, when inter-regional trade is approached in a different way – specifically by means of the Gravity-RAS methodology – other than the one implemented from the residual (15), the result does not improve. Therefore, it can be concluded that the technique for estimating inter-regional trade using the Jahn methodology is preferable when there is no prior input-output framework to serve as a basis for implementation.

However, it must be emphasized that, due to the small number of regions considered and the few differences in the accuracy of the methods analyzed, the results obtained must be interpreted with caution.

A third conclusion to be drawn from the previous results is the greater suitability of hybrid procedures for estimating the demand for domestic intermediate inputs and inter-regional trade. Thus, the use of 2D-LQ bi-dimensional location quotients and the Jahn methodology is considered more appropriate for estimating the household intermediate input table. In contrast, in the estimation of the inter-regional coefficient matrix, it cannot be concluded that one technique is superior to the other. In fact, for each of the three regions estimated, the best approximation to the inter-regional coefficient matrix is obtained with a different methodology.

Regarding the estimation of output multipliers, the Jahn methodology also remains clearly superior to the GRAS methodology, even though in the case of the smaller region the Gravity RAS methodology offers a better goodness-of-fit.

In summary, the choice of the procedure method used in the estimation of regional input-output tables should be subject to obtaining the best goodness of fit, and under this premise, it can be concluded that the Jahn methodology is, within the hybrid procedures, an efficient methodology; its application in the field of regionalisation is highly recommended in situations where no prior input-output framework is available.

References

Arto, I., Rueda-Cantuche, J. M., & Peters, G. P. (2014). Comparing the gtap-mrio and wiod databases for carbon footprint analysis. *Economic Systems Research*, *26*(3), 327–353. https://doi.org/10.1080/09535314.2014.939949

Bakhtiari, S., & Dehghanizadeh, M. (2012). Proposing a new version of location quotients for estimating regional input-output coefficients: A case study of Iran's Yazd province. *African Journal of Business Management*, *6*(23), 6903–6909. https://doi.org/10.5897/AJBM11.2934

Boero, R., Edwards, B. K., & Rivera, M. K. (2018). Regional input-output tables and trade flows: an integrated and interregional non-survey approach. *Regional Studies*, *52*(2), 225–238. https://doi.org/10.1080/00343404.2017.1286009

Bonfiglio, A. (2009). On the parameterization of techniques for representing regional economic structures. *Economic Systems Research*, *21*(2), 115–127. https://doi.org/10.1080/09535310902995727

Boomsma, P., & Oosterhaven, J. (1992). A double-entry method for the construction of bi-regional input-output tables. *Journal of Regional Science*, *32*(3), 269–284. https://doi.org/10.1111/j.1467-9787.1992.tb00186.x

Buendía, J.D., Martínez, R., & Sánchez, M.M. (2022). A new proposal to model regional input–output structures using location quotients. An application to Korean and Spanish regions. *Papers in Regional Science*, 101(5), 1201-1237. https://doi.org/ 10.1111/pirs.12692

Cai, M. (2020). Doubly constrained gravity models for interregional trade estimation. *Papers in Regional Science*, 100(2), 455-474. https://doi.org/10.1111/pirs.12581

Cai, M. (2022). A calibrated gravity model of interregional trade. *Spatial Economic Analysis*. https://doi.org/10.1080/17421772.2022.2081715

Dietzenbacher, E., & Miller, R.E. (2009). Ras-ing the transactions or the coefficients: It makes no difference. *Journal of Regional Science*, *49*(3), 555–566. https://doi.org/10.1111/j.1467-9787.2008.00598.x

Flegg, A. T., Webber, C. D., & Elliott, M. V. (1995). On the Appropriate Use of Location Quotients in Generating Regional Input—Output Tables. *Regional Studies*, *29*(6), 547–561. https://doi.org/10.1080/00343409512331349173

Flegg, A. T., & Webber, C. D. (1997). On the Appropriate Use of Location Quotients in Generating Regional Input-Output Tables: Reply. *Regional Studies*, *31*(8), 795–805. https://doi.org/10.1080/713693401

Flegg, A. T., & Tohmo, T. (2013). Regional Input-Output Tables and the FLQ Formula: A Case Study of Finland. *Regional Studies*, *47*(5), 703–721. https://doi.org/10.1080/00343404.2011.592138

Flegg, A. T., & Tohmo, T. (2019). The regionalization of national input-output tables: A study of South Korean regions. *Papers in Regional Science*, *98*(2), 601–620. https://doi.org/10.1111/pirs.12364

Flegg, A. T., & Webber, C. D. (1997). On the appropriate use of location quotients in generating regional input-output tables: reply. *Regional Studies*. https://www.tandfonline.com/doi/abs/10.1080/713693401

Flegg, A. T., & Webber, C. D. (2000). Regional size, regional specialization and the FLQ formula. *Regional Studies*, *34*(6), 563–569. https://doi.org/10.1080/00343400050085675

Flegg, A. T., Webber, C. D., & Elliott, M. V. (1997). On the appropriate use of location quotients in generating regional input–output tables. *Regional Studies*, *31*(8), 795–805. https://rsa.tandfonline.com/doi/abs/10.1080/00343409512331349173

Flegg, A. T, Huang, Y., & Tohmo, T. (2014). *Cross-hauling and regional input-output tables: the case of the province of Hubei, China* (No. 1310; Economics Working Paper Series).

Flegg, A. T, Lamonica, G. R., Chelli, F. M., Recchioni, M. C., & Tohmo, T. (2021). A new approach to modelling the input – output structure of regional economies using non - survey methods. In *Journal of Economic Structures*. Springer Berlin Heidelberg. https://doi.org/10.1186/s40008-021-00242-8

Flegg, A. T, Mastronardi, L. J., & Romero, C. A. (2014). *Empirical evidence on the use of the FLQ formula for regionalizing national input-output tables: The case of the Province of Córdoba, Argentina Economics Working Paper Series Empirical Evidence on the Use of the FLQ Formula for Regionalizing National Input*.

Flegg, A. T., & Tohmo, T. (2019). The regionalization of national input-output tables: A study of South Korean regions: Papers and Proceedings. *Papers in Regional Science*, *98*(2), 601–620. https://doi.org/10.1111/pirs.12364

Fournier, J. G. (2020). On the accuracy of gravity-RAS approaches used for inter-regional trade estimation: evidence using the 2005 inter-regional input–output table of Japan. *Economic Systems Research*, *5314*(4), 521–539. https://doi.org/10.1080/09535314.2020.1753662

Greaney, T. M., & Kiyota, K. (2020). *The Gravity Model and Trade in Intermediate Inputs* (Issue January).

Günlük-Senesen, G., & Bates, J. M. (1988). Some Experiments with Methods of Adjusting Unbalanced Data Matrices. *Journal of the Royal Statistical Society. Series A (Statistics in Society)*, *151*(3), 473. https://doi.org/10.2307/2982995

Hermannsson, K. (2016). Beyond Intermediates: The Role of Consumption and Commuting in the Construction of Local Input–Output Tables. *Spatial Economic Analysis*, *11*(3), 315–339. https://doi.org/10.1080/17421772.2016.1177194

Holý, V., & Šafr, K. (2022) Disaggregating input–output tables by the multidimensional RAS method: a case study of the Czech Republic. *Economic Systems Research*, https://doi/10.1080/09535314.2022.2091978

Holt, J. (2017). *Approaches to estimating regional input-output tables June 2017* (Issue June).

Huang, W., Kobayashi, S., & Tanji, H. (2008). Updating an input–output matrix with sign-preservation: Some improved objective functions and their solutions. *Economic Systems Research*, 20, 111– 123.

Isard, W. (1951). Interregional and regional input-output analysis: a model of a space-economy. *The Review of Economics and Statistics*. https://www.jstor.org/stable/1926459

Isard, W. (1960). *Methods of Regional Analysis: an Introduction to Regional Science*. The Technology Press of MIT and John Wiley and Sons.

Isard, W., Azis, I. J., Drennan, M. P., Miller, R. E., Saltzman, S., & Thorbecke, E. (2017). Methods of Interregional and Regional Analysis. In *Methods of Interregional and Regional Analysis*. Routledge. https://doi.org/10.4324/9781315249056

Jackson, R. W., & Murray, A. T. (2004). Alternative input-output matrix updating formulations. *Economic Systems Research*, *16*(2), 135–148. https://doi.org/10.1080/0953531042000219268

Jahn, M. (2017). Extending the FLQ formula: a location quotient-based interregional input–output framework. *Regional Studies*, *51*(10), 1518–1529. https://doi.org/10.1080/00343404.2016.1198471

Jahn, M., Flegg, A. T., & Tohmo, T. (2020). Testing and implementing a new approach to estimating interregional output multipliers using input–output data for South Korean regions. *Spatial Economic Analysis*, *1772*(2), 165–185. https://doi.org/10.1080/17421772.2020.1720918

Junius, T., & Oosterhaven, J. (2003). The solution of updating or regionalizing a matrix with both positive and negative entries. *Economic Systems Research*, *15*(1), 87–96. https://doi.org/10.1080/0953531032000056954

Kowalewski, J. (2015). Regionalization of National Input–Output Tables: Empirical Evidence on the Use of the FLQ Formula. *Regional Studies*, *49*(2), 240–250. https://doi.org/10.1080/00343404.2013.766318

Kratena, K., Streicher, G., Temurshoev, U., Amores, A. F., Arto, I., Mongelli, I., Neuwahl, F., Rueda-Cantuche, J. M., & Andreoni, V. (2013). FIDELIO 1: Fully Interregional Dynamic Econometric Long-term Input-Output Model for the EU27. In *JRC Scientific and Policy Reports*. https://doi.org/10.2791/17619

Krebs, O. (2020). RIOTs in Germany - Constructing an interregional input-output table for Germany. In *papers.ssrn.com*. https://doi.org/10.15496/publikation-40407

Lamonica, G. R., & Chelli, F. M. (2018). The performance of non-survey techniques for constructing sub-territorial input-output tables. *Papers in Regional Science*, *97*(4), 1169–1202. https://doi.org/10.1111/pirs.12297

Lampiris, G., Karelakis, C., & Loizou, E. (2019). Comparison of non-survey techniques for constructing regional input–output tables. *Annals of Operations Research*, *294*(1–2), 225–266. https://doi.org/10.1007/s10479-019-03337-5

Lemelin, A. (2009). A GRAS variant solving for minimum information loss. *Economic Systems Research*, 21, 399– 408.

Lenzen, M., Gallego, B., & Wood, R. (2006). A flexible approach to matrix balancing under partial information. *Journal of Applied Iput–Output Analysis*, 11 & 12, 1– 24.

Lenzen, M., Gallego, B., & Wood, R. (2009). Matrix balancing under conflicting information. *Economic Systems Research*, *21*(1), 23–44. https://doi.org/10.1080/09535310802688661

Lenzen, M., Moran, D. D., Geschke, A., & Kanemoto, K. (2014). A non-sign-preserving RAS variant. *Economic Systems Research*, 26(2), 197– 208.

Lenzen, M., Wood, R., & Gallego, B. (2007). Some comments on the GRAS method. *Economic Systems Research*, 19(4), 461– 465.

Mastronardi, L. J., & Romero, C. A. (2012). *A non-survey estimation for regional input-output tables. An application for Buenos Aires City*. http://mpra.ub.uni-muenchen.de/37006/

McCann, P., & Dewhurst, J. H. (1998). Regional size, industrial location and input-output expenditure coefficients. In *Regional Studies* (Vol. 32, Issue 5).

Miller, R. E., & Blair, P. D. (2009). *Input-Output Analysis Foundations and Extensions Second Edition*. https://books.google.es/books?hl=es&lr=&id=viHaAgAAQBAJ&oi=fnd&pg=PR24&dq=location+quotient+input+output&ots=grBketkUaV&sig=Hm7oeCN4HiqUTtPq58BtTz9dWZI

Mínguez, R., Oosterhaven, J., & Escobedo, F. (2009). Cell-corrected RAS method (CRAS) for updating or regionalizing an input-output matrix. *Journal of Regional Science*, *49*(2), 329–348. https://doi.org/10.1111/j.1467-9787.2008.00594.x

Oosterhaven, J., Piek, G., & Stelder, D. (1986). Theory and practice of updating regional versus interregional interindustry tables. *Papers in Regional Science*, *59*(1), 57–72. https://doi.org/10.1111/j.1435-5597.1986.tb00982.x

Paelinck, J., & Waelbroeck, J. (1963). Etude empirique sur l'évolution de coefficients 'input–output': essai d'application de la procédure RAS de Cambridge au tableau industriel belge. *Economie Appliquee*, 16, 81– 111.

Pereira-López, X., Carrascal, A., & Fernández, M. (2013). Advances in updating input-output tables: its relevance for the analysis of regional economies. *Revista Portuguesa de Estudos Regionais*, *33*, 3-12.

Pereira-López, X., Sánchez-Chóez, N.G., & Fernández-Fernández, M. (2021) Performance of bidimensional location quotients for constructing input–output tables. *Journal of Economic Structures* 10, 7. https://doi.org/10.1186/s40008-021-00237-5

Pereira-López, X., Sánchez-Chóez, N.G., & Fernández-Fernández, M. (2022). Spotting Error Patterns in Input–Output Projections Using Location Quotients. *Mathematics*, 10, 1474. https://doi.org/10.3390/math10091474

Ramos, M. C. (1998). *Estimación Indirecta de Coeficientes Input-Output* (153/98).

Riddington, G., Gibson, H., & Anderson, J. (2006). Comparison of gravity model, survey and location quotient-based local area tables and multipliers. *Regional Studies*, *40*(9), 1069–1081. https://doi.org/10.1080/00343400601047374

Round, J. I. (1983). Nonsurvey Techniques: A Critical Review of the Theory and the Evidence. *International Regional Science Review*, *8*(3), 189–212. https://doi.org/10.1177/016001768300800302

Sargento, A. L. M. (2007). Empirical examination of the gravity model in two different contexts: Estimation and explanation. *Jahrbuch Fur Regionalwissenschaft*, *27*(2), 103–127. https://doi.org/10.1007/s10037-007-0013-8

Sargento, A. L.M. (2009). *Regional input-output tables and models Interregional trade estimation and input-output modelling based on total use rectangular tables*. Universidade de Coimbra.

Sargento, A. L. M., Ramos, P. N., & Hewings, G. J. D. (2012). Inter-regional trade flow estimation through non-survey models: An empirical assessment. *Economic Systems Research*, *24*(2), 173–193. https://doi.org/10.1080/09535314.2011.574609

Schaffer, W. A., & Chu, K. (1969). Nonsurvey techniques for constructing regional interindustry models. *Papers of the Regional Science Association*, *23*(1), 83–101. https://doi.org/10.1007/BF01941876

Stone, R. (1961). Input-Output and National Accounts. *Paris. Organization for Economic Cooperation and Development.*

Temursho, U., Oosterhaven, J., & Cardenete, M. A. (2021). A multiregional generalized RAS updating technique. *Spatial Economic Analysis*, *16* (3), 271-28.

Temurshoev, U., Miller, R. E., & Bouwmeester, M. C. (2013). A note on the GRAS method. *Economic Systems Research*, *25*(3), 361–367. https://doi.org/10.1080/09535314.2012.746645

Temurshoev, U., & Timmer, M. P. (2010). *Joint Estimation of Supply and Use Tables Joint Estimation of Supply and Use Tables* * (GD-116). www.wiod.org

Temurshoev, U., & Timmer, M. P. (2011). Joint estimation of supply and use tables. *Papers in Regional Science*, *90*(4), 863–882. https://doi.org/10.1111/j.1435-5957.2010.00345.x

Thissen, M., Di Comite, F., Kancs, A., & Potters, L. (2014). *Modelling Inter-Regional Trade Flows Data and Methodological Issues in RHOMOLO Working Papers A series of short papers on regional research and indicators produced by the Directorate-General for Regional Policy*. https://doi.org/10.2776/871154

Többen, J., & Kronenberg, T. H. (2015). Construction of multi-regional input–output tables using the charm method. *Economic Systems Research*, *27*(4), 487–507. https://doi.org/10.1080/09535314.2015.1091765

Valderas-Jaramillo, J. M., Rueda, J. M., Olmedo, E., & Beutel, J. (2019). Projecting supply and use tables: new variants and fair comparisons. *Economic Systems Research*, *31(3)* 423-444 https://doi.org/10.1080/09535314.2018.1545221

Valderas- Jaramillo, J.M., Amores, A.F., Oliva-Mora, J.R., & Boniquito-Fernández, S. (2019). Provincialización de tablas de Origen y Destino mediante SUT-RAS tridimensional (3D-SUT-RAS): desarrollo y aplicación al caso andaluz. VIII Jornadas de Análisis input–output, organized by the Hispanoamerican Society of input–output Analysis (SHAIO), https://doi.org/10.13140/RG.2.2.25012.37763

Valderas-Jaramillo, J. M. (2015). *Actualización de marcos input-output a través de métodos de proyección. Estudio, aplicación y evaluación empírica en tablas de origen y de destino a precios básicos de vario países de la Unión Europea*. Universidad de Sevilla.

Valderas-Jaramillo, J. M., & Rueda-Cantuche, J.M. (2021). The multidimensional nD-GRAS method: Applications for the projection of multiregional input–output frameworks and valuation matrices. *Papers in Regional Science, 100*, 1599–1624.

Wang, Z., & Canning, P. (2004). *A Flexible Modeling Framework to Estimate Interregional Trade Patterns and Input-Output Accounts*. The World Bank. https://doi.org/10.1596/1813-9450-3359

Yamada, M. (2015). Construction of a multi-regional input-output table for Nagoya metropolitan area, Japan. *Journal of Economic Structures*, *4*(1). https://doi.org/10.1186/s40008-015-0022-7

Zhao, X., & Choi, S. G. (2015). On the regionalization of input–output tables with an industry-specific location quotient. *Annals of Regional Science*, *54*(3), 901–926.

ORCID

José Daniel Buendía Azorín https://orcid.org/0000-0001-9302-7971
Rubén Martínez Alpañez https://orcid.org/0000-0002-3663-3171
María del Mar Sánchez de la Vega https://orcid.org/0000-0002-8647-6215

Appendices

A1. Complementary statistics on the matrix of interregional coefficients

Region	Methodology	WAPE		ρ-SWAPE		WASE	
		FLQ	2DLQ	FLQ	2DLQ	FLQ	2DLQ
Andalusia	JAHN	1.0010	0.8720	0.7050	0.6590	0.2090	0.2358
	GRAS	1.1720	0.8940	0.5694	0.6388	0.2545	0.2453
	Gravity - RAS	1.0640	0.9440	0.6717	0.6362	0.2222	0.2489
Community of Navarra	JAHN	0.9580	0.7840	0.7193	0.7176	0.0707	0.0872
	GRAS	0.8380	0.7760	0.7223	0.7181	0.0819	0.0899
	Gravity - RAS	2.0190	0.9920	0.6446	0.7068	0.0997	0.0789
Basque Country	JAHN	1.3440	0.9080	0.7290	0.7504	0.1048	0.0964
	GRAS	0.8130	0.8760	0.7127	0.7501	0.1198	0.0982
	Gravity - RAS	0.8670	0.8240	0.6840	0.7333	0.1294	0.1095

Source: authors' calculations for Spain regional IOTs.

A1. Complementary statistics on multipliers

Region	Methodology	WAPE		ρ-SWAPE		WASE	
		FLQ	2DLQ	FLQ	2DLQ	FLQ	2DLQ
Andalusia	JAHN	0.6070	0.5580	0.8377	0.8433	0.0289	0.0323
	GRAS	0.6850	0.5980	0.8068	0.8083	0.0319	0.0423
	Gravity - RAS	0.6180	0.5920	0.8067	0.8123	0.0403	0.0413
Community of Navarra	JAHN	0.7160	0.6820	0.8035	0.8044	0.0462	0.0474
	GRAS	0.6940	0.6740	0.7852	0.8033	0.0539	0.0491
	Gravity - RAS	0.6990	0.6750	0.8032	0.8164	0.0481	0.0445
Basque Country	JAHN	0.6340	0.6080	0.8209	0.8253	0.0372	0.0369
	GRAS	0.8090	0.6080	0.6670	0.8191	0.0630	0.0384
	Gravity - RAS	0.8220	0.6660	0.6570	0.7697	0.0643	0.0479

Source: authors' calculations for Spain regional IOTs.

A2. Classification of Economic Activities of Spanish Input-Output Table. 2015

NACE (Rev_2)	Activities
01	Crop and animal production. hunting and related service activities
02	Forestry and logging
03	Fishing and aquaculture
05–09	Mining and quarrying
10-12	Manufacture of food products. beverages and tobacco
13-15	Manufacture of textiles. wearing apparel. leather and related products
16	Manufacture of wood and of products of wood and cork. except furniture; manufacture of articles of straw and plaiting materials
17	Manufacture of paper and paper products
18	Printing and reproduction of recorded media
19	Manufacture of coke and refined petroleum products
20	Manufacture of chemicals and chemical products
21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
22	Manufacture of rubber and plastic products
23	Manufacture of other non-metallic mineral products
24	Manufacture of basic metals
25	Manufacture of fabricated metal products. except machinery and equipment
26	Manufacture of computer. electronic and optical products
27	Manufacture of electrical equipment
28	Manufacture of machinery and equipment n.e.c.
29	Manufacture of motor vehicles. trailers and semi-trailers
30	Manufacture of other transport equipment
31-32	Manufacture of furniture and other manufacturing
33	Repair and installation of machinery and equipment
35	Electricity. gas. steam and air conditioning supply
36	Water collection. treatment and supply
37-39	Sewerage; Waste collection. treatment and disposal activities; materials recovery; Remediation activities and other waste management services
41-43	Construction of buildings; Civil engineering; Specialised construction activities
45	Wholesale and retail trade and repair of motor vehicles and motorcycles
46	Wholesale trade. except of motor vehicles and motorcycles
47	Retail trade. except of motor vehicles and motorcycles
49	Land transport and transport via pipelines
50	Water transport
51	Air transport
52	Warehousing and support activities for transportation
53	Postal and courier activities
55-56	Accommodation; Food and beverage service activities

A2. Classification of Economic Activities of Spanish Input-Output Table. 2015 CONT.

NACE (Rev_2)	Activities
58	Publishing activities
59-60	Motion picture. video and television programme production. sound recording and music publishing activities; Programming and broadcasting activities
61	Telecommunications
62-63	Computer programming. consultancy and related activities; Information service activities
64	Financial service activities. except insurance and pension funding
65	Insurance. reinsurance and pension funding. except compulsory social security
66	Activities auxiliary to financial services and insurance activities
68	Real estate activities
69-70	Legal and accounting activities; Activities of head offices; management consultancy activities
71	Architectural and engineering activities; technical testing and analysis
72	Scientific research and development
73	Advertising and market research
74-75	Other professional. scientific and technical activities; Veterinary activities
77	Rental and leasing activities
78	Employment activities
79	Travel agency. tour operator and other reservation service and related activities
80-82	Security and investigation activities; Services to buildings and landscape activities; Office administrative. office support and other business support activities
84	Public administration and defence; compulsory social security
85	Education
86	Human health activities
87-88	Residential care activities; Social work activities without accommodation
90-92	Creative. arts and entertainment activities; Libraries. archives. museums and other cultural activities; Gambling and betting activities
93	Sports activities and amusement and recreation activities
94	Activities of membership organisations
95	Repair of computers and personal and household goods
96	Other personal service activities
97-98	Activities of households as employers of domestic personnel; Undifferentiated goods- and services-producing activities of private households for own use

A2 Correspondence of the different branches of activity between the different territories

Spain NACE_Rev 2	Spain-Andalusia NACE_Rev 2	Spain-Basque Country NACE_Rev 2	Spain-Navarra NACE_Rev 2)
01	01	01	01
02	02	02	02-03
03	03	03	
05–09	05–09	05–09	05 a 09. 19
10-12	10-12	10-12	10-12
13-15	13-15	13-15	13-15
16	16	16	16
17	17	17	17
18	18	18	18
19	19	19	
20	20	20	20
21	21	21	21
22	22	22	22
23	23	23	23
24	24	24	24
25	25	25	25
26	26	26	26
27	27	27	27
28	28	28	28
29	29	29	29
30	30	30	30
31-32	31-32	31-32	31-32
33	33	33	33
35	35	35	35
36	36	36	36
37-39	37-39	37-39	37-38-39
41-43	41-43	41-43	41-42-43
45	45	45	45
46	46	46	46
47	47	47	47
49	49	49	49-50-51
50	50 - 51	50	
51		51	
52	52	52	52
53	53	53	53
55-56	55-56	55-56	55-56
58	58	58	58

A2 Correspondence of the different branches of activity between the different territories CONT.

Spain NACE_Rev 2	Spain-Andalusia NACE_Rev 2	Spain-Basque Country NACE_Rev 2	Spain-Navarra NACE_Rev 2)
59-60	59-60	59-60	59-60
61	61	61	61
62-63	62-63	62-63	62-63
64	64	64	64
65	65	65	65
66	66	66	66
68	68	68	68
69-70	69-70	69-70	69-70
71	71	71	71
72	72	72	72
73	73	73	73
74-75	74-75	74-75	74-75
77	77	77	77
78	78	78	78
79	79	79	79
80-82	80-82	80-82	80-82
84	84	84	84
85	85	85	85
86	86	86	86
87-88	87-88	87-88	87-88
90-92	90-92	90-92	90-92
93	93	93	93
94	94	94	94
95	95	95	95
96	96	96	96
97-98	97-98	97-98	97-98