Metafísica y Persona. Filosofía, conocimiento y vida
Año 16, Núm. 31, Enero-Junio, 2024, ISSN: 2007-9699

On Sommersian Concept Analysis

Sobre el análisis sommersiano de conceptos

J.-Martín Castro-Manzano[1]
UPAEP Universidad, México
josemartin.castro@upaep.mx

Abstract

In this contribution we explore in what sense Sommers' tree theory, a novel philosophical tool, could be useful for Formal Concept Analysis. Basically, we argue that Sommers' theory is instrumental for the latter insofar as it helps avoid category mistakes. To reach this goal we start by recalling the basic notions of Formal Concept Analysis, then we provide a primer on Sommers' tree theory and, finally, we informally explore what we call Sommersian Concept Analysis.

Keywords: ontology, category mistake, semantic tree.

Resumen

En esta contribución exploramos en qué sentido la teoría arborescente de Sommers, una herramienta filosófica novedosa, podría ser útil para el Análisis Formal de Conceptos. Básicamente, argumentamos que la teoría de Sommers resulta útil para este último en la medida en que ayuda a evitar errores categoriales. Para alcanzar este objetivo, comenzamos recordando las nociones básicas del Análisis Formal de Conceptos, luego brindamos una introducción a la teoría arborescente de Sommers y, finalmente, exploramos de manera informal lo que llamamos Análisis de Conceptos Sommersiano.

Palabras clave: ontología, error categorial, árbol semántico.

Introduction

Broadly construed, Formal Concept Analysis (FCA) is a mathematical method of data analysis that studies conceptual structures by describing relations between objects and attributes. According to Wille, FCA had its origins

[1] ORCID: https://orcid.org/0000-0003-2227-921X

Recepción del oiginal: 10/06/2023
Aceptación definitiva: 21/07/2023

in the restructuring of order and lattice theory, and only after more than a decade of development, its connection with philosophy became clearer.[2]

This connection with philosophy is noticeable. For Wille, mathematics is a discipline with cultural import insofar as it is able to assist our rational communication. Since FCA deals with concepts and concepts are prerequisites for the proper formation of statements –Wille claims– the aim and reach of FCA is to support our rational communication by mathematically developing conceptual structures.[3]

The study of conceptual structures, however, has long been a subject of philosophy, specially within the realms of logic and ontology, through the thick concept of category. And so, traditionally, categories have been understood as conceptual tools that help us classify objects into partitions according to predication. Following this rather short description, we say a category system is a theory of categories, an ontology as it were. Thus, category systems are ubiquitous ontological tools that help us classify objects and build taxonomies, hence developing conceptual structures; but in doing so, they warn us not to commit category mistakes.

A category mistake occurs when an item belonging to a certain category is assigned an attribute belonging to another category. Ryle coined the term in *The Concept of Mind* and suggested a now famous *Gedankenexperiment* to explain it: suppose some person visits Oxford for the first time and is shown a number of colleges, libraries, playing fields, museums, scientific departments, and administrative offices.[4] At the end of the visit they ask: "But where is the University? I have seen where the members of the Colleges live, where the Registrar works, where the scientists experiment and the rest. But I have not yet seen the University in which reside and work the members of the University." In doing so, they insert the University, an item belonging to the class of institutions, into the class of buildings, thus conflating ontological categories, hence committing a category mistake.

In a seminal paper entitled *The Ordinary Language Tree*, Fred Sommers introduced a theory for understanding the structure of language that is particularly wary of category mistakes.[5] In this contribution we would like to explore in what sense this novel philosophical theory could be useful for FCA

---

[2] Wille, Rudolf, Restructuring lattice theory: An approach based on hierarchies of concepts, in Ivan Rival, editor, Ordered Sets, Dordrecht, 1982, Springer Netherlands, pp. 445-470; Wille, Rudolf, *Formal Concept Analysis as Mathematical Theory of Concepts and Concept Hierarchies*, pp. 1-33, Springer Berlin Heidelberg, Berlin, Heidelberg, 2005..
[3] Wille, Rudolf, *Formal Concept Analysis...*
[4] Ryle, Gilbert, The Concept of Mind, London: Hutchinston House, 1951.
[5] Sommers, Fred, The ordinary language tree, Mind, vol. 68, núm. 270, 1959, pp. 160-185.

(and vice versa). Basically, we argue that Sommers' theory is instrumental for FCA insofar as it helps avoid category mistakes within conceptual structures. To reach this goal we start by recalling some basic notions of FCA, then we provide a summary of Sommers' tree theory and, finally, we informally explore what we call Sommersian Concept Analysis (SCA). We hope to make the connection between FCA and philosophy even closer.

1. Formal Concept Analysis

In FCA –as in traditional logic, we might add– we say a concept has extension and intension, and satifies a subconcept-superconcept relation. Broadly, being a subconcept of a superconcept means the extension (respectively, intension) of the subconcept is contained in the extension (resp. intension) of the superconcept. Formally, this can described with the aid of a formal context –the cornerstone of FCA.

A formal context is a structure K = $<G, M, I>$ where G and M are sets, and $I \subseteq G \times M$. The elements of G, and M are called objects (*Gegenstände*), and attributes (*Merkmale*), respectively, and gIm (i.e. $(g, m) \in I$) is an incidence relation read as "the object g has the attribute m." In order to define what is a concept within a formal context K, the following operators are defined for arbitrary $X \subseteq G$ and $Y \subseteq M$:

$$X \rightarrow X^I := \{m \in M \mid gIm \text{ for all } g \in X\}$$

$$Y \rightarrow Y^I := \{g \in G \mid gIm \text{ for all } m \in Y\}$$

A formal concept in a formal context K, then, is defined as a pair (A, B) such that $A \subseteq G$, $B \subseteq M$, $A = B^I$, and $B = A^I$; A and B are called the extent and the intent of the formal concept (A, B), respectively, and the subconcept-superconcept relation is defined by $(A_1, B_1) \leq (A_2, B_2) :\Longleftrightarrow A_1 \subseteq A_2$ ($\Longleftrightarrow B_1 \supseteq B_2$). Thus, given a formal context, formal concepts are partially inclusion-ordered with respect to their extents (resp. intents). To offer an example of a formal context consider Table 1.

In Table 1 we see how each object is related to some attribute. By the Basic Theorem of Lattice Theory, these relations can be deployed by using a diagram (Figure 1) in which each concept is represented by a node so that its extension (resp. intension) consists of all the objects (resp. attributes) whose names can be reached by a descending (resp. ascending) path from that node. We will refer to these notions latter.

Table 1: Some smart people.

	Attribute				
Object	Female	Male	Mathematician	Philosopher	Smart
Aristotle		x		x	x
Gilbert Ryle		x		x	x
Fred Sommers		x		x	x
Rudolf Wille		x	x		x
Simone Weil	x			x	x
Edith Stein	x			x	x
Hannah Arendt	x			x	x
Sofya Kovalevskaya	x		x		x

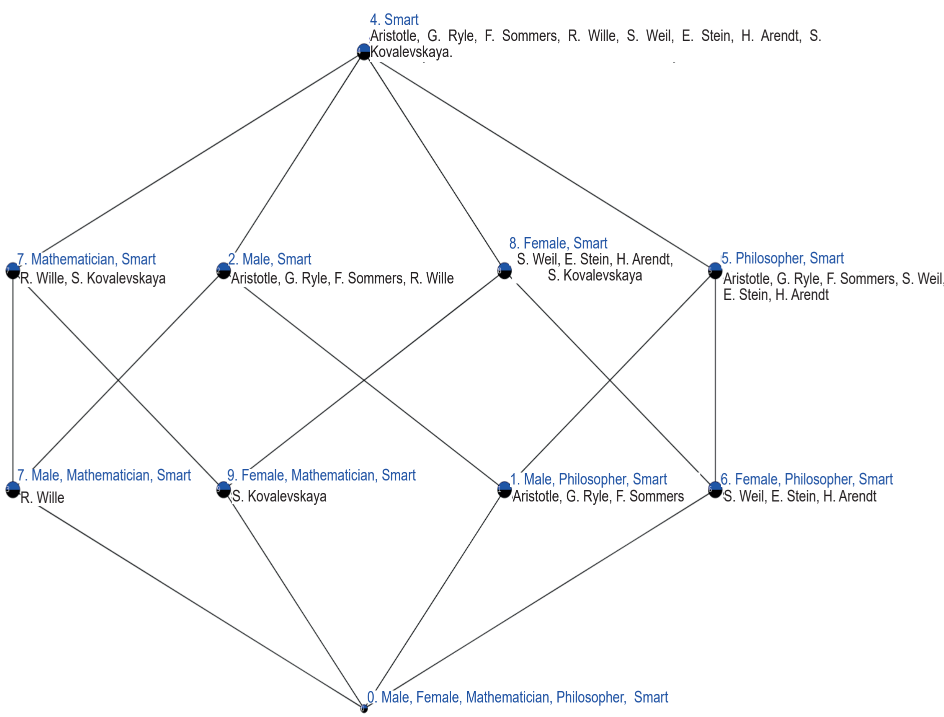


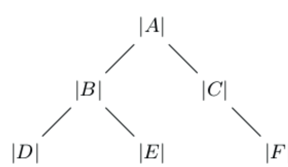
Fig. 1: Some smart people concept lattice (using LatViz).

2. Sommers' tree theory

According to Englebretsen,[6] in order to understand the structure of language, Sommers took the predicable terms of any language to come in logically charged, positive or negative, pairs; for example, the term "red" comes in positive and negative charges, say "red" and "nonred;" while the term "being in Mexico" comes as "being in Mexico" and "not being in Mexico."

According to this assumption, any declarative statement –which puts together a couple of terms that express some concept– may be true, false, or senseless (i.e. a category mistake). So, a term can be predicated sensibly (truly or falsely) or not of some given individual. When this condition is met, the term is said to span such an individual.

For example, *red* spans Aristotle, a car or a wall, but it does not span number π, Kepler's laws or Chomsky's dreams. Notice, however, that if a term spans an individual so does its oppositely charged term, that is, *nonred* spans whatever *red* spans, and it fails to span whatever *red* fails to span: π cannot sensibly be said to be either red or nonred. Using the notation $|T|$ to indicate the absolute value of a term T, as in mathematics, |*red*| would be either *red* (positive charge) or *nonred* (negative charge). The set of individuals spanned by a given term, such as |*red*|, is a category. Given these preliminaries, we can say pairs of (absolute) terms that can be joined to form sensible subject-predicate statements are said to be U-related ("U" for "use"). Pairs of terms that cannot be so joined are N-related ("N" for "nonsense"). Every possible pair of terms in a language, then, will either be U-related or N-related. The U and N relations are symmetric and reflexive, but not transitive, and a model of these sense relations is a model of the categorial sense structure of a language. According to Sommers' theory, there is a small number of rules governing this sense structure that results in the production of binary, reticulating, single-apex trees, for example, as follows:



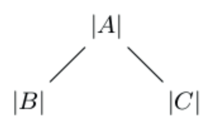
In a tree model of this kind, two terms are connected if and only if they are in the same language. Two conditions of connectedness hold: any two

6 Englebretsen, George, *Robust Reality: An Essay in Formal Ontology*, De Gruyter, 2013.

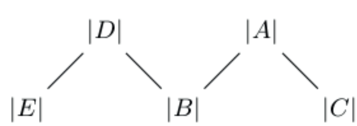
terms U-related to a third term are connected; any two terms connected to a third term are connected. A language, then, is the largest set of mutually connected terms.

Any two terms that are connected on a language tree such that a continuous upward or downward path of line segments leads from one to the other are themselves U-related. It might be thought that a structural requirement would be transitivity: any two terms U-related to a third must be U-related to each other. But this relation does not hold. A counterexample would be *person* and *prime*, both of which are U-related to *interesting* but are not U-related to each other.

The structural principle governing the sense structure of a language is what Sommers called the law of category inclusion, which can be stated as follows: given two N-related terms that are U-related to a third term, there can be no other term that is U-related to one of the first two but N-related to the third. In other words, if two categories share any member in common, then at least one of them must be included in the other. As an example, suppose the first two terms are the N-related pair, $|B|$ and $|C|$, and the third term, to which they are both U-related, is $|A|$. This can be pictured on a tree segment as follows:

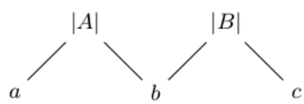


Now let $|D|$ be the fourth term. Since it is U-related to one of the first two terms, say $|B|$, but N-related to $|A|$, there must be a fifth term, say $|E|$, that is U-related to $|D|$ but not to $|A|$:

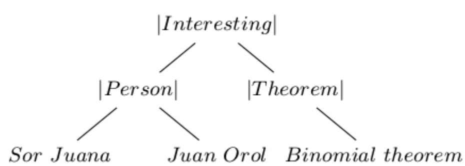


This array of terms is known as the M configuration, and the law of category inclusion forbids it. As a consequence, no path of sense relations can change its upward or downward progression; once a path of U-relations begins to descend it continues downward. And given that the language is finite, a further consequence is that there will be a single top node on the tree and a finite number of tips. Now, categories can be mutually exclusive or one can include the other; but they cannot overlap. The salient feature of terms at the bottom nodes of a tree is that they are U-related to every term that is above them on the path they terminate. This means that the category determined by a bottom term is included in each of the categories determined by the terms to which that term is U-related. Such bottom node categories are types. While a car and Aristotle both belong to the category |*red*|, they do not belong to the same type.

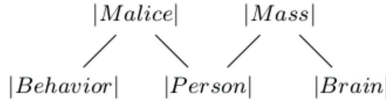
If two individuals belong to the same type, then any term that spans one will span the other. In other words, all members of a given type are spanned by all of the same terms. Just as categories constitute a subset of sets, types constitute a subset of categories: types never include one another. Letting uppercase letters represent (absolute) terms, lowercase letters individuals, and line segments spanning relations, the rule enjoins against the following:



Now, following our exposition pattern, let us put this theory to the test. First, consider a *bona fide* example:



and compare it to a categorially incoherent theory, namely, one that allows an M configuration:

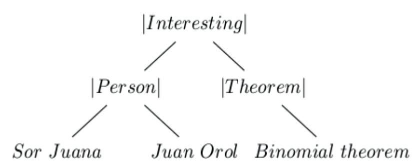


This last theory, for example, would be behind a claim like the next one: "people's brains, which are physical things, are responsible for people's wrong-doings." The problem with a claim like this is mass (a physical property) spans both brains and people, but not behaviors; whereas malice spans both behaviors and people, but not brains. In a case like this, categorial coherence can be regained either by denying some statements or by enforcing ambiguity on some terms. We will refer to these notions later.

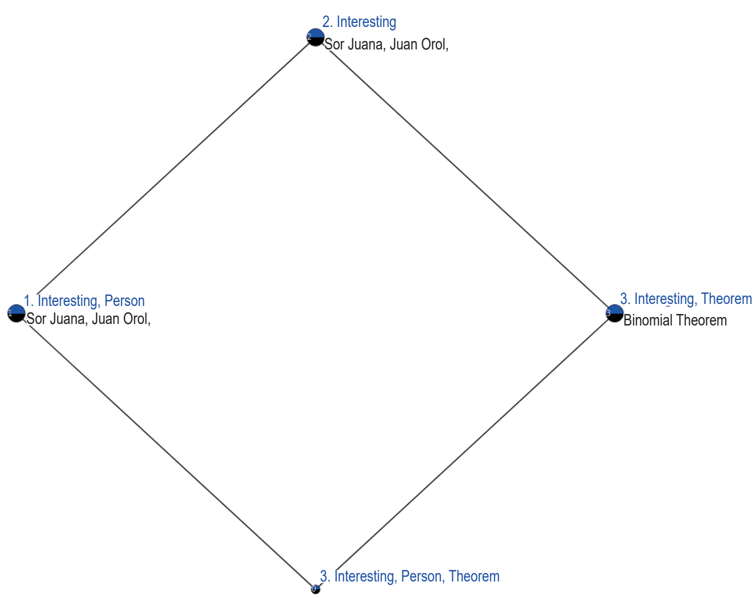
3. Sommersian Concept Analysis

Given the previous notions, what we want to do now is to informally explore what we call Sommersian Concept Analysis. So, in order to capture what we mean by SCA, consider the following examples:

Example 1:



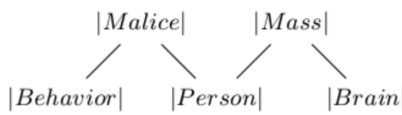
A coherent concept tree with no M configuration



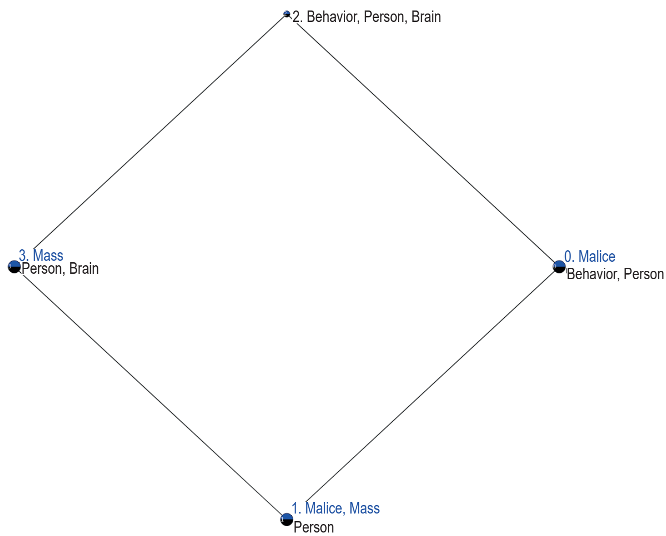
The corresponding concept lattice with a full top vertex and an empty extension bottom vertex

Notice how a coherent tree is related to a concept lattice with certain features, namely, a concept lattice with a full top vertex and an empty extension bottom vertex that has an Eulerian path. Now consider an incoherent tree and observe some features of the corresponding concept lattice:

Example 2:



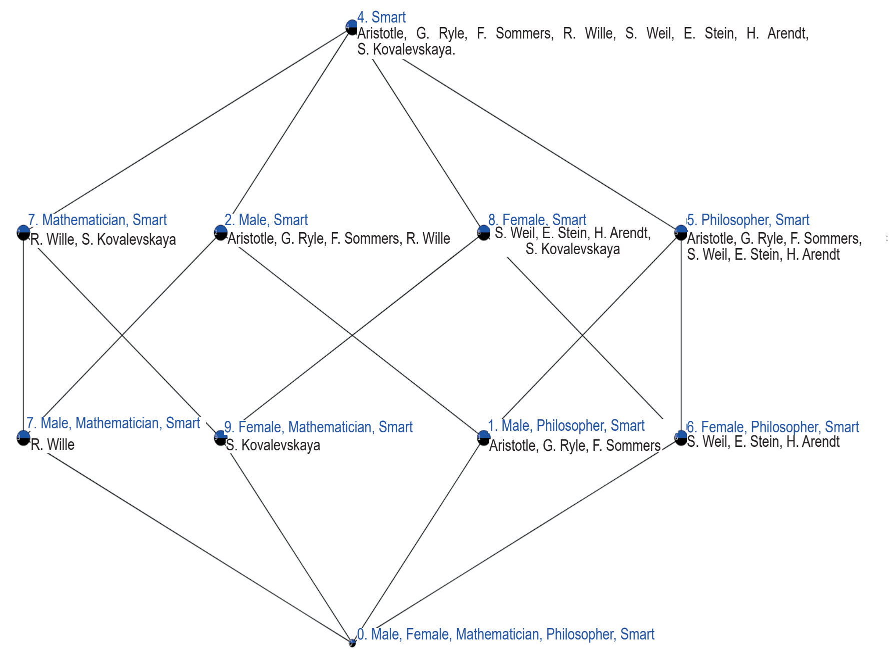
An incoherent tree of concepts with M configuration



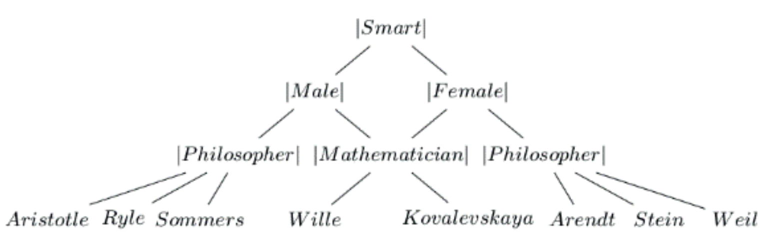
The corresponding concept lattice with an empty top vertex with respect to intension

Now, recall our very first example, which seems quite simple and coherent:

Example 3:



A concept lattice with a non empty top vertex but an empty bottom vertex with respect to extension

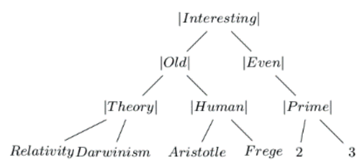


The corresponding concept tree with an M configuration

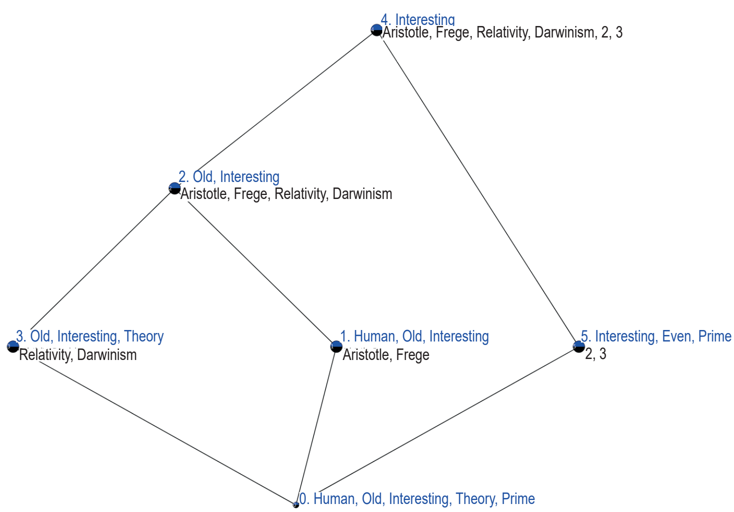
However, it turns out that the tree is incoherent since it includes an M configuration between |*Male*| and |*Female*| (thus showing |*Male*| and |*Female*|, and |*Philosopher*| and |*Mathematician*|, are not so simple categories and that, in order to restore coherence, we should find a hidden, intermediate category, or we should enforce ambiguity in some categories, for instance, by distinguishing between |*Philosopher*$_1$| and |*Philosopher*$_2$| on the grounds that there are different sorts of philosophers), and notice the lattice has no Eulerian path. In this particular example, Sommersian Concept Analysis would urge us to reconsider our typical concept lattice, and so SCA may help FCA in order to be wary of category mistakes.

Finally, consider another example:

Example 4:



A concept tree with no M configuration



The corresponding concept lattice with a non empty top vertex but an empty bottom vertex with respect to extension

Notice how in this last example we have a concept tree with no M configuration and a corresponding concept lattice with a non empty top vertex but an empty bottom vertex with respect to extension that has en Eulerian path. These examples and considerations lead us to the next:

> *Conjecture*: Let *C* be a concept lattice with a full top vertex with respect to extension and an empty bottom vertex with respect to extension, and let *T(C)* be the concept tree associated to *C*, then if *T(C)* is categorically coherent, *C* has an Eulerian path.

We find this conjecture interesting because it would explain a relevant link between SCA and FCA, thus fullfilling Wille's *desideratum* of connecting philosophy and mathematics once again.

Concluding remarks

In this contribution we explored in what sense Sommers' tree theory could be useful for Formal Concept Analysis. Basically, we argued that Sommers' theory is instrumental for the latter insofar as it helps avoid category mistakes. Our future work, however, consists in checking the last conjecture and offering a full-fledged theory about the link between SCA and FCA. In the meantime, let us reconsider our intuitions about predicables and concepts.

References

Boehner, Philoteus, *Medieval Logic: An Outline of Its Development from 1250 to c. 1400,* Wipf & Stock Publishers, 2007.

Chisholm, Roderick, *On Metaphysics*, University of Minnesota Press, 1989.

Englebretsen, George, *Bare Facts and Naked Truths: A New Correspondence Theory of Truth*, Taylor & Francis, 2017.

Englebretsen, George, *Robust Reality: An Essay in Formal Ontology,* De Gruyter, 2013.

Hartmann, Nikolai, *Der Aufbau der realen Welt: Grundriß der allgemeinen Kategorienlehre*, De Gruyter, 1940.

Hoffman, J. and Rosenkrantz, G.S., *Substance Among Other Categories*. Cambridge Studies in Philosophy, Cambridge University Press, 1994.

Ingarden, Roman, *Time and Modes of Being*, Thomas, 1964.

Johansson, I., *Ontological Investigations: An Inquiry into the Categories of Nature, Man and Society*. De Gruyter, 2013.

Lowe, E.J., *The Four-Category Ontology: A Metaphysical Foundation for Natural Science*, Clarendon Press, 2006.

Ryle, Gilbert, *The Concept of Mind*, London: Hutchinston House, 1951.

Sommers, Fred, The ordinary language tree, *Mind*, vol. 68, núm. 270, 1959, pp. 160-185.

Wille, Rudolf, *Formal Concept Analysis as Mathematical Theory of Concepts and Concept Hierarchies*, pp. 1-33, Springer Berlin Heidelberg, Berlin, Heidelberg, 2005.

Wille, Rudolf, Restructuring lattice theory: An approach based on hierarchies of concepts, in Ivan Rival, editor, *Ordered Sets* (pp. 445-470), Dordrecht, 1982, Springer Netherlands.