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The world's productivity distribution and optimal knowledge absorption

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Abstract

An optimal growth model is devised and subject to dynamic analysis. The model is a standard intertemporal utility maximization framework, with a one-dimensional constraint characterizing the pace of productivity growth. In this setting, knowledge acquisition depends not only on the investment in the adoption of existing technologies, but also on the pre-determined shape of the world's productivity distribution. The configuration of the distribution will determine the probability of successful imitation and, therefore, the extent in which the economy might approach the world's technology frontier. The dynamic analysis points to the formation of a saddle-path stable equilibrium. The type of distribution is decisive in defining the exact location of the stable trajectory and of the equilibrium point.

Keywords: knowledge diffusion, productivity distribution, intertemporal optimization, balanced growth, local dynamics

JEL Classification Codes: O33, O41, C61

1. Introduction

Recent literature on the theory of technology adoption and knowledge absorption has highlighted the role of the productivity distribution in shaping patterns of catching-up and falling-behind. The basic idea is that any given agent (country, region, firm, or worker) placed over the productivity distribution will benefit from the interaction with those who hold relatively higher levels of productivity. In this type of setting, productivity growth will be driven not only by the investment made in the acquisition of knowledge but also by the shape of the productivity distribution and by the location of the agent in such distribution (the further away the agent's productivity is from the knowledge frontier, the faster it might grow due to

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imitation or absorption, i.e., the agent benefits from backwardness). Relevant references on the subject include Benhabib *et al*. (2014, 2021), Buera and Lucas (2018), Konig *et al*. (2016, 2022), Perla and Tonetti (2014), and Perla *et al*. (2021).

Inspired in the aforementioned literature, this research note devises a standard intertemporal utility maximization model, with productivity growth driven both by the investment in knowledge acquisition and by the shape of the productivity distribution. In the proposed setting, a large number of regions compose the world economy, and the model is approached from the perspective of the representative agent of one single region. The representative agent faces a world's productivity distribution that is given, that the agent has no capacity to control or influence, and that remains unchanged throughout the individual's planning horizon.

The analysis suggests that it is possible to sort regions by their productivity and to align them from the lowest to the highest productivity rank, under the form of a universal distribution. One should keep in mind that this is a simplifying assumption that obscures relevant features, which, most probably, imply that each region faces a differently shaped distribution: the ability to access and absorb the knowledge of others is associated with a multiplicity of drivers, including geography (Audretsch, 2003), cultural and linguistic proximity (Felbermayr and Toubal, 2010), specialization profiles (Cooke, 2012), and institutional arrangements (Rodriguez-Pose, 2002). Hence, the most sensible interpretation of what the productivity distribution signifies is to construe it as a unique representation of the economy's position with relation to all other economies regarding how much, if anything, it can learn from them, given the set of features that were enumerated above.

The devised framework generates a balanced growth path (BGP), which corresponds to a unique set of equilibrium values for the level of investment per unit of knowledge and for the productivity ratio (defined as the quotient between the agent's productivity level and the world's knowledge frontier). Although a unique equilibrium is derived, regardless of the features of the productivity distribution, the BGP levels of the endogenous variables will depend on the specific distribution one considers.

To illustrate the outcome of the model, three different distributions are taken: uniform, exponential, and Gumbel. There is a practical reason why these are the distributions selected for the analysis; namely, as presented, they involve the consideration of no additional parameters beyond those associated with the structure of the optimality problem (i.e., efficiency parameters, the rate of time preference, and the exogenous growth rate of the technology frontier), what is useful to maintain the analysis at a tractable level.

Under any of the assumed distributions, the equilibrium is saddle-path stable, meaning that the economy converges to the BGP through the one-dimensional stable trajectory that is formed. This trajectory is negatively sloped, but its exact position will differ with the type of distribution. The type of the distribution does not change the nature of the result (convergence to a unique BGP); however, it is decisive regarding the location of the equilibrium point, and the location of the trajectory followed to get there.

The remainder of this note is organized as follows. Section 2 characterizes the structure of the model. Section 3 presents the BGP and studies the local dynamics. Section 4 introduces the three distributions and confirms the saddle-path stability outcome for any of the distributions. In section 5, a numerical example is explored with the purpose of illustrating results. Section 6 concludes.

2. The Knowledge Acquisition Model

This section makes a formal presentation of the analytics of the model. The proposed theoretical framework is a simplified version of the innovation-imitation models of Benhabib *et al*. (2014, 2021) and Gomes (2024*a*, 2024*b*), duly adapted to a regional / international economy setting.

Let $Z(t)$ represent the world's knowledge frontier, and let this frontier grow at the exogenous and constant rate $\gamma \geq 0$. The world economy is composed by a large number of regions, which are heterogeneous with respect to their productivity levels. The productivity index associated with a given region a, at date t, is denoted by $A(t) \in [0, Z(t)]$. The ratio $x(t) \equiv A(t)/Z(t)$ measures the distance of the region's productivity relative to the technology frontier (the closer the value of the ratio is to 1, the closer the productivity level is to the frontier).

In this setting, $F(x(t))$ is the cumulative distribution function (CDF) of productivity, thus representing the percentage of geographical locations with a productivity level not higher than $A(t)$. The CDF corresponds to a finite support distribution, such that $F(0) = 0$, $F(1) = 1$, and $F'(x(t)) > 0$.

The representative agent in region *a* maximizes intertemporal utility. The agent's income is a linear function of productivity, $Y(t) = gA(t)$, $g > 0$; and the agent invests an amount $s(t) \geq 0$ per unit of knowledge, in an effort to acquire new knowledge. Hence, the net income available for consumption, at time period *t*, corresponds to $C(t) = [g - s(t)]A(t)$. A constant intertemporal elasticity of substitution utility function is assumed; to simplify computation, this takes the form of a trivial logarithmic function. The agent maximizes, at $t = 0$, intertemporal utility,

$$U(0) = \int_0^{+\infty} e^{-\rho t} \ln\{[g - s(t)]A(t)\} \, dt \tag{1}$$

In Eq. (1), $\rho \geq 0$ represents the rate of time preference. The maximization of (1) is subject to a dynamic constraint that translates the time evolution of productivity. Productivity growth depends on the investment made in absorbing external technology, but the probability of successful investment is contingent on the place that the region's knowledge index occupies in the world's productivity distribution. The larger the number of points in space with a productivity level higher than the productivity at region *a*, the stronger will be the growth of productivity. In summary, the growth of $A(t)$ is positively correlated with $s(t)$ but negatively correlated with the CDF.

Under the above reasoning, the differential equation that represents the constraint of the intertemporal optimization problem is:

$$\dot{A}(t) = B\left[1 - F\big(x(t)\big)\right]s(t)A(t), \;\; B > 0, A(0) = A_0 \text{ given} \tag{2}$$

The optimal control problem approached by the representative agent of region a consists in maximizing (1) subject to (2). The model involves two endogenous variables, the control variable $s(t)$ and the state variable $A(t)$, and one exogenous variable, $Z(t)$. Four parameters are relevant in this simple optimization model, namely ρ, g, B, γ. Later, one will realize that the feasibility of the equilibrium imposes the following constraint on parameter values: $gB > \rho + \gamma$.

To solve the optimization problem, the respective current-value Hamiltonian function is constructed:

$$H[A(t), s(t), p(t)] = \ln\{[g - s(t)]A(t)\} + p(t)B\left[1 - F\big(x(t)\big)\right]s(t)A(t) \tag{3}$$

In Eq. (3), variable $p(t)$ is the shadow-price of $A(t)$. First-order optimality conditions are presented below.

$$\frac{\partial H}{\partial s} = 0 \Rightarrow [g - s(t)]p(t)B\left[1 - F\big(x(t)\big)\right]A(t) = 1 \tag{4}$$

$$\begin{gathered}\dot{p}(t) = \rho p(t) - \frac{\partial H}{\partial A} \Rightarrow \\ \dot{p}(t) = \left\{\rho - gB\left[1 - F\big(x(t)\big)\right] + BF'\big(x(t)\big)s(t)x(t)\right\}p(t)\end{gathered} \tag{5}$$

and the transversality condition

$$\lim_{t\to\infty} A(t)e^{-\rho t}p(t) = 0 \tag{6}$$

The differentiation of Eq. (4) with respect to time yields:

$$\dot{s}(t) = \left[\frac{\dot{p}(t)}{p(t)} + \frac{\dot{A}(t)}{A(t)} - \frac{F'\big(x(t)\big)}{1 - F\big(x(t)\big)}\left(\frac{\dot{A}(t)}{A(t)} - \gamma\right)\right][g - s(t)] \tag{7}$$

Replacing the growth rates of the state and co-state variables, as displayed in Eq. (2) and Eq. (5), into expression (7), an explicit equation of motion is derived for the dynamics of the investment variable,

$$\dot{s}(t) = \left\{\rho - [g - s(t)]B\left[1 - F\big(x(t)\big)\right] + \gamma \frac{F'\big(x(t)\big)}{1 - F\big(x(t)\big)}x(t)\right\}[g - s(t)] \tag{8}$$

Noticing that $\frac{\dot{x}(t)}{x(t)} = \frac{\dot{A}(t)}{A(t)} - \frac{\dot{Z}(t)}{Z(t)}$, the differential equation for ratio $x(t)$ takes the form:

$$\dot{x}(t) = \left\{B\left[1 - F\big(x(t)\big)\right]s(t) - \gamma\right\}x(t) \tag{9}$$

Eqs. (8) and (9) constitute a two-dimensional two-variable dynamic system, whose dynamics are scrutinized in the following section.

3. Balanced Growth Path and Local Dynamics

In this model, the BGP is defined as the long-term scenario in which: (*i*) regional productivity grows at the same rate as the knowledge frontier; (*ii*) the investment in knowledge absorption per productivity unit is constant. Analytically, characterizing the BGP requires the determination of the solution (x^*, s^*) derived from the system of equations $\dot{x}(t) = 0$, $\dot{s}(t) = 0$. Although an explicit general expression for the BGP values of the endogenous variables is not determinable, it is straightforward to prove that the equilibrium exists and that it is unique.

Equilibrium conditions imply the following equality:

$$gB[1 - F(x^*)] = (\rho + \gamma) + \gamma \frac{F'(x^*)}{1 - F(x^*)}x^* \tag{10}$$

The l.h.s. of Eq. (10) is a decreasing function of x^*, starting at gB for $x^* = 0$, and ending at zero for $x^* = 1$. The r.h.s. of Eq. (10) is an increasing function of x^*, starting at $\rho + \gamma$ for $x^* = 0$ and diverging to infinity as x^* approaches 1. Therefore, as long as condition $gB > \rho + \gamma$ holds, there is one, and only one, intersection point between the two sides of the equation; in this point, we will find a value $x^* \in (0,1)$ that indicates the long-term distance of the region's productivity level to the frontier (in this state, the two levels of knowledge grow at the same rate and, thus, they indefinitely maintain the distance between them).

Once the BGP knowledge ratio is derived, the equilibrium investment might be obtained taking into consideration Eq. (9), i.e.,

$$s^* = \frac{\gamma}{B[1-F(x^*)]} \tag{11}$$

Convergence towards equilibrium point (x^*, s^*) is assured as long as saddle-path stability holds (due to the presence, in the problem, of a control variable). To investigate whether the equilibrium is saddle-path stable, the analysis proceeds with the inspection of local dynamics. The linearization of system (9)-(8) in the vicinity of the BGP locus yields,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{s}(t) \end{bmatrix} = J \begin{bmatrix} x(t) - x^* \\ s(t) - s^* \end{bmatrix} \tag{12}$$

with J the Jacobian matrix of the system,

$$J = \begin{bmatrix} -\gamma \frac{F'(x^*)}{1-F(x^*)} x^* & B[1-F(x^*)]x^* \\ j_{21} & \rho + \gamma \frac{F'(x^*)}{1-F(x^*)} x^* \end{bmatrix} \tag{13}$$

and $j_{21} = \frac{\rho + \gamma \frac{F'(x^*)}{1-F(x^*)} x^*}{B[1-F(x^*)]} \left\{ \left[(\rho+\gamma) + 2\gamma \frac{F'(x^*)}{1-F(x^*)} x^* \right] \frac{F'(x^*)}{1-F(x^*)} + \gamma \frac{F''(x^*)}{1-F(x^*)} x^* \right\}$.

The qualitative nature of the dynamics can be inferred from the signs of the eigenvalues of matrix J. Saddle-path stability requires the existence of one negative and one positive eigenvalues, while instability emerges whenever both eigenvalues are positive. Because the trace of the matrix is $Tr(J) = \rho > 0$, one knows that at least one of the eigenvalues is positive. The other one will be negative if the determinant of the matrix is below zero. The expression of the determinant is:

$$\begin{aligned} Det(J) = & -(\rho+\gamma)^2 \frac{F'(x^*)}{1-F(x^*)} x^* - (2\rho+\gamma)\gamma \left[\frac{F'(x^*)}{1-F(x^*)} \right]^2 (x^*)^2 \\ & - \left[\frac{F'(x^*)}{1-F(x^*)} + \frac{F''(x^*)}{F'(x^*)} \right] \gamma \frac{F'(x^*)}{1-F(x^*)} (x^*)^2 \left[\rho + \gamma \frac{F'(x^*)}{1-F(x^*)} x^* \right] \end{aligned} \tag{14}$$

Eq. (14) indicates that the determinant of the Jacobian matrix is almost surely negative, meaning that the equilibrium is saddle-path stable. The only circumstance in which such result would not hold would be in the scenario in which $F''(x^*)$ is a sufficiently large negative value such that the last term in the expression becomes larger, in absolute value, than the sum of the first two terms. For any of the three distributions considered in the next section, the saddle-path stability outcome unambiguously holds.

4. Uniform, Exponential, and Gumbel Distributions

With a uniform distribution, there is a coincidence between the CDF and the value of the productivity ratio, $F\big(x(t)\big) = x(t)$. In this case, it is possible to derive an explicit expression for the equilibrium level of $x(t)$. This is one of the two solutions of (10), namely the one for which the equilibrium value is lower than 1,

$$x^* = 1 - \frac{\rho}{2gB} - \sqrt{\left(\frac{\rho}{2gB}\right)^2 + \frac{\gamma}{gB}} \tag{15}$$

The productivity ratio in Eq. (15) is positive under the already imposed condition $gB > \rho + \gamma$. Observe that $F'(x^*) = 1$ and that $F''(x^*) = 0$. This implies that the determinant of J, in Eq. (14), is necessarily a negative value and that the saddle-path equilibrium is guaranteed. Consider now an exponential distribution with a finite support in the interval $[0,1]$. The corresponding CDF can be presented under the form:

$$F\big(x(t)\big) = \frac{1 - e^{-x(t)}}{1 - e^{-1}} \tag{16}$$

In this case, there is no closed-form solution for the equilibrium productivity ratio; however, one knows this is a unique value, given the reasoning in the previous section. The first and second derivatives of the CDF are, for x^*, $F'(x^*) = \frac{e^{-x^*}}{1-e^{-1}}$ and $F''(x^*) = -F'(x^*)$. Now, the second derivative of the CDF is a negative value; nevertheless, this does not change the evidence that $Det(J) < 0$ and that the equilibrium is saddle-path stable; this is because the term $\frac{F'(x^*)}{1-F(x^*)} + \frac{F''(x^*)}{F'(x^*)} = \frac{e^{-1}}{e^{-x^*}-e^{-1}}$ is still positive.

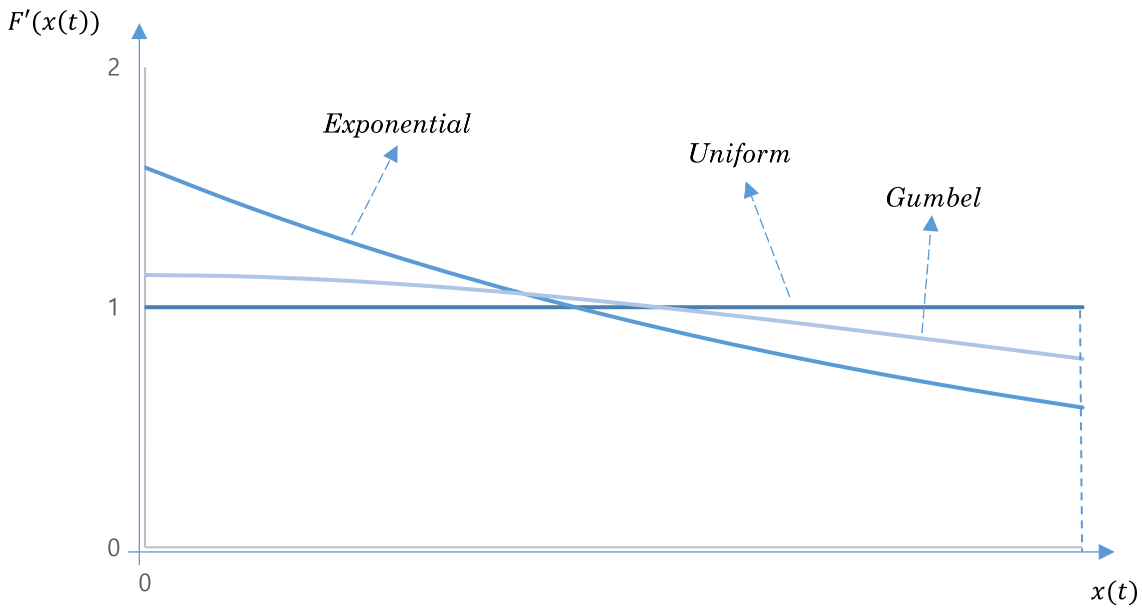
Finally, consider a standard Gumbel distribution. The respective CDF with support in the interval $[0,1]$ is represented as follows:

$$F(x(t)) = \frac{e^{-e^{-x(t)}} - e^{-1}}{e^{-e^{-1}} - e^{-1}} \quad (17)$$

Again, there is no explicit solution for x^*, although one knows that a solution exists for a lower than one positive productivity ratio. First and second derivatives are: $F'(x^*) = \frac{e^{-(x^*+e^{-x^*})}}{e^{-e^{-1}}-e^{-1}}$ and $F''(x^*) = -(1-e^{-x^*})F'(x^*)$. The second derivative of the CDF is negative.

In this case, $\frac{F'(x^*)}{1-F(x^*)} + \frac{F''(x^*)}{F\prime(x^*)} = \frac{e^{-e^{-x^*}}-(1-e^{-x^*})e^{-e^{-1}}}{e^{-e^{-1}}-e^{-e^{-x^*}}}$. The computed expression is a positive value for any value of the productivity ratio between zero and 1, what allows to confirm that, also in the case of the Gumbel distribution, the determinant of the Jacobian matrix is negative and, thus, once again saddle-path stability holds independently of parameter values.
Figure 1 displays the probability distribution function, $F'(x(t))$, for each of the three distributions. It is visible, from the graphic, that each line represents a differently shaped distribution: the probability distribution functions are linear, convex, and concave, respectively, for the uniform, the exponential, and the Gumbel distributions.

Figure 1. Probability distribution functions (uniform, exponential, and Gumbel)



Source: Author's own computation.

5. Numerical Illustration

To illustrate the proposed model, consider the following array of parameter values: $(\rho, g, B, \gamma) = (0.025; 0.2; 1; 0.05)$. These values satisfy condition $gB > \rho + \gamma$. Three distributions have been adopted in the previous section. Given the selected parameter values, it is straightforward to compute the equilibrium for the productivity ratio and for the investment variable, in each of the three cases. Results are displayed in Table 1.

Table 1. Productivity ratio and investment equilibrium values

	Uniform distribution	Exponential distribution	Gumbel distribution
$\boldsymbol{x}^*$	0.434	0.336	0.396
$\boldsymbol{s}^*$	0.088	0.091	0.089

Source: Author's own computation.

To address local dynamics, in the vicinity of the BGP solution, one must compute, for each case, the Jacobian matrix. These are:

- Uniform distribution:

$$J = \begin{bmatrix} -0.0383 & 0.2456 \\ 0.0300 & 0.0633 \end{bmatrix} \tag{18}$$

- Exponential distribution:

$$J = \begin{bmatrix} -0.0346 & 0.1843 \\ 0.0285 & 0.0596 \end{bmatrix} \tag{19}$$

- Gumbel distribution:

$$J = \begin{bmatrix} -0.0374 & 0.2223 \\ 0.0300 & 0.0623 \end{bmatrix} \tag{20}$$

As previously highlighted, the trace of each of the matrixes in Eqs. (18) to (20) is equal to the rate of time preference, and the determinant is, in each case, negative, meaning that, associated with each one of the matrixes is one negative and one positive eigenvalues. The presence of a negative eigenvalue determines the existence of a one-dimensional stable arm in the two-dimensional space of the system. Therefore, one may compute the expression of the stable trajectory for each of the distributions. The control variable will initially adjust towards the stable trajectory, and this is then jointly followed by the two endogenous variables until the steady state result is attained.

The expression of the stable trajectory is:

$$s(t) - s^* = \varrho[x(t) - x^*] \tag{21}$$

In Eq. (21), ϱ represents the slope of the stable trajectory, which corresponds to the ratio between the elements in the eigenvector associated with the negative eigenvalue of matrix *J*. For each of the distributions, in this numerical example, the stable trajectories are as follows:

- Uniform distribution:

$$s(t) - s^* = -0.199[x(t) - x^*] \Rightarrow s(t) = 0.175 - 0.199x(t) \tag{22}$$

- Exponential distribution:

$$s(t) - s^* = -0.214[x(t) - x^*] \Rightarrow s(t) = 0.163 - 0.214x(t) \tag{23}$$

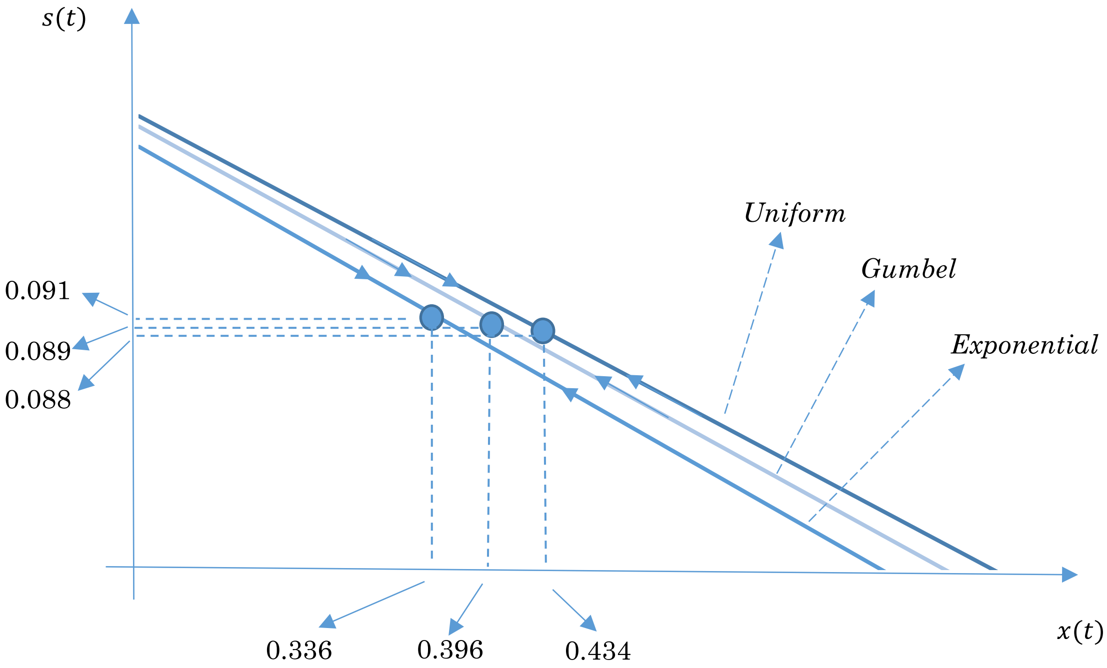
- Gumbel distribution:

$$s(t) - s^* = -0.206[x(t) - x^*] \Rightarrow s(t) = 0.171 - 0.206x(t) \tag{24}$$

All the saddle-path trajectories are negatively sloped. The largest slope is unveiled for the exponential distribution and the smallest slope for the uniform distribution. The different position of the curves implies also different locations for the equilibrium point, with the exponential distribution implying the highest investment in equilibrium and the lowest productivity ratio; the opposite occurs for the uniform distribution, as already displayed in Table 1. Figure 2 graphically represents the position of each saddle-curve for each distribution, as well as the corresponding equilibrium points.

The numerical illustration confirms the generic results in section 3: the endogenous variables follow a stable trajectory towards a steady state that remains undisturbed unless some perturbation in any of the model's parameter values occurs. Hence, the long-term investment in knowledge acquisition and the long-term distance of the economy's technology to the worldwide knowledge frontier are fully dependent on the four parameters in the framework. The question one might raise is whether the selected numerical values truly reflect real-world scenarios and whether changing them significantly disturbs the obtained results.

Figure 2. Saddle-path trajectories (uniform, exponential, and Gumbel distributions)



Source: Author's own computation.

The chosen values for the rate of time preference ($\rho = 0.025$) and for the growth rate of the knowledge frontier ($\gamma = 0.05$) are reasonable, well-adjusted to empirical evidence, and slight changes on them are not particularly impactful in what concerns the model's outcomes. A special attention, though, should be given to the other two parameters: g represents all the factors besides productivity that influence the generation of income (e.g., factor endowments or the scale of production) and B sets the pace at which productivity grows (given the investment in knowledge acquisition and the shape of the productivity distribution). Most probably, these values vary in space and time. They are different across regions, what implies that economies will not converge to a same steady state (as in growth theory, we can conjecture that convergence is not absolute but conditional). They are also prone to change over time, i.e., an economy that becomes more competitive may observe increases in both g and B, what has a direct positive impact in the productivity convergence process. On the contrary, a localized deep recession, an armed conflict, or another negative shock, may reduce the values of such parameters, making the economy progressively lag behind relative to the world's frontier.

6. Conclusion

In this research note, the trivial optimal control problem that highlights the intertemporal trade-off between consumption and investment has been reassessed. The novelty in the analysis consists in assuming that the acquisition of knowledge is contingent not only on the investment made in the absorption of external knowledge, but also on the shape of the world's productivity distribution. Because agents (regions) can only absorb knowledge from those holding higher productivity levels than the one they hold, it matters to know the shape of the distribution and where the agent locates on such distribution.

The solution of the problem indicates the existence of a BGP, where the relevant endogenous variables (investment and productivity ratio) acquire constant values. The equilibrium level of the productivity ratio will be lower than one, a value that indicates how far the productivity of the region remains from the technology frontier (in the BGP, these grow at the same rate). The equilibrium is, most likely, saddle-path stable and, thus, the level of investment, which is the control variable of the problem, initially adjusts to the stable trajectory, which is then followed until the BGP locus is accomplished.

To illustrate the dynamics of the model, three differently shaped productivity distributions were taken, none of them bringing additional parameters into the analysis. Under each of the three distributions, the saddle-path stable equilibrium is confirmed. However, the shape of the distribution matters for the exact location of the stable trajectory and of the equilibrium point. The uniform distribution is the one allowing for the lowest investment level and the highest productivity ratio in equilibrium; the opposite result is found for the exponential distribution. The Gumbel distribution delivers intermediate results.

An important caveat is that the action of the representative agent in a specific region exerts no influence on the shape of the world's productivity distribution. In the proposed setting, this is exogenous and immutable over time. Possible extensions of the analysis include the possibility of endogenizing the productivity distribution (and its dynamics), and also the possibility of introducing other types of distributions (besides the adopted three) in search for eventually distinct dynamic outcomes. Moreover, exploring further and deeper the determinants of productivity differences across regions is vital to strengthen the explanatory capability of the framework, along the lines mentioned in the introduction: surely geography, culture, industry profiles, and institutions, are all central in shaping the productivity distribution and, as a consequence, in shaping the time evolution of the productivity ratio and of the investment in knowledge acquisition.

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