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The Integration of Digital Technology in Task-Design on Eye-Tracking Studies in Geometry

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Abstract

Eye-tracking (ET) method provides a promising channel for educational researchers to connect learning outcomes to cognitive processes. The main principle of ET is that our gaze and our focus of attention are connected. Due to the advent of digital technologies, eye tracking studies are increasingly growing in different fields and in mathematics education. We assume that the provided task is one of the ways to analyse deeply cognitive processes, and we wonder how ET studies are dealing with the integration of digital technology in task-design in geometry. Starting from Strohmaier et al. (2020) we found three studies, one focused on geometric proofs and two based on construction of geometric objects. We highlight the contributions of integrating digital technology in learning and instruction, and in research in geometry.

Keywords

Eye tracker, digital devices, integration technology, task-design, geometry

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La Integración de la Tecnología Digital en el Diseño de Tareas en Estudios de Rastreo Ocular en Geometría

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Resumen

El método del rastreo ocular (RO) proporciona un canal prometedor para que los investigadores educativos conecten los resultados del aprendizaje con los procesos cognitivos. El principio fundamental del RO es que nuestra mirada y nuestro foco de atención están conectados. Debido a la llegada de las tecnologías digitales, los estudios de seguimiento ocular están creciendo cada vez más en diferentes campos y en la educación matemática. Asumimos que la tarea proporcionada es una de las formas de analizar procesos cognitivos profundos, y nos preguntamos cómo las investigaciones en RO están abordando la integración de la tecnología digital en el diseño de tareas en geometría. A partir de Strohmaier et al. (2020) encontramos tres estudios, uno centrado en pruebas geométricas y dos basados en la construcción de objetos geométricos. Destacamos los aportes de la integración de la tecnología digital en el aprendizaje y la enseñanza, y en la investigación en geometría.

Palabras clave

Rastreador ocular, dispositivos digitales, integración de tecnología, diseño de tareas, geometría

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Eye movements are related to a subject's cognitive process, suggesting that the direction of human gaze and the focus of attention are connected, and visual attention can indicate levels of competence in intellectual tasks (Garcia Moreno-Esteva, Kervinen, Hannula, & Uitto, 2020). In mathematics education, with the advancement of digital technologies, from 2014 onwards, the number of published studies using different eye-track devices significantly increased (Strohmaier, MacKay, Obersteiner, & Reiss, 2020).

Strohmaier et al. (2020) reviewed 161 eye-tracking studies published between 1921 and 2018 to assess what domains and topics were addressed, how the method was used, and how eye movements were related to mathematical thinking and learning. As mathematics education researchers, we agree with Campbell et al. (2009) that we develop cognitive frameworks about mathematical thinking to help better understand matters concerning learning, instruction, and assessment.

According to Hannula et al. (2019) we need to study mathematical behaviour in ecologically valid ways, but how does research consider the role of the task in ecological geometric learning processes? One of the most crucial challenges in eye-tracking research is to properly link eye movements to these assumed underlying cognitive processes (Strohmaier et al., 2020). Based on such cognitive challenge, we wonder: How studies on eye-tracking in geometry deal with cognitive aspects, mainly task-design, in particular the following aspects: 1) Which geometry domain/topic are integrating digital technology? 2) What eye-tracking methods highlight in geometric learning with such integration? 2) In this article[1], we revisited 22 papers from Strohmaier et al. (2020) focused on geometry, shape, and form domain, and we analyzed 3 studies.

Theoretical Framework

Eyes are part of the body, and we have known for a long time that human cognition is embodied (Glenberg, 2008; Radford, 2014; Wilson & Golonka, 2013). So, human cognition is grounded in the body, and in its location in space and time (Andrà et al., 2009). Eye-tracking movement integrate our sensory-motor (Nemirovsky, Kelton, & Rhodehamel, 2013) and our gaze behaviour is situational in nature (Haataja, Moreno-Esteva, Toivanen, & Hannula, 2018).

Eye tracking technology is used to understand individuals' non-conscious, moment-to-moment processes during video-based learning. It is a kind of non-verbal way to gather data and a specialized data analysis program to examine the positions and movements of an individual's eyes as it happens (Deng & Gao, 2022). Longer fixations are associated with more effortful cognitive engagement with a task: a higher mean fixation duration on a given location indicates a higher level of cognitive engagement with the information at that location (Hodds, Alcock, & Inglis, 2014).

In the case of task integrating the use of Digital Geometry Software (DGS) suggests that the long fixations are linked to both instrument manipulation and cognitive processes (Hannula, Toivanen, & Garcia Moreno-Esteva, 2019). According to these authors, these results suggest that using a digital tool increases the amount of both automatic scanning fixations and long fixations related to more elaborated processing. However, the qualitative examination of the long gazes in context suggests that a significant amount of these long fixations is related to interacting with DGS, for example when selecting an option from a drop-down menu or using mouse to place an object in the coordinate system. For instance, concerning ET focusing on reading when the input is text, dwell time and number of fixations have values that make text close to formulas (namely, quite low), but fixation duration is like graph (low). The authors infer that text in the input is attended for a shorter time, both in terms of duration and in terms of the number of times the students come to see it (Andrá et al., 2015).

Another example regarding fixations showed that tables concentrated students' attention on the dependent variable data, whereas diagrams distributed students' attention evenly across the numeric and visual elements of the task (Xolocotzin, Inglis, & Medrano, 2020). The authors claim that an interpretation of results from a cognitive load framework would have been problematic and that diagrams should have produced less functional answers because they have more information and require more cognitive resources than table.

Research by Soares, Lukasova, Carthery-Goulart, and Sato (2021) presented studies showing differences between people with high- or low-performance strategies during a reasoning task. Although they observed low performance students had difficulties in both reading and specific mathematics contents, they claim that the grade does not necessarily represent performance. According to them ET can facilitate interpreting the most prominent explanation for students' difficulties and, without this equipment, some teachers would not have identified such problems.

Eye-tracking method provides a promising channel for educational researchers to connect learning outcomes to cognitive processes (Lai et al., 2013) and in agreement with Wilson and Golonka (2013) research programs based on embodiment cognition should follow - as a first step - a task design analysis. Considering the advance of digital technology and the devices to gather a great amount and detailed data from gaze we think it could be important to reflect about the kind of proposed task.

These results suggest that using a digital tool increases the amount of both automatic scanning fixations and long fixations related to more elaborated processing. However, the qualitative examination of the long gazes in context suggests that a significant amount of these long fixations is related to interacting with GeoGebra, for example when selecting an option from a drop-down menu or using a mouse to place an object in the coordinate system (Hannula et al. 2019).

Since eye tracking seemed particularly beneficial for studying processes rather than outcomes, for revealing mental representations, and for assessing subconscious aspects of mathematical thinking (Strohmaier et al. 2020) in this paper we address issues concerning geometric learning provided by tasks with integration of technology in its design and implementation.

In our previous analysis we rely on Stein's framework to categorize the tasks provided in the selected papers regarding problem solving (Bairral & Aldon, submitted). The framework of the cognitive demand of the tasks (Stein & Lane, 1996; Stein & Smith, 1998; Stein et al. 2000) was neither based on ET studies nor even included digital technologies on the criteria of task identification. For future research we suggest the possibility of creating a typology of task demand with description including technological issues. Another point of questioning is that such a framework mixing gaze observation and problem-solving does not contain many tasks based on reasoning and proof processes, particularly in geometry. In another review – based on ET studies in geometry from Strohmaier et al. (2020) – we identified predominance of tasks focused on identification of geometric shapes, conceptualization, and definition (Bairral et al. 2024).

The relevance of the integration of digital technology in task-design and the necessity of implementing more tasks based on reasoning and proof processes was highlighted in both reviews. From observing that the geometry program needs even more attention from curriculum professional developers and researchers, we assert that the content of our paper and the provision of such analysis could enrich models focused on integrating digital technology on the provided tasks. Assuming integration as protagonist - as a process to think and solve the task differently – in this paper we illustrate and analyze tasks in three geometry ET studies.

State-of-art concerning ET geometry studies integrating technology

So far, we found two literature reviews the mathematics education field. Lilienthal and Schindler (2019) focused on papers published in the proceedings of The International Group for the Psychology of Mathematics Education (PME) of the last ten years. Strohmaier et al. (2020) reviewed 161 eye-tracking studies published between 1921 and 2018 to assess what domains and topics were addressed, how the method was used, and how eye movements were related to mathematical thinking and learning. We decided to elaborate our review starting from Strohmaier et al. (2020) as it is an exhaustive review that included other publication contexts and investigated a large variety of research areas on mathematics education. In Bairral et al. (2024) we revisited 22 papers from Strohmaier et al. (2020) focused on geometry, shape, and form domain as summarized on Table 1.

Table 1

Brief Description of 22 Papers Using ET Method in a Geometrical Domain (Bairral et al. 2024)

Topic[2]	Authors	Specific topics
Geometric proofs	Alqassab, Strijbos & Ufer (2018)	Properties of triangles, parallelograms, transversals lines and quadrilaterals
	Lin, Wu & Sommers (2012)	Geometric proof reading (true or false situation), congruence of figures (example with square and circle)
	Muldner & Burleson (2015)	Triangle with inscribed rhombus
	Schindler et al. (2016)	Measurement of angles in an equilateral hexagon
Analyzing mental rotation tasks	Chen & Yang (2014)	Not specified
	Fry (1988)	Indirect association with spatial abilities
	Merschmeyer-Brüwer (2001)	Two-dimensional cubes, spatial relationships.
	Roach et al. (2016)	Rotation of blokes
	Wang, Chen & Lin (2013)	Solid and flat figures

Topic[2]	Authors	Specific topics
Construction of geometric objects	Epelboim & Suppes (2001) Hannula & Williams (2016) Lee & Wu (2017) Schimpf & Spannagel (2011) Shvarts (2018)	Angles in geometric figures Construction of triangles, calculation of area, solving non-routine problems Geometrical descriptions Polygons and measuring angles Triangle manipulation in an interactive computer environment
The perception of objects in a Cartesian plane	Krichevets, Shvarts e Chumachenko (2014) Shvarts (2018)	Points in Cartesian plane Cartesian plane, focusing on location and distance
Geometric calculations in dynamic problems	Lin & Lin (2014) Lin & Lin (2013)	Similar triangles and their properties Triangle exterior angle theorem
Processing of vector fields and geometric shapes	Klein et al. (2018) Ögren, Nyström & Jarodzka (2017) Haataja et al. (2018) Verdine et al. (2017)	Not specified Vectorial calculus concepts Not specified Geometric shapes

Table 1 reveals a diversity of topics and provides a rich view of the teaching and learning scenario in geometry with ET method. However, from these 22 studies we found only 3 integrating digital technology on the provided task. The integrating of digital technology on the provided task was the criteria of inclusion of the papers. All the three included paper worked with students, one in geometric proofs (Muldner & Burleson, 2015) and two in the construction or analyzing geometric objects (Schimpf & Spannagel, 2011; Shvarts 2018). In this paper we illustrate and analyze the provided tasks – or part of them – in these 3 papers, summarized on Table 2.

Table 2

Papers Using ET Method in a Geometrical Content with Task-Design Integrating Digital Technology

Authors	Theme	Device	Subjects	Proposed task(s)
Muldner & Burleson (2015)	Multiple solution task	Sensing devices, including a skin conductance bracelet, and an EEG sensor	21 university level students having taken at least one math course.	Students are asked to generate as many proofs to the target problem as they could think of.
Schimpf e Spannagel (2011)	The angle sums of triangle (quadrilateral, pentagon, hexagon, and heptagon)	Cinderella	171 students from 8th grade.	Two experiments were conducted to investigate the effects of reducing graphical user interfaces considering time taken by users to find given icons with full and reduced interfaces.
Shvarts	Problem solving	GeoGebra	4 student-tutor.	The task is proposed in two

Authors	Theme	Device	Subjects	Proposed task(s)
(2018)	(disclosing a parabola)		School level is not given, and prior analysis of the task was not done.	stages: first, it is a question of highlighting a curve based on a geometric property (locus of points equidistant from a point and a line) and then of finding the functional relationship associated with this curve.

Results

The Use of ET and the Integration of DT in Task Concerning Geometric Proofs and Construction of Geometric Objects

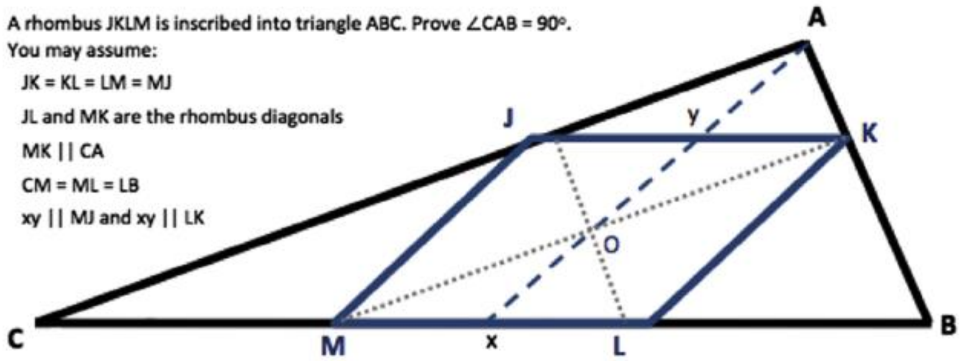
The three studies implement different tasks to analyze participants' performance. With geometric tests, ET helps measure fixation time in specific areas during problem solving, highlighting the importance of feedback in understanding problems and creativity in generating geometric tests. Subjects' feedback, stimulation of creativity, increased student engagement and the influence of solution quality are considered and show how subjects process visual and textual information in specific tasks. In investigations focusing on geometric objects, they address the mental construction of objects, showing how subjects tend to construct mental objects to solve problems and how this construction reflects higher-level reasoning about the problem. Collaborative geometric problem solving is also explored, highlighting student engagement, visual attention, and the importance of constructing geometric objects in problem solving.

Geometric Proofs: Feedback and Creativity

Muldner and Burleson (2015) explore the creativity of students through the resolution of a multiple solution task, in these cases a geometrical problem that can be tackled in several ways. They used the problem shown in Figure 1.

Figure 1

The Target Problem Studied in Muldner & Burleson (2015, p. 130)



The authors highlight the choice of a geometric problem that allows multiple solutions, which stimulates students' creativity and divergent thinking. The use of a sensor helps in collecting data to model the creativity of subjects in generating geometric proofs. It served as the main platform for resolving the problem. Participants interacted with the app during the activity, while data from sensors such as eye trackers, skin conductance bracelets, and EEG sensors recorded their interactions and behaviors. This integration of the application with sensor devices enabled a detailed analysis of students' creativity during the activity. The analysis of individual creativity was focused on fluency, flexibility, and originality, and on the combination among them. The results revealed the ability of the algorithms to establish two levels - low and high - of creativity based on sensor data. Muldner and Burleson (2015) highlight how these technologies can be useful in assessing and modeling student creativity.

In this study, the benefits of the use of eye-tracking devices and EEG sensors were manifold: the stimulation to creativity, by allowing the students to explore various solutions to the problems, the students' higher level of commitment and participation in the solving of geometry challenges, the personalized feedback provided based on data collected by sensors, the more accurate evaluation of students creativity and the demonstration of the potential of the devices to enhance the learning of geometry. These results demonstrate how the combination of the application of geometry with the sensors can improve the students' learning and offer new approaches for the teaching of geometry.

Construction of Geometric Objects: Design Interface and Visual Attention

The study by Schimpf and Spannagel (2011) aimed to explore the effects of simplifying the graphical interface of a dynamic geometry system on user interactions, particularly among novices, and investigate whether this simplification could promote learning. The methodology involved carrying out two experiments with eighth grade students in two secondary schools in Germany. Participants were divided into two groups: one using the complete interface (Figure 2a) of the Cinderella environment, with 48 icons, and the other using a simplified interface

(Figure 2b) containing only six icons essential for the proposed tasks, as illustrated in Figures 2a and 2b.

Figure 2a
Complete Interface

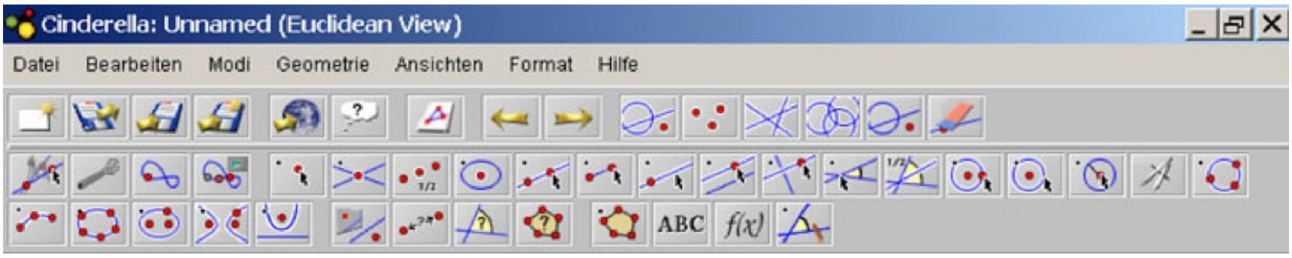


Figure 2b
Reduced Interface

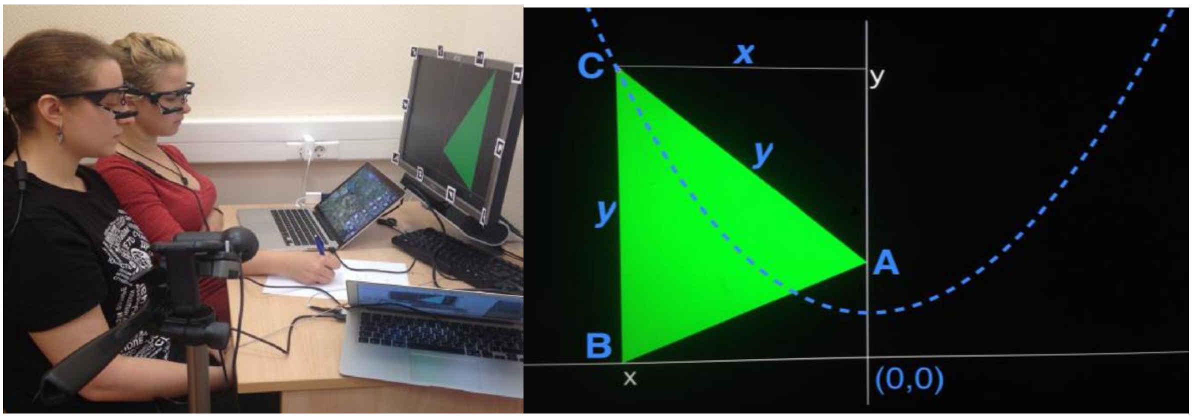
Source. Schimpf & Spannagel (2011, p. 394)

The approach involved a combination of constructing figures, measuring angles, solving problems, and exploring geometric properties. Data on performance, interaction time, self-confidence in computational skills and participants' perception were collected. The results revealed that simplifying the graphical interface did not have a significant impact on users' ability to locate familiar or new icons. Furthermore, no significant differences in performance were observed between the groups that used the full and simplified interface. Participants' self-confidence in their computational skills influenced the time dedicated to introducing and using the help but did not affect performance on the proposed tasks. These findings suggest that educators have the flexibility to choose between a full or simplified interface depending on specific educational needs. The study showed that simplifying Cinderella's graphical interface did not harm students' performance in geometry tasks and offered insights into how simplifying the interface can be an effective strategy to facilitate interaction, especially for novice users, with dynamic geometry systems.

Shvarts (2018) elaborated a computer-based interactive activity for disclosing a parabola. The study explored the transition from joint visual attention to joint mental attention during collaboration. The geometry content covered involved the manipulation of triangles and the resolution of geometric problems related to spatial properties and relationships. Participants interacted with an environment that presented triangles and allowed geometric manipulations, such as moving vertices and identifying specific properties of triangles. Students were challenged to find the formula for a curve drawn by the vertex of a manipulated triangle, expressing the position of a vertex in terms of Cartesian coordinates. This involved the application of geometric concepts, such as distances and relationships between the vertices of the triangle, to solve the proposed problem.

The results indicated a high temporal and spatial coordination of the tutors' perception with the students' actions, allowing sustained joint visual attention during the students' manipulations. Throughout the interactions between tutor and student, alternating episodes of dialogue and explanation were observed, with the tutor offering broader support when necessary. The transition from joint visual attention to mental attention was evidenced during collaboration, and the technology enabled detailed analysis of the coordination of joint visual attention between tutor and student and offered valuable insights into how visual communication and interaction influence construction of mathematical meanings during collaboration, as shown in Figure 3 concerning second stage of interactive learning material. Vertex C is manipulated by the student, while markers X and Y run along the axes. Triangle ABC turns green when CB=CA. Blue inscriptions are provided for clarity only and are not visible to the student

Figure 3
Dual Eye Tracking Experimental Setup



Source. Shvarts (2018, p. 4)

The use of double ET made it possible to synchronously record the eye movements of two participants, opening new perspectives in the analysis of visual attention in joint actions. Eye trackers were attached to create environments in which two people share a common space and can gesture and discuss manipulations on a shared monitor. The original software was developed to allow superimposition of the screen video with the two gaze paths, bringing innovation compared to the static images under the gaze paths in previous studies. Since visual attention of learners is correlated with dialogues and gestural expressions, this technical solution made it possible to combine qualitative analyses to look frame by frame at the videography of gestures and verbal expressions.

Approaching the Three Studies

The studies by Muldner and Burleson (2015), Schimpf and Spannagel (2011), and Shvarts (2018) present convergences and particularities in relation to the geometric tasks and digital technologies used. In terms of convergence, all these studies share the perspective of integrating technology into geometric activities with the aim of enriching students' learning experience. Each research proposes activities that encourage the comprehensive and interactive resolution of geometric problems, thus promoting the subjects' creativity and understanding.

Regarding particularities, the study by Muldner and Burleson (2015) focuses on generating proofs for a specific geometric problem, the research by Schimpf and Spannagel (2011) and Shvarts (2018) explores the construction of geometric objects and the manipulation of triangles, respectively. Furthermore, each study integrates, differently, digital technology in the design of the task (sensors, bracelets, and dynamic geometric environment) and ET in the form of data production. From the three studies and their specificities, we have established that the use of ET enhances integrated and detailed views on the subjects' cognitive development. Particularly, the potential of technological integration and combination (task-design + data production) the results of the three studies indicate contributions for learning and instruction, and for research in geometry field.

Table 3

Contributions of Integrating Digital Technology in ET in Geometry

Domain/topic	Learning and instruction	Research
Geometric proofs (Muldner & Burleson, 2015)	-stimulate creativity -increase engagement when solving geometric challenges -offer personalized feedback	- assess and model creativity in generating geometric proofs -serve as the main platform for problem solving -record interactions and behaviors that enable detailed analysis of creativity during the activity
Construction of geometric objects (Schimpf & Spannagel 2011; Shvarts, 2018)	-consider flexibility in choosing a full or simplified interface of a device in task design -allow detailed analysis of joint visual attention -jointly observe the temporal and spatial coordination of perception -identify comprehension difficulties -promote non-verbal communication -stimulate collaboration between those involved	- observe synchronously record the eye movements with other expressions and language aspects -combine qualitative analyses to look frame by frame at videography of gestures and verbal expressions

Final Remarks

In this article, we presented and analysed a sub-set of studies in eye-tracking regarding geometrical domain. Starting from Strohmaier et al. (2020) we found three studies, one focused on geometric proofs and two based on the construction of geometric objects. In all three papers, the improvement of geometric knowledge throughout eye-tracking methods is underlined. In these papers that eye-tracking methods are useful to go further in the articulated and intertwined analysis of students' cognitive process and collaboration.

We can observe how researchers are improving their way of gathering data designing tasks using digital resources. Although we identified detailed information regarding data collection setting, we think that studies could provide details (aims, affordances, constraints etc.) of the proposed tasks regarding the analysis and learning gains etc. If we are interested in promoting other learning aspects (besides identification, observation etc.) we must pay attention to the type of the designed task.

As eye tracking is a multidisciplinary field, we need to consider the focus of study between psychology, neuroscience, mathematics education and so on. But it is necessary to cross-analyse with other frameworks (didactic, psychological, pedagogical, etc.) to establish the link between ET parameters and cognitive functions. Another important point would be to consider tasks in a digital environment comprising not only the effective task but also the technology as a tool, as it changes the task-design and the mathematics content and related procedures. For further analysis we suggest cross-analysis concerning digital integration technology in considering the other domains/topics in ET studies.

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Notes

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[2]As described in Strohmaier (2020, p. 152).

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