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A REAL ARGUMENT TO DEFY CLASSICAL LOGIC

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Abstract. In this paper, an argument by Fisher (2004) is formalized and evaluated by means of some tools of classical logic. The argument presented by Fisher is a version of a piece of reasoning of great historical importance known as Pascal's Wager. According to Fisher, "this is a fascinating piece of reasoning. It is complex and important and hard to handle". Here is shown that, although formal logical analysis has limitations to evaluate everyday deductive arguments, it is perfectly capable to formalize and evaluate Fisher's argument. This should show that its usefulness is undeniable.

Keywords: classical logic • real arguments • Pascal's wager • evaluation of arguments • deductive reasoning

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In this work, it is formalized and evaluated an everyday argument taken from Fisher's *The Logic of Real Arguments* known as Pascal's Wager after the French philosopher and mathematician:

> Either there is a Christian God or there isn't. Suppose you believe in His existence and live a Christian life. Then, if He does exist you will enjoy eternal bliss and if He doesn't exist you will lose very little. But suppose you don't believe in His existence and don't live a Christian life. If He doesn't exist you will lose nothing, but if He does exist you will suffer eternal damnation! So, it is rational and prudent to believe in God's existence and to live a Christian life. (2004, p.2)

According to Fisher, "this is a fascinating piece of reasoning. It is complex and important and hard to handle. In this case furthermore, it is the sort of argument which tends to stop the non-believer in his tracks: *if it is right* it seems to provide a very compelling reason for reforming his ways because the consequences of his being mistaken are so appalling" (2004, p.2). Fisher argues that the techniques of logic seem to be of very little help in handling this argument and other similar ones. Therefore, it is interesting to examine what some methods of deduction of propositional calculus and predicate logic can provide to help us evaluate the argument.

First of all, the conclusion, "it is rational[1] and prudent to believe in God's existence and to live a Christian life", rests on the subsidiary conclusion that doing these two things leads to eternal happiness or, at worst, to losing very little, while if you do not believe and do not lead a Christian life you will suffer eternal damnation or at best you will lose nothing.

In predicate logic, the first premise of the argument can be formalized as the disjunction $G_1 \vee \sim G_1$, with $G_1 : \exists x G(x)$, where $G(x) = x$ is a Christian God.

However, the variable for the first disjunct of the premise can be formalized simply as G : There is a Christian God. In a similar way, "You will lose nothing" can be formalized in predicate logic as: $\sim\exists x T(x)$, where $T(x) = x$ is a thing you will lose. But this expression can be formalized as N : You will lose nothing. So, the entire argument can be formalized using only propositional variables:

G : There is a Christian God
B : You believe in the existence of a Christian God
C : You live a Christian life
E : You will enjoy eternal bliss
L : You will lose very little
N : You will lose nothing
S : You will suffer eternal damnation

Using this dictionary, the argument with the subsidiary conclusion can be formalized as:

1. $G \vee \sim G$
2. $(B \wedge C) \rightarrow ((G \rightarrow E) \wedge (\sim G \rightarrow L))$
3. $(\sim B \wedge \sim C) \rightarrow ((\sim G \rightarrow N) \wedge (G \rightarrow S))$
   Therefore, $((B \wedge C) \rightarrow (E \vee L)) \wedge ((\sim B \wedge \sim C) \rightarrow (N \vee S))$.

The resulting argument with this alternative conclusion can be evaluated in the propositional calculus in various ways. A simple way is with the reductio ad absurdum method by assigning truth values. We first separate the conjuncts of the conclusion, and each of the two resulting arguments is evaluated separately. The formalization of the first argument is as follows:

1. $G \vee \sim G$
2. $(B \wedge C) \rightarrow ((G \rightarrow E) \wedge (\sim G \rightarrow L))$
3. $(\sim B \wedge \sim C) \rightarrow ((\sim G \rightarrow N) \wedge (G \rightarrow S))$
   Therefore, $(B \wedge C) \rightarrow (E \vee L)$

In the reductio ad absurdum method of assigning truth values,[2] it is assumed that the conclusion is false and the premises are true, that is, it is assumed that the argument is invalid and if we reach a contradiction, then it is concluded that the argument cannot be invalid, so it is valid. Given that the conclusion is a conditional, if the conclusion is false, the antecedent is true and the consequent is false, so the truth values of the variables B, C, E and L are determined: since the conjunction of the antecedent is true, B is true and C is also true, and in the case of the consequent, since the disjunction is false, the variables E and L are false.

If these truth values are assigned uniformly in premise 2, it follows that the premise 2 must be false, since it is a conditional with a true antecedent and its consequent is a conjunction with at least one false conjunct (if G is true, the first conjunct is false, and if G is false, the second conjunct is false):

1. $G \vee \sim G$
2. $(B^t \wedge C^t) \rightarrow ((G \rightarrow E^f) \wedge (\sim G \rightarrow L^f))$
3. $(\sim B \wedge \sim C) \rightarrow ((\sim G \rightarrow N) \wedge (G \rightarrow S))$
   Therefore, $(B^t \wedge C^t) \rightarrow (E^f \vee L^f)$

Therefore, the argument cannot have all its premises true and its conclusion false, so it is valid: the conclusion follows from the premises. The second resulting argument has the same form, so it is also valid. This can be shown by applying the same method, but now to premise 3.

1. $G \vee \sim G$
2. $(B \wedge C) \rightarrow ((G \rightarrow E) \wedge (\sim G \rightarrow L))$
3. $(\sim B^t \wedge \sim C^t) \rightarrow ((\sim G \rightarrow N^f) \wedge (G \rightarrow S^f))$
   Therefore, $((\sim B^t \wedge \sim C^t) \rightarrow (N^f \vee S^f))$

Since in each of the two resulting arguments the conclusion follows and both have the same premises, the conjunction of their respective conclusions also follows. Therefore, the original argument is valid.

Another way to evaluate the validity of the argument is using rules of natural deduction. From premise 2 it is reasonable to conclude that if you believe in the existence of God, you live a Christian life, and God exists, then you will enjoy eternal bliss; but if you believe in the existence of God, you live a Christian life, and God does not exist, you will lose very little. In fact, they are two ways of expressing the same idea (import-export rule). This can be formalized thus:

$(B \wedge C) \rightarrow ((G \rightarrow E) \wedge (\sim G \rightarrow L))$
Therefore, $(((B \wedge C) \wedge G) \rightarrow E)) \wedge ((B \wedge C) \wedge \sim G) \rightarrow L)$

$((B \wedge C) \wedge G) \rightarrow E)) \wedge (((B \wedge C) \wedge \sim G) \rightarrow L)$
Therefore, $(B \wedge C) \rightarrow ((G \rightarrow E) \wedge (\sim G \rightarrow L))$

In a similar way, the premise 3 $(\sim B \wedge \sim C) \rightarrow ((\sim G \rightarrow N) \wedge (G \rightarrow S))$ is equivalent to $((\sim B \wedge \sim C) \wedge \sim G) \rightarrow N)) \wedge (((\sim B \wedge \sim C) \wedge G) \rightarrow S)$.

Although these equivalences seem intuitive, it is well known that intuition is not enough for them to be correct. However, their validity can be easily proved with the method of assigning values used before and with others. This is very advantageous, since an inference cannot be left only in the hands of intuition, for this often deceives us. With these equivalences in mind, the premises of the resulting argument can be stated as follows:

1. $G \vee \sim G$
2. $((B \wedge C) \wedge G) \rightarrow E$
3. $((B \wedge C) \wedge \sim G) \rightarrow L$
4. $((\sim B \wedge \sim C) \wedge \sim G) \rightarrow N$
5. $((\sim B \wedge \sim C) \wedge G) \rightarrow S$

Although it might seem little progress has been made by breaking down these equivalences, it is a way that allows us to clearly see that there are some possibilities that are not contemplated in the premises of the argument. For example, in the actual context of the discussion, it seems reasonable to accept the truth of these presuppositions which are not included in the premises:

$((B \wedge \sim C) \wedge G) \rightarrow S$
$((B \wedge \sim C) \wedge \sim G) \rightarrow L$

In contrast, the following possibilities are not compatible, given the usual doctrines of Christianity, because no one can live a Christian life without believing in the Christian God

$((\sim B \wedge C) \wedge G)$
$((\sim B \wedge C) \wedge \sim G)$

Therefore, adding the two presuppositions (6 and 7 bellow) the complete information in the premises of the argument can be stated as follows:

1. $G \vee \sim G$
2. $((B \wedge C) \wedge G) \rightarrow E$
3. $((B \wedge C) \wedge \sim G) \rightarrow L$
4. $((\sim B \wedge \sim C) \wedge \sim G) \rightarrow N$
5. $((\sim B \wedge \sim C) \wedge G) \rightarrow S$
6. $((B \wedge \sim C) \wedge G) \rightarrow S$
7. $((B \wedge \sim C) \wedge \sim G) \rightarrow L$

If someone objects that the subsidiary conclusion is not the best option or that it is not clearly justified, the answer is that it is not necessary to consider that specific conclusion in the analysis, it is possible to examine the inferences that can be made directly from the premises.

From premises 5 and 6, it is concluded that, if God exists and I do not live a Christian life, then it does not matter if I believe in the existence of God or not, either way I will suffer eternal damnation:

$((\sim B \vee B) \wedge (\sim C \wedge G)) \rightarrow S$, which can be reduced to $(\sim C \wedge G) \rightarrow S$

So, what is relevant to suffering eternal damnation is not so much disbelieving in God, but rather not leading a Christian life. Belief alone does not make any difference.

From 3 and 7, it follows that if I believe in God and he does not exist, whether or not I lead a Christian life, I lose very little.

From 4 and 7, it can be concluded that if God does not exist and I do not lead a Christian life, then if I believe, I lose very little, and if I do not believe, I lose nothing.

What is being contrasted in the original argument is not only the mere belief or disbelief in the Christian God but the belief in God and living a Christian life compared to the opposite: not believing in God and not leading a Christian life. This explains why belief in God is only a necessary condition but not a sufficient one to attain eternal bliss. On the other hand, the lack of belief in God is a sufficient condition for not leading a Christian life, making it sufficient to lead to eternal damnation. Thus, disbelief in God has significantly more profound negative consequences than the positive consequences of belief in God.

It is possible to disagree with the degree of certainty of certain premises or presuppositions, but the usefulness of logical analysis to consider all the relevant information in the context and to identify where there may be points of controversy is clear.

One must be aware that the formal logical analysis has limitations. For example, the first premise is a tautology, so it does not play any role in the evaluation of the validity of the argument. But it does suggest that the disjuncts are two disjoint and exhaustive alternatives, that is, one excludes the other, but at least one must occur. It could be thought that these features of being disjoint and exhaustive are inherited to the options presented in the disjunction that appears in the consequent of the conditional $(B \wedge C) \rightarrow (E \vee L)$.

At first glance, the following inference in general may seem correct, since if Y occurs, its negation $\sim Y$ cannot occur:

$X \rightarrow ((Y \rightarrow W) \wedge (\sim Y \rightarrow Z))$, therefore, $X \rightarrow (W \bar{\vee} Z)$, where $\bar{\vee}$ represents the exclusive disjunction.

However, Y and its negation are only a sufficient condition for W and Z respectively, not a necessary condition, so although Y and its negation cannot occur simultaneously, W and Z can. Again, the method used to show the formal invalidity of the reasoning in question can be used. It would be useful to know what non-logical methods can offer to help make this type of distinction and avoid incurring in fallacies of this sort.

However, once it is determined that the argument is invalid or valid, one can go to the content to see what additional information can be obtained. In this case, it is the very content of the propositions what shows that N and S are incompatible. But, although it is not obvious, it seems that E and L are compatible. Therefore, in the conclusion of the original argument, the disjunction "$N \vee S$" is an exclusive disjunction, while the disjunction "$E \vee L$" is inclusive. This strengthens the conclusion that "it is rational and prudent to believe in God's existence and to live a Christian life", because it can lead us to receive eternal bliss or, in the worst case, to lose very little, with the possibility of both things happening and thus the loss is compensated to the maximum. In contrast, if the opposite occurs, there is only one of two options: eternal damnation or no loss in the best case.

Note that this last inference is not based only on the content of the propositions, but also on knowledge of the logical connectives, thus vindicating the usefulness of logical analysis of the examined argument. In general, deductive methods of classical logic are useful to evaluate real deductive arguments and they can be complemented by those of informal logic. (Hernández 2017).

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Notes

[1]According to Jeff Jordan, Pascal's Wage is the most prominent example of a theistic pragmatic argument: "The Wager is not an argument that God exists. That sort of argument, the appeal to evidence, whether empirical or conceptual, is the domain of the other theistic arguments. Pascal's Wager is an argument that belief in God is pragmatically rational, that inculcating a belief in God is the response dictated by prudence. To say that an action is pragmatically rational implies that it is in one's interests to do that action". (2006, p.2).

[2]Copi, Cohen and McMahon claim that "This *reductio ad absurdum* method of assigning truth values is often the quickest method of testing arguments, but it is more readily applied in some arguments than in others, depending on the kinds of propositions involved". (2014, p.423).