ARISTOTLE'S SEA BATTLE, EXCLUDED MIDDLE AND BIVALENCE

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Abstract.   In this paper, I present a formal reconstruction of the classical argument for fatalism set forth by Aristotle in *On Interpretation* 9. From there, I expose two different formal solutions for avoiding the unwanted conclusion based on the traditional interpretation of Aristotle's rejection of the Principle of Bivalence: On the one hand, Łukasiewicz's three-valued logic and, on the other hand, supervaluation semantics. I also address some criticisms made against these two proposals. To finish, I remark on some alternative interpretations of Aristotle's intentions maintaining that the Stagirite philosopher rejected fatalism without abandoning the Principle of Bivalence.

Keywords: fatalism• future contingents • bivalence • three-valued logic • supervaluationism

1. The argument for fatalism

Fatalism is the view that we have no control over future events. It does not matter what we do, all events are destined to happen. Aristotle discussed the argument for fatalism in *On Interpretation* 9 where he states that if statements about future events are true or false, then those events turn out to be necessary or impossible. In Aristotle's words:

Again, if a thing is white now, it was true before to say that it would be white, so that of anything that has taken place it was always true to say 'it is' or 'it will be'. But if it was always true to say that a thing is or will be, it is not possible that it should not be or not be about to be, and when a thing cannot not come to be, it is impossible that it should not come to be, and when it is impossible that it should not come to be, it must come to be. All, then, that is about to be must of necessity take place. It results from this that nothing is uncertain or fortuitous, for if it were fortuitous it would not be necessary. (Edghill 1926, 18b p. 10-16).

In order to illustrate this situation, Aristotle's famous example is about a sea battle that is going (or is not going) to take place tomorrow. Let us consider a reconstruction of Aristotle's argument:

1. Either there will be a sea battle tomorrow or there will not be a sea battle tomorrow.

Apparently, (1) can be assumed to be true. From there, it seems that at least one of the following sentences must be true:

2. There will be a sea battle tomorrow.
3. There will not be a sea battle tomorrow.

If (2) is true and (3) is false, it would be settled today that tomorrow will be a sea battle, consequently, the sea battle would be necessary. Otherwise, if (2) is false and (3) is true, it would be settled today that tomorrow would not be a sea battle, consequently, the sea battle would be impossible.

Thus, by accepting the previous reasoning, we are obliged to infer the following fatalist conclusion:

4. Either it is necessary that there will be a sea battle tomorrow or it is impossible that there will be a sea battle tomorrow.

There is a general consensus that Aristotle intended to argue against this fatalist consequence (Gaskin 1995, p. 12). Unfortunately, the agreement among scholars ends here. There exist different interpretations about how Aristotle proposed to block the above derivation. However, I only consider here the traditional view. According to this interpretation, with the purpose of maintaining the openness of the future, the Stagirite philosopher held that future contingents- statements about future events that can occur or not occur, such as (2) and (3)- are neither true nor false (Byrd 2010, p. 160). Hence, Aristotle's solution seems to rely on abandoning the logical principle that states that every sentence is either true or false, namely, the Principle of Bivalence. But before moving forward, let us consider a formal reconstruction of the derivation from (1) to (4) to make explicit the logical principles involved.

For the sake of clarity, firstly, I define the symbols of the formal language: "q" stands for statements about past or present events, while " p " stands for statements about future contingent events. Besides, "T p " symbolizes the statement that it is (or it was) true that p . The principles required for the derivation are the following:

Principle of Bivalence (PB): $Tp \lor T\neg p$

Fixity of past and present events (Fix): $q \supset \Box q$[1]

Necessity of future contingent events when they are (or were) true (Tness): $\Box(Tp \supset p)$

The rest of the logical principles employed in the proof are well-known modal or propositional logic laws.

(i)	$Tp \lor T\neg p$	PB
(ii)	Tp	assumption
(iii)	$Tp \supset \Box Tp$	Fix
(iv)	$\Box(Tp \supset p)$	Tness
(v)	$\Box Tp \supset \Box p$	Distribution: (iv)
(vi)	$Tp \supset \Box p$	Transitivity: (iii), (v)
(vii)	$\Box p$	Modus ponens: (ii), (vi)
(viii)	$\Box p \lor \neg\Diamond\neg p$	Addition (vii)
(ix)	$T\neg p$	assumption
(x)	$T\neg p \supset \Box T\neg p$	Fix
(xi)	$\Box(T\neg p \supset \neg p)$	Tness
(xii)	$\Box T\neg p \supset \Box\neg p$	Distribution: (xi)
(xiii)	$T\neg p \supset \Box\neg p$	Transitivity: (x), (xii)
(xiv)	$\Box\neg p$	Modus ponens: (ix), (xiii)
(xv)	$\neg\Diamond\neg p$	Definition of $\Diamond$: (xiv)
(xvi)	$\Box p \lor \neg\Diamond\neg p$	Addition: (xv)
(xvii)	$\Box p \lor \neg\Diamond\neg p$	Proof by cases: (i), (ii)–(viii), (ix)–(xvi)

In this way, from PB, we derived that any future contingent event is either necessary or impossible.

2. Two formal solutions to avoid fatalism

As I mentioned earlier, according to the traditional interpretation, Aristotle's proposal against the fatalist conclusion (4) rests on rejecting PB (Gaskin 1995, p. 12; Byrd 2010, p. 160). Hence, if statements about future contingent events do not yet have a truth value, it is possible to avoid the conclusion that everything occurs out of necessity. I present below two formal developments that reject PB: Łukasiewicz's three-valued logic (Łukasiewicz's 1968) and supervaluation semantics (van Fraasen 1966; Thomason 1970).

2.1. Łukasiewicz's three-valued logic

According to Jan Łukasiewicz, Aristotle advocated for introducing a third logical value for sentences about future contingent events (Jarmuźek 2018, p. 140). Thus, his proposal for avoiding fatalism relies on the development of a truth-functional semantics with three truth values: 1 for "true", 0 for "false" and ½ for "indeterminate". If a sentence takes this third value ½, then it is not settled whether it is true or false (Łukasiewicz 1968, p. 48). The following truth tables depict the meaning of the logical connectives in Ł$_3$:

¬	
1	0
½	½
0	1

∧	1	½	0
1	1	½	0
½	½	½	0
0	0	0	0

∨	1	½	0
1	1	1	1
½	1	½	½
0	1	½	0

⊃	1	½	0
1	1	½	0
½	1	1	½
0	1	1	1

PB is not valid in this framework because statements can take the third value. So, not every sentence is either true or false. Without step (i), the formal derivation is blocked and we are not obliged to embrace fatalism. However, Łukasiewicz's proposal has a weak spot: The Principle of Excluded Middle (EM) does not hold in his system. Let us consider sentences (2) and (3) again. If (2) is indeterminate, its negation (3) is also indeterminate according to the definition of ¬. Then, (1) is indeterminate as well as the disjunction of two indeterminate sentences is indeterminate: if $p = $ ½ and $\neg p = $ ½, $p \vee \neg p = $ ½.

It has been argued that Aristotle intended to retain EM because according to him the disjunction of a sentence and its negation is necessarily true, even in the case of statements about future contingent events such as $p \vee \neg p$ (Haack 1996, p.85; Priest 2008, p.133). Thus, despite rejecting PB, Aristotle seemed to accept the validity of EM. Consequently, Ł$_3$ does not seem to be the optimal solution when avoiding fatalism.

It is worth noting that PB and EM are two different principles. On the one hand, PB states that every sentence is either true or false. On the other hand, EM states that it is the case of s or $\neg s$, formally expressed as $s \vee \neg s$, being "s" any statement of a formal language. Systems that validate EM also validate PB (Haack 1996, p. 67). Nevertheless, as we will see in the following subsection, it is possible to build formal systems that reject PB while maintaining EM.

2.2. Supervaluation semantics

To deal with the problem of truth assignments to sentences containing non-denoting terms, van Fraansen (1966) devised supervaluation semantics. Afterwards, Thomason (1970) put this idea together with Prior's (1967) branching time model and developed a formal framework for future contingents that rejects PB but preserves EM.

Supervaluationism admits truth-value gaps, i.e., sentences that lack a truth value: they are neither true nor false. This is the case of sentences such as (2) and (3), which are *gappy* sentences. The reason why (2) and (3) are gappy is that there are some possible futures in which (2) is true and (3) is false and others in which (2) is false and (3) is true. It is notable that, in contrast to Ł$_3$, (2) and (3) are not indeterminate, they do not have a truth value at all. Hence, a fundamental supposition of this proposal is that sentences about future contingent events can be considered true or false relative

to possible futures or histories. In some possible futures, tomorrow, there will be a sea battle, but in others, there will not be a sea battle. Consequently, (2) is true in the first possible history but false in the second. The opposite works for (3).

Time is seen as a line branching from the present into a range of possible histories that, in turn, branch again and again. Regarding the formal framework, a tree is a world, a history is a line along a tree representing a way the world may have gone or may go and a moment is a point along the line. Sentences are evaluated as true or false in a moment m relative to a history h. Truth in a moment m is defined as truth in all histories passing through m. Thus, p is true at m if $p = 1$ in all histories passing through m and p is false at m if $p = 0$ in all histories passing through m. Nevertheless, if there exist histories passing through m in which $p = 1$ and others in which $p = 0$, then p is neither true nor false at m. Consequently, PB does not hold. Thus, the fatalist derivation is blocked.

Furthermore, EM holds in supervaluationism because at any given moment m $p \vee \neg p$ is true, as $p \vee \neg p = 1$ in all histories passing through m. Even though p lacks a truth value, $p \vee \neg p$ is assigned true by all the classical valuations. This leads to a non-truth-functional semantics since the disjunction of two future contingent sentences, such as (1), can be true even when neither of its disjuncts is true. Thus, this second formal solution can avoid the unwanted fatalist conclusion by rejecting PB while holding EM in agreement with Aristotle's intention.

A drawback of this proposal is that as future contingents lack a truth value it is not easy to determine whether they can be correctly asserted. However, we frequently assert statements about future contingent events in everyday language. This challenge is known as the assertion problem and has been faced by devising alternative formal frameworks (Belnap & Green 1994).

3. Final remarks

It has been claimed that Aristotle did not intend to reject PB or EM in *On Interpretation* 9. On the contrary, he refuted fatalism by distinguishing between the truth value of sentences about future contingent events and their necessity. According to this interpretation, Aristotle rebutted fatalism by blocking the derivation from truth to necessity (Gaskin 1995, p. 14). Thus, he rejected Tness, which was applied in steps (iv) and (xi) in the formal derivation.

Moreover, it has been stated that PB does not entail fatalism. According to this view, there exists a misconception in the idea of truth as correspondence with reality. It is erroneously believed that the truth or falsity of future contingent sentences, such as (2) and (3), involved the existence of some fact or event today. The relation of correspondence can be obtained between terms that represent events existing at

different times. Thus, (2) and (3) can be either true or false although there does not exist yet the event that makes them true or false (Iacona 2007, p. 56). This line of response gave rise to a series of solutions based on revisiting the Theory of Truth.

The present discussion about Aristotle's intentions in *On Interpretation* 9 and the formal solutions to avoid fatalism is far from being resolved. Instead, a heated debate continues. In keeping with the spirit of this special issue, this paper aimed to show the usefulness of logic and formal methods for exposing philosophical problems clearly and, from there, evaluate the adequacy of different solutions.

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Notes

[1]This and the following principle were proposed by Priest (2008) to respond to Haack's (1996) charge that Aristotle's argument rests on a modal fallacy.