

# QUANTIFIERS AND EXISTENCE

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**Abstract.** There are some sentences that include expressions that refer to entities that do not exist. One example is this: Mary is in terror of werewolves. Some argue that this sentence cannot be translated into predicate logic. This may be seen as a flaw in predicate logic. Against this, I argue in this paper that the problem is not predicate logic, but rather our commitments to the existence and nature of certain things. By revising some of these commitments, we can see that predicate logic is perfectly capable of dealing with the problematic sentences.

**Keywords:** quantifiers • scope • formalisation • existence • ontology

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## 1. Introduction

Quantified expressions like “all”, “some”, “not all”, “both”, “none” and many others are ubiquitous in the English language.<sup>1</sup> Native speakers normally do not have any problem using and understand these expressions, and usually use them correctly in reasoning. However, there are many cases in which the normal speaker may overlook differences in meaning when using these expressions. Predicate logic may prove useful in these cases. Take just one example connected with the *scope* of quantified expressions. Consider the following sentence:

(1) Some editor reads every manuscript

The reader may note an ambiguity in this sentence. With the usual procedure to translate this expression in predicate logic, we can explain this difference in meaning more clearly. First, we define the predicate letters:

$Ex$ :  $x$  is an editor

$Mx$ :  $x$  is a manuscript

$Rxy$ :  $x$  reads  $y$

So, let us translate these two senses in the usual way, using ‘!’ to indicate uniqueness: ‘there is one and only one’:

Subject-wide scope:



(1a)  $(\exists!x)(Ex \wedge \forall y(My \rightarrow Rxy))$ .

Object-wide scope:

(1b)  $(\forall x)(Mx \rightarrow \exists y(Ey \wedge Ryx))$

(1a) says that there is one and only one editor that reads every manuscript. (1b) says instead that every manuscript is read by at least one editor. This is compatible with the possibility that different editors read different manuscripts. It is expected that this translation, in this case, helps us to see the two possible meanings of (1) and better understand better our use of the quantifier.

These examples are simple, but others can be more complex and trickier. Other cases of scope ambiguity may occur when the quantifiers interact with other logical expressions like negation and even modal operators, as the following sentences show:

(2) Each runner did not finish the race.<sup>2</sup>

(3) It is possible that a kid is playing in the street.

(2) may have two different meanings, depending on which expression has wide scope. We define again our predicate letters:

$Rx$ :  $x$  is a runner

$Fx$ :  $x$  finished the race

Now, we have two possible readings:

Quantifier-wide scope:

(2a)  $\forall x(Rx \rightarrow \neg Fx)$

Negation-wide scope:

(2b)  $\neg \forall x(Rx \rightarrow Fx)$

(2a) says that every single runner in the race did not finish it. (2b) says that not every runner finished the race. This is compatible with the possibility that some finished it, or none finished it. In the case of (3), we can explain the ambiguity this way:

$Kx$ :  $x$  is a kid.

$Px$ :  $x$  is playing in the street.

And we use ‘ $\Diamond$ ’ for ‘it is possible that’ or ‘Possibly...’:

Quantifier-wide scope:

(3a)  $(\exists x)(Kx \wedge \Diamond Px)$

Modal operator-wide scope:

$$(3b) \quad \Diamond((\exists x)(Kx \wedge \Diamond Px)$$

(3a) says that there is a kid, and it is possible that this kid is playing in the street. This is compatible with the fact that this kid is not actually playing in the street. (3b) only says that it is possible that some kid is playing in the street. It is compatible with the fact that there is no kid at all. Again, it is expected that these cases would show that predicate logic is useful to disentangle the ambiguity, and this contributes greatly to our understanding of the quantifiers. And if we press the point, we would like to say that formal logic does it better than any other alternative way to explain and clarify the same ambiguity. This may be enough to show to some extent the importance of predicate logic in shedding light on our use of quantifiers in our normal life. And its importance can be further seen when we consider that sentential logic cannot show the intuitive validity of the following simple argument:

Argument 1

Fritz the cat sleeps on the bed.

So, there is something that sleeps on the bed.

We may see that the conclusion of the argument follows from the premise. But it is not clear how to show it by using sentential logic. We define the sentence letters:

*C*: Fritz the cat sleeps on the bed

*D*: there is something that sleeps on the bed

The translation we get is this:

*C*

$\therefore D$

Seen this way, the argument does not appear valid. So, we need some other tool to unpack the hidden content of the argument. The usual way requires predicate logic:

*f*: Fritz the cat

*Sx*: *x* sleeps on the bed.

We get:

Argument 1\*

*Sf*

$\therefore (\exists x)Ex$

This way, we can notice that the argument is valid.<sup>3</sup> From this example and the previous one, we may see that quantifiers and arguments that include them are better understood thanks to the use of the modern tools for formalisation in the first order quantified logic.

All these things considered, there is still the impression that formal logic does very little to help us understand our argumentative practices in the real world. There is a series of problematic sentences that involve quantifiers and some issues of existence that may pose challenges when translated into predicate logic. Despite the difficulties, I want to show that the problem is not predicate logic, but something else.

## 2. Quantifiers and existence

The issue I want to address can be illustrated in a very simple way by the following example (Simpson 1999, p. 155):

(4) Mary is in terror of werewolves

By appealing to the usual way to formalize some expressions, we can translate this sentence into predicate logic as follows:

$m$ : Mary

$Wx$ :  $x$  is a werewolf

$Txy$ :  $x$  is in terror of  $y$

By using either the existential quantifier or the universal quantifier, the resulting translation does not seem to be adequate:

(4a)  $(\exists x)(Wx \wedge Tmx)$

This formula says that there is at least one thing that is a werewolf and Mary is in terror of it. In principle, this seems to be inadequate because the formula asserts the existence of werewolves and, as far as we know, there are no werewolves. The first conjunct of the formula is false, so the entire conjunction is false. The formula is false no matter if Mary is in terror or not of werewolves. The universal quantifier does not seem to produce a better result:

(4b)  $(\forall x)(Wx \rightarrow Tmx)$

This formula says that, if something is a werewolf, then Mary is in terror of it. Given that werewolves do not exist, the antecedent of the conditional is false, so the whole formula is trivially true independently of whether the original sentence is true or false about Mary. So, how are we supposed to translate (4)?

Simpson's solution to this problem of translation is avoiding any existential commitment to suspect entities (1999, p. 155). He suggests translating the sentence as follows:

$m$ : Mary

$T$ : is in terror of werewolves

So, we get:

(4c)  $Tm$

(4c) does not have the previous problems. It is not trivially true, and its truth-value will depend on whether Mary is terrified of werewolves or not. However, with restrictions of this kind, we lose one of the advantages of predicate logic, which was meant to unpack the content of propositions. Moreover, it is difficult to show that the following argument is valid:

Argument 3

Mary is in terror of werewolves

So, Mary is in terror of something.

How are we supposed to translate the conclusion of the argument? We may introduce a new predicate letter, but the result is not convincing:

$m$ : Mary

$Tx$ :  $x$  is in terror of werewolves.

$Pxy$ :  $x$  is in terror of  $y$ (?)

We get:

Argument 3\*

$Tm$

$\therefore (\exists x)(Pmx)$ (?)

What can we do in this case? Are we to say then that predicate logic is not as useful as some logicians want us to believe? The issue here does not seem to be in the formal apparatus, but rather with more fundamental questions about the nature of certain expressions and what they seem to refer to, as well as with questions about what kind of things the quantifiers are meant to quantify over.

### 3. A solution?

The expression “werewolves” is problematic because it refers to something that do not exist.<sup>4</sup> First-order quantifiers do not quantify over non-existing things.

So, one part of the problem is the specification of the domain of quantification. We are not sure whether we should allow for certain mysterious things in the domain: are fictional characters allowed? Things that do not exist? Mere *possibilia*?<sup>5</sup> There is not a clear agreement on this. But the point I want to stress here is that whatever resolution we reach in the debate, formal logic may be useful to understand the different views in question. In this sense, formal logic is a tool always available for every metaphysical view that one wants to endorse. What we must get right is our metaphysics.

Do we solve the problem only by allowing, for example, a specification of the domain that allow us to include the problematic entities? Surely, it is not enough, for it may seem reasonable to expect good arguments that show that the discourse of the entities in question makes sense, and it is in a good standing. Maybe there are no such arguments, but whatever resolution we have about this matter, formal logic is there to deal with the appropriate way to analyse the resulting expressions and arguments.

Consider an analogous case. The previous sentence: (4) ‘Mary is in terror of werewolves’ was problematic because, when formalised, it says that there are werewolves, but these do not exist. In contrast, the following sentence may seem less problematic:

(5) Mary is in terror of ducks

(6) Mary is in terror of green chairs

These sentences say something about Mary and some ordinary things. However, there are views that call into question the reality of things like natural kinds (Ludwig 2018, Hacking 2007), colours (Hardin 1988, Ch. II, Maund 2011, 2012) and ordinary objects (van Inwagen, 1990, Ch 12; see Thomasson, Ch. 9 for a discussion of eliminativist views). According to these views, natural kinds, colours, and ordinary objects are not less controversial than werewolves or mere *possibilia*. There are deep metaphysical questions about the nature or even existence of all these ordinary things: do ducks really exist? Or only exist particles arranged duckwise?) Does the green colour exist or is it only a qualia, an experience, a phenomenological event? What about chairs? Is there only particles arranged chairwise? From this perspective, translating (5) and (6) into predicate logic may produce the same problematic results as in the case of (4).

*m*: Mary

*D*\_: \_ is a duck

$G\_:$   $\_$  is green

$C\_:$   $\_$  is a chair

$T\_:$   $\_$  is in terror of  $\_$

We get:

(5\*)  $(\exists x)(Dx \wedge Tmx)$

(6\*)  $(\exists x)((Gx \wedge Cx) \wedge (Tmx))$

If one holds sceptics views on the existence of ducks and green chairs, one may have some scruples to assent to these formulas. They are false independently of whatever relation Mary has with ducks and green chairs. The universal quantifier is inadequate as in the previous case:

(5\*\*)  $(\forall x)(Dx \rightarrow Tmx)$

(6\*\*)  $(\forall x)((Gx \wedge Cx) \rightarrow (Tmx))$

(5\*\*) and (6\*\*) are trivially true if we call into question the existence of ducks and green chairs. Again, this does not seem to capture the meaning of the original sentences, for their truth-value must depend on whether Mary is terrified of ducks and green chairs. So, we are in a similar predicament as in the case of werewolves. What do we do now?

An important point here is to note that the formal apparatus of logic may not be decisive to find a solution to deeper problems about what kind of things exist, and what their nature is. When nobody calls into question the existence or nature of ducks and green chairs, formulas (5\*) and (6\*) are perfectly and unrepachable translations of (5) and (6) into predicate logic. (4\*) is problematic because mentions werewolves, and in this case, it seems that everybody would question their existence. But if what I have argued in previous paragraphs is correct, it seems that this is due more to some kind of picky attitude in choosing what things we find agreeable and which things are not. Similar suspicions about the existence and nature of werewolves can be raised for ducks and green chairs. So, if we are to doubt about the former, we should also doubt about the latter.

However, we may do the opposite: we can stop doubting the existence of things we can perfectly talk about. A position of this kind is put forward by Jonathan Shaffer, although his interest is directed more to a revival of the Aristotelian view on metaphysics as the study of what grounds what (in contrast with a Quinean view) (2009). In general, we can say that there are two projects or activities implicit in many philosophical views in connection with ontology and metaphysics, and sometimes they are confused. On the one hand, ontology deals with what thing there are. On the other hand, metaphysics can be seen as dealing with questions about the ultimate nature of

things, their essence or, as Schaffer says, what ground what and what is fundamental. The first project is easy, for ontology is cheap<sup>6</sup>: everything exists. Werewolves, natural kind, colours, chair exist. But in saying this, I am not committing myself to any substantive view on the nature of each of these entities. Are werewolves concrete things? Mental entities? Concepts? Or what? I do not know; we need to do metaphysics to answer this question.

In this sense, existential claims are trivial. When we use the existential quantifier, we do assert their existence, but in doing so, we only state that they do exist. However, this claim doesn't commit us to any substantial view regarding what kind of thing they are. This is not a simple solution, for it appears to trivialise a series of classical discussions about the existence of various things. This may be so, which is why we have metaphysics to deal with these deeper questions. So, the previous difficulties in translating sentences are not due to an intrinsic flaw in predicate logic, but rather to our metaphysical views. But if we distinguish the ontological project from the metaphysical project and we keep ontology cheap, we can do logic just fine.<sup>7</sup>

#### 4. Conclusion

I have discussed a potential problem for predicate logic when it deals with some sentences that make references to a certain kind of things whose existence and nature are at least questionable. Given the nature of these entities, we may find difficulties to find suitable translations into the language of predicate logic. However, I have tried to show that the logic involved has ways to translate the sentences in question, and the apparent difficulties derive from a completely different source: deeper and more general questions about the existence and nature of certain things. In this sense, the apparent limitation of the formal apparatus of predicate logic is not really of problem of logic, but rather a problem of ontology and metaphysics. Although it was not my intention to defend any view about the deeper metaphysical nature of the problematic entities (fictional characters, non-existing things, mere *possibilia*, natural kinds, colours, ordinary objects like chairs), I presented a way to keep ontology cheap and separated from more substantive question about the ultimate nature of things. Questions about existence are relatively simple: everything exists; the metaphysical nature of the things that exists is a completely different question, for which no cheap answer is available. In any event, logic should work with a cheaper ontology, where answers to questions of existence do not have to be committed to deep metaphysics.



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## Notes

<sup>1</sup>In this paper, we focus only on what Keenan and Paperno, following Partee (1995), call D-quantifiers (2012, p. 2): expressions that bind arguments of predicates. This type of quantified expressions distinguishes from A-quantifiers: expressions that combine with predicates to form complex predicates. Adverbs are clear examples of this kind of quantifiers: ‘once’, ‘twice’, ‘sometimes’, ‘three times’, ‘occasionally’, ‘often’, ‘frequently’, ‘rarely’, ‘seldom’, ‘never’, ‘a lot’. The example I consider next is taken from this work by Keenan and Paperno (2012, p. 16), but it is easy to come up with many other similar examples.

<sup>2</sup>This phrase may sound unnatural in English, although it is not ungrammatical. I will use it to illustrate how ambiguities of scope arises when quantifiers interact with negation.

<sup>3</sup>Some arguments with quantifiers may be shown to be valid by using only sentential logic. Take this argument:

Argument 2

All cars built before 2000 are subject to driving restrictions.

My car was built in 1998.

So, my car will be subject to driving restrictions.

With the appropriate definition of sentential letters, we can translate the argument this way:

*P*: It is car built before 2000.

*Q*: It is subject to driving restrictions.

*R*: My car was built in 1998.

We get:

Argument 2\*

$P \rightarrow Q$

$R \rightarrow P$

*R*

$\therefore Q$

The translation shows that the argument is valid. However, we cannot expect to be able to do this for every argument that includes quantifiers.

<sup>4</sup>The difficulty does not seem to be, in principle, the fact that the sentence involves an opaque context: an attitude toward something. If the sentence were rather ‘Mary is in terror of spiders’, the corresponding translations into predicate logic would not be problematic, for spiders do exist.

<sup>5</sup>A parallel issue here is whether certain modal claims like ‘There could have been things other than there actually are’ and ‘Everyone who is actually rich could have been poor’ cannot be properly translated into first-order modal logic. I will not be able to address this issue, but some of the ideas defended here can be extended to this case with the appropriate adjustments. For a discussion of this issue, see (Kocurek 2018).

<sup>6</sup>I take this term from Agustín Rayo (2012, p. 4). Although he may have something different in mind and he does not explain it in more detail, I take it to mean that ontological questions have simple answers: do number exist? Do fictional characters exist? Do mere *possibilia* exist? My answer to these questions and many others is yes, of course, they do. In the same vein, by accepting the existence of numbers, I am not immediately committed to any substantive view about the ultimate nature of numbers. So, my ontological commitments are compatible with any metaphysical theory that turns out to be the right one. I differ from Rayo in the sense that the metaphysical project (metaphysicalism as he calls it (2013)) may be something worth being invested in. Rayo seems to think that this is more likely a waste of time or theoretically unrequired.

<sup>7</sup>Another issue that I will not be able to address, related to the problem at hand, is presented by Stephen K. McLeod (2011)). The following sentences present some problems of translation for predicate logic:

(7) Everything exists.

(8) Something exists.

(9) Some things do not exist

One issue has to do with the right way to translate them. One way is to use a predicate of existence ‘*Ex*’ like this:

$$7^* (\forall x)(Ex)$$

$$8^* (\exists x)Ex$$

$$9^* (\exists x)\neg Ex$$

Other way is by using identity like this:

$$(7^{**}) (\forall x)[x = x]$$

$$(8^{**}) (\exists x)[x = x]$$

$$(9^{**}) (\exists x)[\neg(x = x)]$$

Either option may face a more general issue, which connects with what I have said so far. Should we interpret the quantifiers as having existential import or not? At first sight, it looks like that if they do have existential import, we get in trouble as  $9^*$  and  $9^{**}$  would seem contradictory. Adopting a cheap ontology may help, but we will still need to elaborate a more precise proposal to translate these sentences.