

BLACK-TIE OPTIONAL SORITES

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Abstract. Sorites is the paradox which exploits the tolerance of vague language to get an absurd conclusion. The present note argues that, contrary to some other approaches, formalizing the antinomy does not serve the purpose of elucidation.

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1. Introduction

Formal languages have different kinds of communicative purposes. One of them is to impart orders to computers, which lack a natural way to communicate, and thus need mediation from a programming language. Let us call this a *computational* purpose. One may in particular want to impart *fuzzy* orders to a computer, or expect fuzzy answers from them. That task probably includes some way of incorporating the vague vocabulary we use into a code the machine can understand, but it does not assume that the relation between the imprecise language and the formal one is more intimate than necessary to do its job.

Another communicative purpose is to convey some very complicated and abstract facts, since formal languages can be easier to understand because of their syntactical and lexical simplicity. Paradigmatically, a theorem might be much easier to follow and check when its proof is presented formally. Let us call this an *elucidation* purpose.

The extensive use of formal languages in philosophy papers regarding the Sorites paradox, implies some people believe it is the sort of abstract matter for which formalization can fulfil this elucidation purpose. This paradox can be presented as an argument to an absurd conclusion, generated from (almost?) any vague predicate and some form of the Tolerance principle, which Crispin Wright famously characterized as follows:

A predicate F is tolerant with respect to [a concept] ϕ if there is also some positive degree of change in respect of ϕ insufficient ever to affect the justice with which F is applied to a particular case. (Wright 1975, p. 334)



Take as an illustration the predicate “being dawn”, where the dimension of change with respect to *dawnness* is time, and where the above principle mandates tolerance with respect to one second:

At 6 a.m. was the break of dawn.

If it was dawning at the time, then one second earlier it was already dawning.

Therefore, it was dawning at midnight.

A first order formalization of this argument would be:

$$\frac{\begin{array}{c} D(21600) \\ \forall x(D(x) \rightarrow D(x + 1)) \end{array}}{D(64800)}$$

where the numerals stand for names of times of the day, expressed in seconds. Conclusion follows by a series of applications of Modus Ponens and Universal Instantiation.

According to the old lore, championed by Russell (1923), vagueness is an evil that must be purged, and formalization is its wooden stake. It serves the double purpose of exposing the incoherence of natural language and replacing it by a purified substitute. This stance, however, is at odds with the aforementioned task of implementing vague computer languages, which presupposes the cognitive and semantical importance of vagueness.

The new lore, the one which lets vagueness wander among mortals, still insists on formalization to contain the evil within. Formalization as an elucidation tool is supposed to show us where our reasoning went astray in a manner that was not evident before. It is then assumed that the formal counterpart captures something *essential* that is veiled in natural language, in contraposition to the looser translation or modelling needed in programming. Let us in what follows briefly examine whether this is what is happening with the Sorites.

2. Formalization Without Syntax

The somewhat widespread consensus that a logical analysis is needed to tackle Sorites can be quite puzzling because there seems to be little syntactically relevant about the Sorites. Compare the situation with The Liar paradox. There is no unique referent to that name, given that infinitely many liar sentences can be generated, both in natural language and in formal theories. For one, there is no upper bound to how many sentences one can baptize by stating:

(*n*) the sentence *n* is not true.

Also, any such sentence is logically equivalent to infinitely many sentences, thus generating infinitely many Liars, each of them with a different logical form. However, this does not mean that the Liar is formally intractable, because all these sentences arguably have at least three common, minimal features: the truth predicate, negation, and names for sentences (in particular, self-reference). Thus, logical form (in a broad sense) is relevant in order to understand the root of the paradox. Also, formal tools will be useful in order to provide a new theory of truth (and/or negation, reference, etc.) to replace the naïf one.

Sorites, on the other hand, cannot be characterized by a single sentence, and among the arguments which are called “soritical”, one can find very different, non-equivalent logical forms: there is no particular predicate or names involved; no specific logical vocabulary; there may or may not be an explicit Tolerance Principle, which can be expressed generally, or singularly, as all required (usually finite) instances; the conclusion can be taken to be the false claim that dawn breaks at midnight, or the plainly contradictory claim that dawn does and does not break at midnight, or the counterintuitive claim that there is a sharp boundary between night and dawn. More importantly, none of these arguments is particularly complex in its logical form.

Moreover, there are also good reasons to doubt that the formalization stands in the appropriate relation to the informal paradox, so that one can extrapolate a lesson from it. For instance, Sainsbury (1990) argues that formalizing vague languages does a big disservice to them, since what is essential to vagueness gets lost in mathematical precision, and cannot even be regained by proposing more complex non-classical systems: “you do not improve a bad idea by iterating it” (Sainsbury 1990, p. 255).

On top of that, the soritical phenomenon can be appreciated without appealing to any argument whatsoever. We can present someone with a series of tokens, seamlessly shifting from yellow to orange, and ask them to classify them by putting them next to one or the other pole of the series. This person will find themselves in trouble, without language being involved in any direct way. This mute Sorites illustrates how vagueness is, indeed, very deeply, a non-syntactical matter.

3. Formality Without Formalization

It seems, then, that the philosophical use of formal tools to elucidate Sorites serves a very limited purpose. The question now is whether there are uses other than strict elucidation that can be of epistemological interest. Let us point to two of them.

The first one is to study the behavior of determinacy operators. These operators are probably not just the formal translation of naïf concepts found in natural language, but a highly theoretical version thereof. Truths involving many (even infinite)

iterations of them, and interactions with other types of operators, are much more tractable formally than in natural language—see for instance Bacon (2020).

The second one is to put to test some non-classical logics. As we pointed out, when modelling vague language, few choose to remain in the classical realm. All the soritical-type arguments can only be deemed valid but unsound by classical standards (and some of them even valid *and* sound). Deviant logics, on the other hand, may consider some of them to be invalid, or classify as unsound some of the classically sound ones –being substructural logics such as the ones in (Cobrerros et al 2019) the latest trend. From this point of view, and contrary to the idea of formalization as elucidation, it is Sorites that serves as an adequacy criterion for formal theories, and not the other way around.

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