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LOGIC TAKING CARE OF ITSELF: THE CASE OF CONNEXIVE LOGIC

LUIS ESTRADA-GONZÁLEZ
Universidad Nacional Autónoma de México, MÉXICO
loisayaxsegrob@comunidad.unam.mx

Abstract. Logic is an excellent tool for reasoning about most philosophical topics, including logical issues themselves. Discussions about the validity or otherwise of certain principles have been widespread throughout the history of logic. This paper exemplifies that with the analysis of the debate surrounding connexive logics. In connexive logics, certain principles involving mainly negation and implication hold good, whereas they are not valid in most well-known logics. Despite their intuitiveness, the connexive principles quickly lead to contradictions and even to triviality, i.e. to the truth of every proposition. This chapter surveys the main arguments against the connexive principles and discusses some prospects for challenging those arguments and endorsing the connexive principles.

Keywords: connexive Logic • contradiction • triviality • simplification • contraposition • detachment

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The distinctive mark of connexive logics is that they include among their logical truths some formulas, involving mainly negation and implication, intended to express the ideas that a proposition cannot imply its negation and that all propositions are self-compatible. Among those principles one can find the following ones:

$$\begin{array}{cc}
A \circ A & \sim(A \circ \sim A) \\
\sim(\sim A \to A) & \sim(A \to \sim A) \\
(A \to B) \to \sim(A \to \sim B) & (A \to \sim B) \to \sim(A \to B) \\
\sim((A \to B) \wedge (A \to \sim B)) & \sim((A \to B) \wedge (\sim A \to B)) \\
\sim(A \to \sim B) \to (A \to B) & \sim(A \to B) \to (A \to \sim B) \\
(A \to B) \to \sim(\sim A \to B) & (\sim A \to B) \to \sim(A \to B)
\end{array}$$

$\sim$, $\to$ and $\wedge$ stand for a negation, an implication and a conjunction, respectively, and $\circ$ is a compatibility connective.[1] Schemas like these have been motivated and investigated on various grounds. For general overviews, see McCall (2012), Wansing (2022) and Francez (2021). Since $A \circ B$ is usually taken to mean $\sim(A \to \sim B)$, in what follows I will simply use schemas written with $\sim$, $\to$ and $\wedge$. Also, some of these schemas receive special names. The schemas on the second line are collectively called 'Aristotle's Theses'; the ones on the third line are collectively called 'Boethius' Theses'; the first schema in the fourth line is called 'Abelard's Principle'; the second one, 'Aristotle's Second Thesis'.[2] The principles in the fifth line are the converses of Boethius' Theses, vividly defended by Wansing. (See for example Wansing and Skurt 2018.) The last schemas have been thoroughly studied by Francez. (See for example Francez 2021).

Despite their intuitiveness, connexive principles quickly lead to contradictions and even to triviality. I survey here the main arguments against the connexive principles and discuss some prospects for challenging those arguments and endorsing the connexive principles. Although I tried to present a wide survey of the options to challenge the arguments with some detail, it is far from complete. Some of the options to block the anti-connexivist arguments can be presented using very elementary logical tools; in those cases, I present them as comprehensively as possible given the space constraints. In some other cases, the options need more advanced tools, and presenting them would require detours that would complicate and lengthen this survey. In these latter cases, to preserve quick readability, I simply point to useful literature to know more details about them. Moreover, presenting the arguments in a different format –say, using Gentzen sequent calculi, natural deduction Fitch-style or tableaux– might highlight different parts of the arguments, while perhaps hiding others, and that could lead to different diagnoses and proposals. I hope the reader takes this text as what it is: an invitation to know more about connexive logic and the topics in philosophy of logic surrounding it.

To better assess the arguments against the connexive principles and the prospects of blocking them, let me assume the following rather standard evaluation conditions for the connectives:

$\sim A$ is true iff A is false	$\sim A$ is false iff A is true
$A \wedge B$ is true iff A is true and B is true	$A \wedge B$ is false iff A is false or B is false
$A \to B$ is true iff, if A is true then B is true	$A \to B$ is false iff A is true and B is false

Moreover, I assume the following (again, rather standard) definition of logical validity:

An argument with premises $P_1, \ldots, P_n$ and conclusion C is *logically valid in a logic* **L** iff in every interpretation in which the premises are true, the conclusion is true as well. I would write that as $P_1, \ldots, P_n \models_{\mathbf{L}} C$.

Implications are difficult to understand, and that is an understatement. Over the next few pages, different ways of evaluating an implication will appear. I tend to favor the suppositional method, that is, assuming the truth of the antecedent to get the truth of the consequent (via the evaluation conditions assumed). Nonetheless, the more usual ways of evaluating them through the combinations of interpretations of both the antecedent and the consequent will appear, too.

Having said that, let the fun begin. At (*Prior Analytics* 57b3), Aristotle argues that contradictories cannot both entail the same thing, that is, something along the lines of Aristotle's Second Thesis above. But this principle is inconsistent with Simplification, Contraposition and Adjunction:

0.	$\sim((\sim A \to \sim(A \wedge \sim A)) \wedge (\sim\sim A \to \sim(A \wedge \sim A)))$	Aristotle's Second Thesis
1.	$(A \wedge \sim A) \to A$	Simplification
2.	$(A \wedge \sim A) \to \sim A$	Simplification
3.	$\sim A \to \sim(A \wedge \sim A)$	1, Contraposition
4.	$\sim\sim A \to \sim(A \wedge \sim A)$	2, Contraposition
5.	$(\sim A \to \sim(A \wedge \sim A)) \wedge (\sim\sim A \to \sim(A \wedge \sim A))$	3, 4 Adjunction

Let now me review some of the attempts to find a way out of the difficulty.

Aristotle's Second Thesis is invalid. That is the obvious lesson to be drawn by many logicians. Even some connexivists find the principle suspicious, precisely because its inconsistency with some other basic logical principles. Nonetheless, other connexive principles, more entrenched than Aristotle's Second Thesis, are not in a better position. An argument similar to the above can be run for Aristotle's Thesis:

0.	$\sim((A \wedge \sim A) \to \sim(A \wedge \sim A))$	Aristotle's Thesis
1.	$(A \wedge \sim A) \to A$	Simplification
2.	$(A \wedge \sim A) \to \sim A$	Simplification
3.	$\sim A \to \sim(A \wedge \sim A)$	1, Contraposition
4.	$(A \wedge \sim A) \to \sim(A \wedge \sim A)$	2, 3, Transitivity of implication

and Step 4 clearly contradicts Step 0.

Although rejecting the validity of Aristotle's Thesis is the obvious lesson to be drawn for many logicians, this option is simply beyond the question for a connexivist. (Although I will come back later to the option of having Aristotle's Theses without Boethius' ones, or vice versa.) Thus, a connexivist must reject Simplification, Contraposition, Transitivity of the conditional, Adjunction or the idea that contradictions must be avoided. Let me briefly discuss each of these options.

Contraposition is not valid. Contraposition is a usual suspect because it does not seem valid when reasoning about contradictions. Suppose that one has an implication $A \to B$ with an antecedent that is just true and a consequent that is contradictory, in particular, that is both true and false. Presumably the implication is both true and false: it is true, because both antecedent and consequent are true, but it is false because the antecedent is true and the consequent is false. But then $\sim B \to \sim A$ seems just false: the antecedent is both true and false, but the consequent is just false. Hence, the truth of the antecedent leads to the falsity of the consequent, it cannot be otherwise.

Even if one grants that line of reasoning to reject Contraposition, it seems that the particular instance used in the argument is safe. Suppose that $A \wedge \sim A$ is true, as it asked by the antecedent in 2. Then, plausibly each of the conjuncts, A and $\sim A$, is true. Hence, if $A \wedge \sim A$ is true then $\sim A$, is true. If $\sim A$ is true, then A is false. If A is false, then $A \wedge \sim A$ is false too. But if $A \wedge \sim A$ is false, $\sim(A \wedge \sim A)$ is true. Hence, if $\sim A$ is true, $\sim(A \wedge \sim A)$ is true.

Transitivity of implication is not valid. There have been reasons to reject the validity of Transitivity of implication. Here I will only present one, based on the assumption that it is wrong to say that the falsity of an implication's antecedent is sufficient for the truth of the whole implication; in symbols, that the schema $\sim A \to (A \to B)$ should not be logically valid.

The problem is that $\sim A \to (A \to B)$ seem to follow from very reasonable assumptions:

1.	$\sim A \to \sim(A \wedge \sim B)$	Monotonicity of falsity
2.	$\sim(A \wedge \sim B) \to (A \to B)$	Definition of $A \to B$
3.	$\sim A \to (A \to B)$	1, 2 Transitivity of implication

The first step expresses that A's falsity is sufficient for the falsity of the conjunction of A with anything else. The second one expresses that the falsity of conjoining A's truth and B's falsity is sufficient for the truth of 'A implies B'. However, the last step expresses that A's falsity is sufficient for the truth of 'A implies B'. But this cannot be. The truth value of only one of the components of an implication should not be enough to determine the value of an implication, their values must be more tightly correlated.

Rejecting Transitivity of implication is an option more radical than the previous ones because it could lead to modify not only the properties of some connectives, but also the accepted properties of logical validity itself. Indeed, if one accepts that if the implication $A \to B$ is valid then the argument with premise A and conclusion B is valid too, the invalidity of

$$A \to B, B \to C;\ \text{therefore } A \to C$$

leads to the invalidity of

$$A,\ \text{therefore } B;\ B,\ \text{therefore } C.\ \text{Hence, } A \text{ therefore } C.$$

For people who think that the meta-language (dealing with, among other things, validity) is more basic, in a sense that need not be specified here, than the object language (dealing with, among other things, implication connectives), this conclusion is unacceptable, unless they have independent reasons to reject Transitivity of validity. Read (2012) thinks, for example, that the argument is invalid because there is an ambiguity in the middle term. To better see it, let me rewrite the argument using disjunctions:

1*.	$\sim A \to (\sim A \vee B)$	Addition
2*.	$(\sim A \vee B) \to (A \to B)$	Definition of $A \to B$
3*.	$\sim A \to (A \to B)$	1, 2 Transitivity of implication

According to Read, if 2* is valid, there must be a connection between $\sim A$ and B to guarantee that from their disjunction one can obtain $A \to B$. But the disjunction guaranteeing that is not one that validates Addition as in 1*, since it allows introducing completely unrelated disjuncts. And vice versa: a disjunction satisfying Addition, as in 1*, is not one that can guarantee that there is an implication relation between its disjuncts, as 2* would require. That ambiguity would lead to a similar ambiguity in the conjunction occurring in the original argument: the one needed to validate Monotonicity of falsity might not be the same connective as the one used to define the implication. Thus, there seems to be no good arguments against Transitivity of implication.

Simplification is not valid. Maybe this is much to swallow, too. After all, if $A \wedge B$ is true, then, by the evaluation conditions for conjunction, both A and B are true. Hence, it seems safe to go in any case from the truth of $A \wedge B$ to that of A, that is, $(A \wedge B) \to A$ seems valid. But notice that it is a very specific instance of Simplification which is involved here, namely, Simplification of a contradiction. Maybe this form of Simplification is what should be restricted.

One reasoning to block Simplification of contradictions is as follows. Since A and $\sim A$ are contradictory, their conjunction cannot be true, not even by hypothesis. Suppose even that they are false in all interpretations.[3] And since the truth of an implication requires passing in some way from the truth of the antecedent to the truth of the consequent, that cannot be achieved since there is no truth to begin with in any case. Hence, neither $(A \wedge \sim A) \to A$ nor $(A \wedge \sim A) \to \sim A$ are true, let alone valid. Right, they are not false either, since the falsity of an implication requires having a true antecedent and a false consequent and, once again, there is no truth in the antecedent to begin with.

Nonetheless, connexivists that support the option of restricting Simplification of contradictions but still accept the validity of Aristotle's Thesis, Contraposition and Transitivity of implication must face

ALBERIC'S ARGUMENT:

0.	$\sim((A \wedge \sim B) \rightarrow \sim(A \wedge \sim B))$	Aristotle's Thesis
1.	$A \rightarrow B$	Hypothesis
2.	$\sim B \rightarrow \sim A$	1, Contraposition
3.	$(A \wedge \sim B) \rightarrow A$	Simplification
4.	$\sim A \rightarrow \sim(A \wedge \sim B)$	3, Contraposition
5.	$(A \wedge \sim B) \rightarrow \sim B$	Simplification
6.	$(A \wedge \sim B) \rightarrow \sim A$	2, 5 Transitivity of implication
7.	$(A \wedge \sim B) \rightarrow \sim(A \wedge \sim B)$	4, 6, Transitivity of implication

Everett Nelson, perhaps the first great connexivist of the 20th century, had misgivings about Simplification in general, not restricted to the simplification of contradictions. His treatment of implications (or entailments, as he preferred) in Nelson (1930) was not based on truth values at all, but on a primitive notion of incompatibility. According to him, $A \rightarrow B$ is valid if and only if $\sim B$ is incompatible with A.

For Nelson, incompatibilities in general are not reducible to formal contradictions, that is, a pair consisting of the formulas A and $\sim A$, contradictions are their prime examples, and they can be used to give a sense of what he had in mind. Suppose that all non-main implications in a formula can be converted into $\sim A \vee B$, just as in many orthodox approaches. In fact, suppose that any antecedent and consequent can be converted to disjunctive normal forms. Then, $A \rightarrow B$ is valid only if

(case 1) the main connective in the disjunctive normal form of A is a conjunction and every literal in the disjunctive normal form of A forms a contradictory pair with some literal in the disjunctive normal form of $\sim B$, and vice versa;

(case 2) the main connective in the disjunctive normal form of A is a disjunction and some literal in the disjunctive normal form of A forms a contradictory pair with some literal in the disjunctive normal form of $\sim B$, and vice versa.

Although this is far from giving a sound and complete semantics for Nelson's logic—which remains an important open problem in the field– it is possible to illustrate why $(A \wedge B) \rightarrow A$ would not be valid: not all the literals in $A \wedge B$ form a contradictory pair with the negation of the consequent, $\sim A$. It can be easily checked that Aristotle's and Boethius' Theses are valid according to this test.

Nonetheless, and Nelson's thoughts notwithstanding, not many connexivists are ready to give Simplification of contradictions up. The cost is that accepting it while accepting the validity of Boethius' Thesis and Transitivity leads to contradictions as well, as the following argument shows (see Routley et al. 1982, p. 82); the argument was rediscovered by Lenzen (2019)):

ROUTLEY'S ARGUMENT

0.	$((A \wedge {\sim}A) \to A) \to {\sim}((A \wedge {\sim}A) \to {\sim}A)$	Boethius' Thesis
1.	$(A \wedge {\sim}A) \to A$	Simplification
2.	$(A \wedge {\sim}A) \to {\sim}A$	Simplification
3.	${\sim}((A \wedge {\sim}A) \to {\sim}A)$	0, 1 Detachment

And, clearly, 2 and 3 contradict each other.

Detachment is invalid. Again, several options are open to block this argument. I have already discussed Simplification (of contradictions). Doing without Detachment is another option. Detachment cause trouble when the antecedent of the implication $A \to B$ is both true and false and the consequent is just false. In such case, A is (at least) true, and if the implication is taken as (at least) true as well, Detachment would take us from true premises to untrue conclusions because by hypothesis B is just false.[4]

Nonetheless, the application of Detachment in the argument above seems safe, for the consequent in 2 is not that arbitrary. Suppose that $(A \wedge {\sim}A) \to A$ is true, as it asked by the antecedent in 0. This means that from the truth of $A \wedge {\sim}A$ one can obtain the truth of A. Nonetheless, from the truth of $A \wedge {\sim}A$ one can obtain the falsity of A as well, courtesy of the truth of ${\sim}A$. Hence, $(A \wedge {\sim}A) \to A$ is both true and false. By a similar reasoning, $(A \wedge {\sim}A) \to {\sim}A$ would be likewise both true and false, and therefore ${\sim}((A \wedge {\sim}A) \to {\sim}A)$ would be both true and false as well. Then, the implication to be detached is not one where the consequent is just false, so there is no danger of detaching a just false consequent.

Aristotle's Theses are valid, but Boethius's Theses are not. In the context of connexivity, abandoning Boethius' Thesis as a response to Routley's argument would be the most radical option, for at least since McCall (1966) the validity of Boethius' has been a necessary condition for the connexivity of a logic. Moreover, if Aristotle's Thesis is kept together with Simplification (and Detachment), one must recall that there is an argument showing that Aristotle's Thesis, Simplification, Contraposition and Transitivity of the implication entail a contradiction, so Contraposition or Transitivity of the Implication must go anyway.

And although both (groups of) theses seem to have equal intuitive appeal, Aristotle's Thesis and Boethius' Thesis are not equivalent. To prove them equivalent, some non-trivial logical machinery is needed. Thus, for example, to prove that Boethius' Thesis entails Aristotle's one needs Identity and Detachment:

0.	$(A \to A) \to {\sim}(A \to {\sim}A)$	Boethius' Thesis
1.	$(A \to A)$	Identity
2.	${\sim}(A \to {\sim}A)$	0, 1 Detachment

Nonetheless, to prove that Aristotle's Thesis entails Boethius', much more is needed:

0.	$\sim(A \to \sim A)$	Aristotle's Thesis
1.	$(A \to B) \to (\sim B \to \sim A)$	Contraposition
2.	$(A \to B) \to ((A \to \sim B) \to (A \to \sim A))$	1, Prefixing
3.	$(A \to B) \to (\sim(A \to \sim A) \to \sim(A \to \sim B))$	2, Contraposition (in the consequent)
4.	$\sim(A \to \sim A) \to ((A \to B) \to \sim(A \to \sim B))$	3, Permutation
5.	$(A \to B) \to \sim(A \to \sim B)$	0, 4 Detachment

Contradictions are not problematic. If one does not want to mess with the validity of Aristotle's Thesis, Boethius' Thesis or of any other logical principle involved in the proofs, another option is simply to bite the bullet and accept the contradiction without supposing that it will cause any major problem, like triviality. In fact, there seem to be reasons, independent of connexivity, to suppose that there are contradictory logical truths, as the following parallel arguments show:

If $A \wedge \sim A$ is true, both A and $\sim A$ are true.
If both A and $\sim A$ are true, A is true.
Therefore, if $A \wedge \sim A$ is true, A is true.
Hence, $(A \wedge \sim A) \to A$ is true.

If $A \wedge \sim A$ is true, both A and $\sim A$ are true.
If both A and $\sim A$ are true, $\sim A$ is true.
If $\sim A$ is true, A is false.
Therefore, if $A \wedge \sim A$ is true, A is false.
Hence, $(A \wedge \sim A) \to A$ is false.
Hence, $\sim((A \wedge \sim A) \to A)$ is true.

And since the result has been obtained by considering all the possible interpretations in which the antecedent $A \wedge \sim A$ might be true, the conclusions have the force of logical validity.

Orthodox logicians might say that neither the truth, let alone the validity, of an implication is evaluated in that way, but one must consider all the possible interpretations of the subformulas and then check whether $A \to B$ is true in all the interpretations. Nonetheless, that misses the point of implication or, at least, of an important group of implications, which are evaluated on the supposition that the antecedent holds. To know whether an antecedent implies a consequent, one must suppose the (truth of the) antecedent, whatever it is, and then check whether logic alone, for example, as encoded in the evaluation conditions for the connectives, allows us to arrive from it to the (truth of the) consequent.

A	$\sim A$
$\{1\}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$
$\{\ \}$	$\{\ \}$
$\{0\}$	$\{1\}$

$A \wedge B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$	$\{1,0\}$	$\{0\}$	$\{0\}$
$\{\ \}$	$\{\ \}$	$\{0\}$	$\{\ \}$	$\{0\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

Nonetheless, even for those who are used to evaluate implications as certain disjunctions, for example, that take the truth of an implication $A \rightarrow B$ to mean that the antecedent is untrue or the consequent is true, the above contradictions around Simplification may arise. The following tables, introduced by Wansing (2006), show that one can validate both $(A \wedge {\sim}A) \rightarrow A$ and ${\sim}((A \wedge {\sim}A) \rightarrow A)$ without validating everything:

$A \rightarrow B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{\ \}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$
$\{0\}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$	$\{1,0\}$

In this semantics, interpretations are set of truth values. Thus, a formula can be just true, $\{1\}$; both true and false, $\{1,0\}$; neither true nor false, $\{\ \}$; just false, $\{0\}$. The falsity condition for the implication is playing a crucial role in here. It demands that the implication is false if and only if the antecedent is not true or the consequent is false. Since there are plenty of cases that make an implication both true and false, namely, when the antecedent is untrue or the consequent itself is both true and false, it is not surprising that certain contradictions appear.

I have surveyed some of the most important arguments that show that the connexive principles lead to contradictions. Moreover, I have just showed that these contradictions can be embraced without triviality, that is, contradictions need not entail any proposition. To end this paper, let me present one argument that show that connexive principles might lead to triviality if some principles, stronger than Simplification, Contraposition and the others previously analyzed are accepted:

1. ${\sim}(((A \rightarrow B) \wedge A) \rightarrow {\sim}((A \rightarrow B) \wedge A)))$
2. ${\sim}(((A \rightarrow B) \wedge A) \rightarrow {\sim}((A \rightarrow B) \wedge A)) \rightarrow {\sim}((A \rightarrow B) \rightarrow {\sim}A)$
3. ${\sim}((A \rightarrow B) \rightarrow {\sim}A) \rightarrow B$
4. ${\sim}((A \rightarrow B) \rightarrow {\sim}A)$
5. B

B is an arbitrary formula, hence, triviality. Step 1 is an instance of Aristotle's Thesis; 2 is a scheme valid even in weak relevance logics, and 3 is valid in the strongest relevance logic, **R**. 4 follows from 1 and 2 by Detachment, and 5 follows from 3 and 4 by Detachment. I have briefly mentioned the prospects of doing without Detachment, and so the next suspects are 2 and especially 3. Both principles deserve careful examination. As a hint, note that 3, in terms of compatibility, expresses that if an implication is compatible with its antecedent then the consequent is true, but that is surely wrong: the compatibility of an implication with its antecedent does not seem sufficient for the truth of the consequent of that very implication. Nonetheless, I will stop here. I think I have already given a fair picture of the possible dialectics when it comes to discuss the validity of logical principles, connexive ones in particular.

In this paper I have presented a survey of the main arguments that show how the connexive principles lead to contradictions and even to triviality. To avoid the conclusions, some steps must be blocked in a principled way. I have also surveyed the main options to consider those arguments invalid if one wants to give the connexive principles a chance, and they range from accepting the invalidity of Simplification or Contraposition to accepting that some contradictions hold good.

But the lesson to be learned goes well beyond the case study of connexivity. Logic is an excellent tool for reasoning about most philosophical topics, including logical issues themselves. Discussions about the validity or otherwise of certain principles have been widespread throughout the history of logic, it is not a twentieth century eccentricity driven by an excessive focus on formalism.

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Notes

[1]I do not assume that all the occurrences of these signs stand exactly for the same connective each time, but expressing this possibility would have led to a clumsy presentation of the principles.

[2]There is no consensus on to what extent the ancient authors alluded to in the names of the principles really endorsed them. Some think that these authors really said what they seem to say; Sylvan (1989) is a good example of this stance. Others claim that, in the vast majority of cases, such authors defended nothing that could contradict the current logical orthodoxy; Lenzen (2022) is a good example of this latter stance.

[3]But that is not necessary, all what is required is that they are not true. That is compatible with their falsity, but also with their lacking truth value, as in the account of negation as cancellation. For the latter, see Routley et al. (1982, p. 88–92).

[4]One can rightly wonder why $A \rightarrow B$ would be at least true in that case. Since B is just false by hypothesis, there is no way in which one can go from the truth of the antecedent to the truth of the consequent, because the consequent is not true. Considering $A \rightarrow B$ as true in that case seems a bad remnant of the contemporary orthodoxy, taking the falsity of the antecedent as a sufficient condition for the truth of an implication.

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